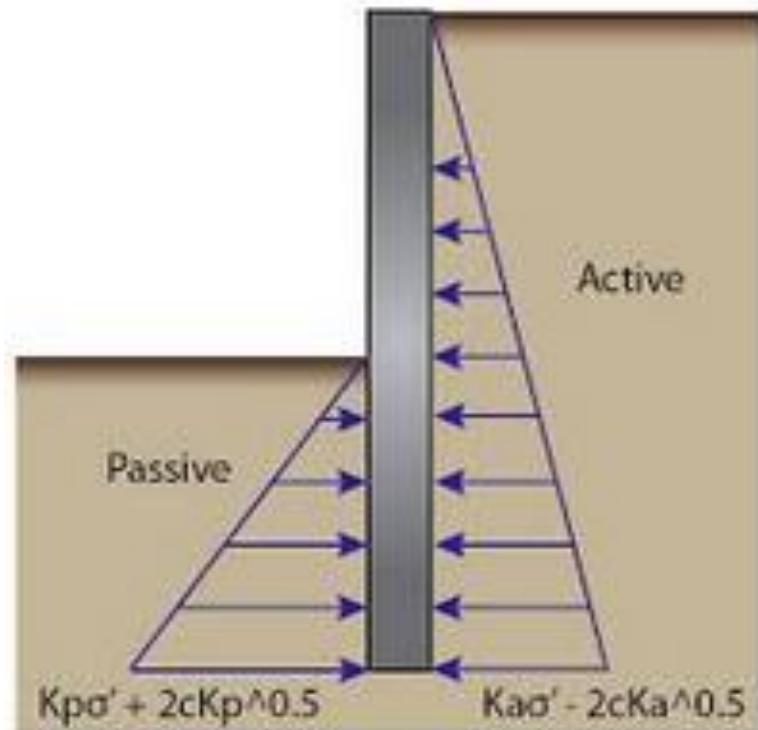


# Lateral Earth Pressure

## CHAPTER 16

Omitted parts:

Sections 16.9, 16.10, 16.16 , 16.17



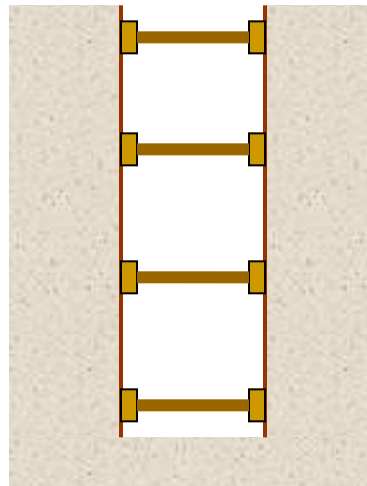
# INTRODUCTION

- Proper design and construction of many structures such as:
  - **Retaining walls** (basements walls, highways and railroads, platforms, landscaping, and erosion controls)
  - **Braced excavations**
  - **Anchored bulkheads**
  - **Grain pressure on silo walls and bins**

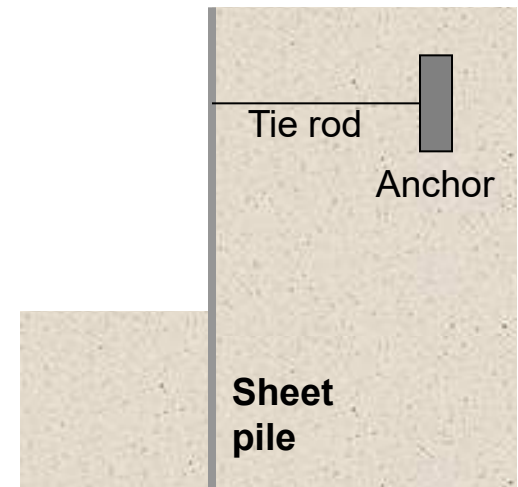
require a thorough knowledge of the **lateral forces** that act between the retaining structures and the soil masses being retained.



**Cantilever  
retaining wall**



**Braced excavation**



**Anchored sheet pile**

# INTRODUCTION

- These lateral forces are caused by lateral earth pressure.
- We have to estimate the lateral soil pressures acting on these structures, to be able to design them.

