King Saud University
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Solution Quiz 1 ACTU 464–474 October 1, 2020

Questions (**5 marks**)

Let X be a random loss with mixed probability distribution given by:

$$f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ \dfrac{1}{4} & \text{for} \quad x = 0. \\ \dfrac{3}{4}xe^{-x} & \text{for} \quad x > 0 \end{cases}$$

1. **(3 marks)** Calculate (a) $\Pi_{\text{SL}}(\frac{1}{3})$, (b) $\Pi_{\text{sd}}(\frac{1}{2})$ and (c) $\Pi_{\text{Exp}}(0.92358)$. (The premium are that of complete insurance X).
2. **(2 marks)** For an Excess of loss reinsurance of X with retention level $M = 1.3$. Calculate Π_{Qua} such that $P(Z > \Pi_{\text{Qua}}) = 0.10$.

Solution

1. (a) We first calculate $E[X]$.

$$E[X] = 0 + \frac{3}{4}\int_0^{\infty} x^2 e^{-x} dx = \frac{3}{2} = 1.5$$

$\Pi_{\text{SL}}(\frac{1}{3}) = (1 + \frac{1}{3})E[X] = \frac{4}{3}\frac{3}{2} = \mathbf{2}$.
(b) And $E[X^2] = \dfrac{3}{4}\int_0^{\infty} x^3 e^{-x} dx = \frac{9}{2} = 4.5$ thus $\text{Var}(Z) = \frac{9}{2} - \frac{9}{4} = \frac{9}{4}$ and $\text{sd}(X) = \frac{3}{2}$.Thus $\Pi_{\text{sd}}(\frac{1}{2}) = \frac{3}{2} + \frac{1}{2}\frac{3}{2} = \frac{9}{4} = \mathbf{2.25}$.
(c) $\Pi_{\text{Exp}}(0.92358) = \frac{1}{0.92358}\ln(M_X(0.92358))$ but

$$M_X(c) = E\left[e^{cX}\right] = \frac{1}{4} + \frac{3}{4}\int_0^{\infty} e^{cx} x e^{-x} dx = \frac{1}{4} + \frac{3}{4}\int_0^{\infty} x e^{-(1-c)x} dx = \frac{1}{4} + \frac{3}{4(1-c)^2}$$

hence

$$\Pi_{\text{Exp}}(0.92358) = \frac{1}{0.92358}\ln\left(\frac{1}{4} + \frac{3}{4(1-0.92358)^2}\right) = \mathbf{5.2592}.$$

2. We know tha $P(Z > \Pi_{\text{Qua}}) = 0.10$ is equivalent to

$$P(Z \leq \Pi_{\text{Qua}}) = 0.9 = P(X \leq \Pi_{\text{Qua}} + 1.3) \qquad (1)$$

and the c.d.f. of X is given for $x > 0$ by

$$P(X \leq x) = \frac{1}{4} + \frac{3}{4}\int_0^{x} s e^{-s} ds = \frac{1}{4} + \frac{3}{4}\left(1 - (x+1)e^{-x}\right),$$

therefore the solution to (1) is given by the solution to $\frac{1}{4} + \frac{3}{4}(1 - (x+1)e^{-x}) = 0.9$, which is 3.524, finally $\Pi_{\text{Qua}} = 3.524 - 1.3 = \mathbf{2.224}$