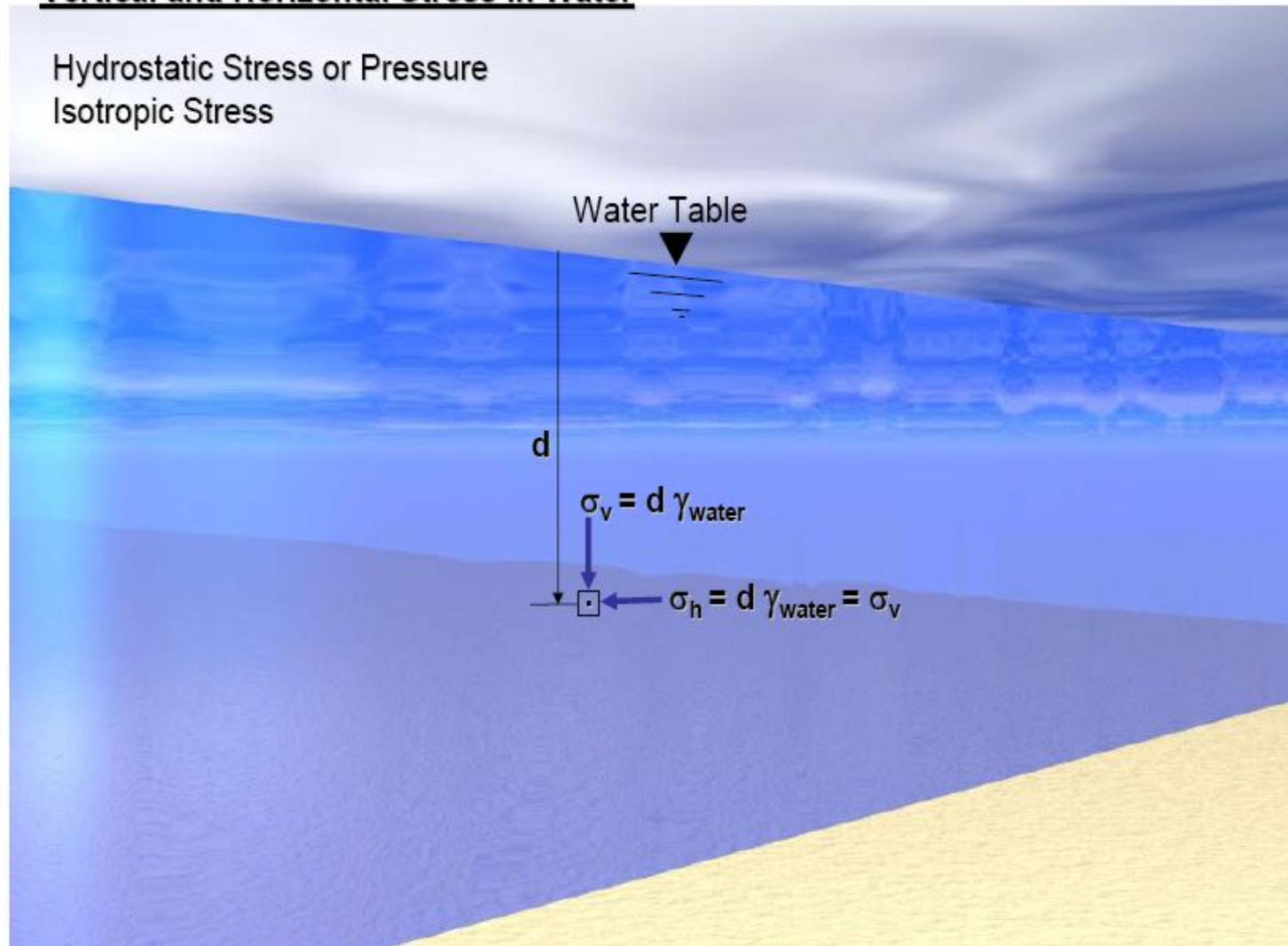
The magnitude and distribution of lateral earth pressure depends on many factors, such as:

- The **shear strength** parameters of the soil being retained,
- The **inclination** of the surface of the **backfill**,
- The **height** and **inclination** of the **retaining wall** at the wall–backfill interface,
- The nature of **wall movement** under lateral pressure,
- The **adhesion** and **friction** angle at the wall–backfill interface.

# Vertical and Horizontal Stress in Water

## Vertical and Horizontal Stress in Water

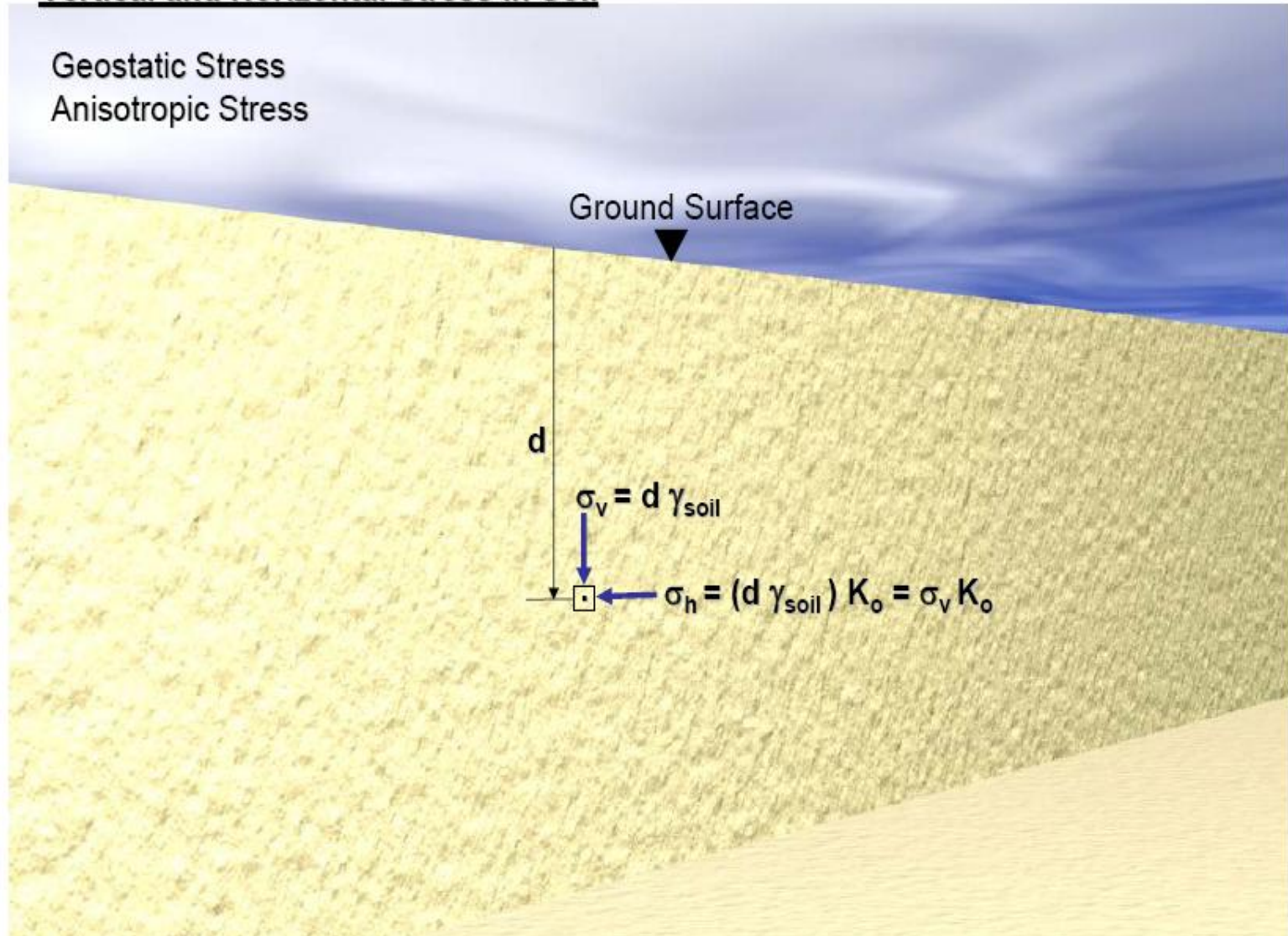
Hydrostatic Stress or Pressure  
Isotropic Stress



# Vertical and Horizontal Stress in Soil

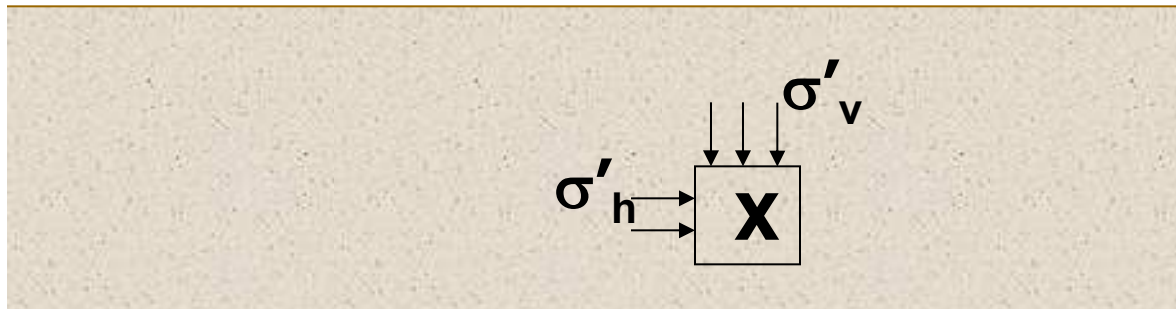
## Vertical and Horizontal Stress in Soil

Geostatic Stress  
Anisotropic Stress



# Coefficient of Lateral Earth Pressure

In a homogeneous natural soil deposit,



GL

The ratio  $\sigma'_h/\sigma'_v$  is a constant known as [coefficient of lateral earth pressure](#).

In other words, it is the ratio of the effective horizontal stress ( $\sigma'_h$ ) to the effective vertical stress ( $\sigma'_v$ ); then

$$K = \frac{\sigma'_h}{\sigma'_v}$$

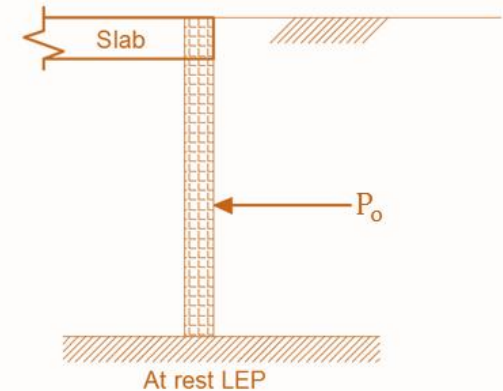
Or in terms of total stresses

$$K = \frac{\sigma_h}{\sigma_v}$$

# Types of Lateral Earth Pressures

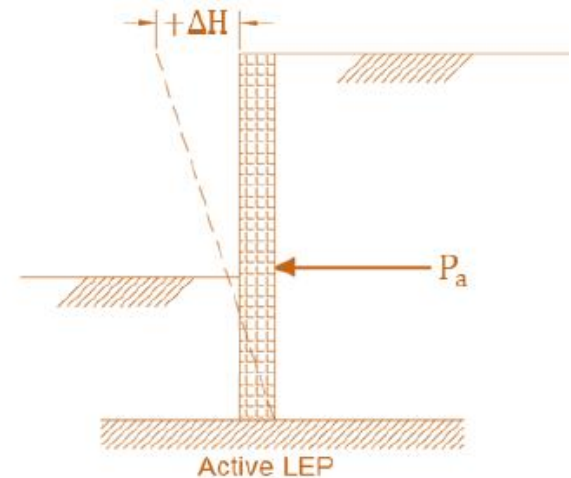
## 1. At Rest Lateral Earth Pressure:

The wall may be **restrained from moving**, for example; basement wall is restrained to move due to slab of the basement and the lateral earth force in this case can be termed as " $P_o$ ".



## 2. Active Lateral Earth Pressure:

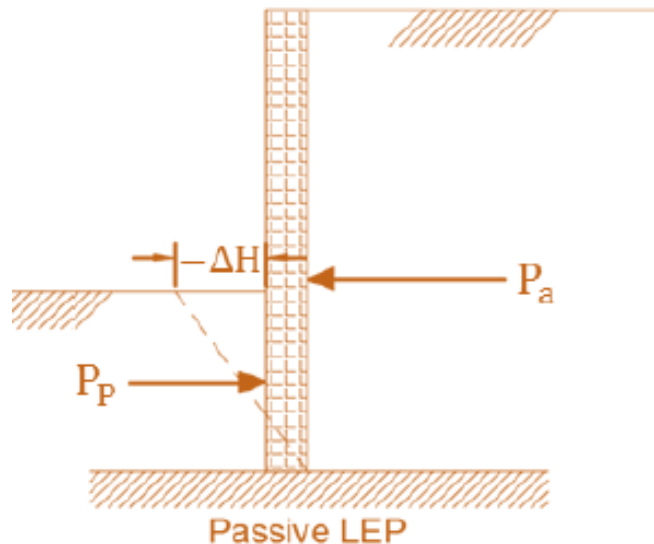
In case of the wall is free from its upper edge (retaining wall), the wall may **move away** from the soil that is retained with distance " $+\Delta H$ " (i.e. the soil pushes the wall away) this means the soil is active and the force of this pushing is called active force and termed by " $P_a$ ".



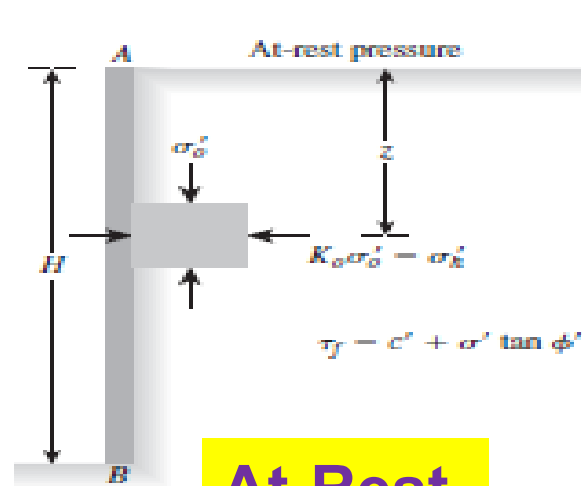
# Types of Lateral Earth Pressures

## 3. Passive Lateral Earth Pressure:

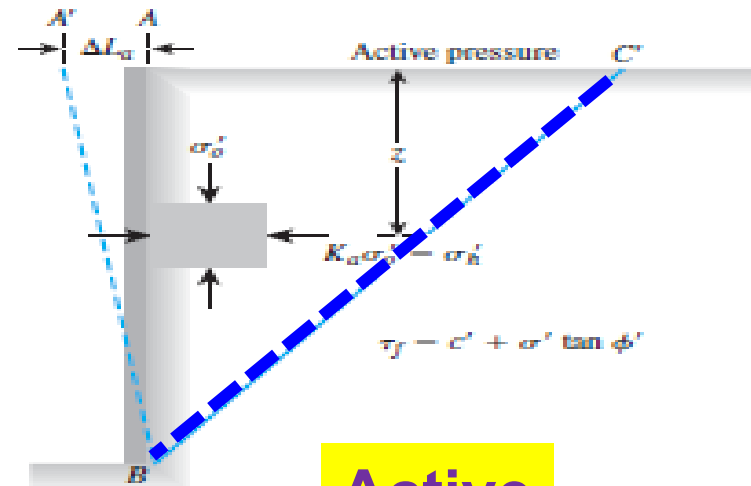
For the wall (retaining wall) in the left side there exist a soil with height less than the soil in the right and as mentioned above the right soil will push the wall away, so the wall will be **pushed into** the left soil (i.e. soil compresses the left soil) this means the soil has a passive effect and the force in this case is called passive force and termed by " $P_p$ ".



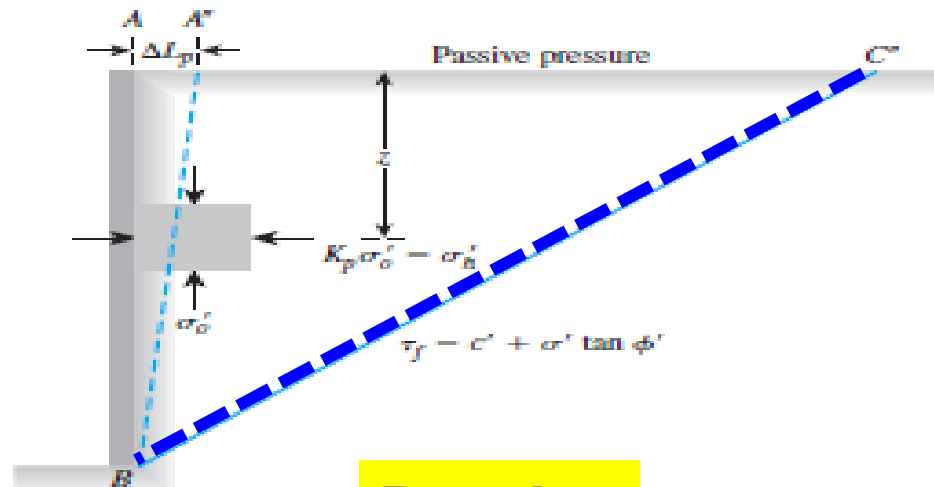
# CASES



**At-Rest**

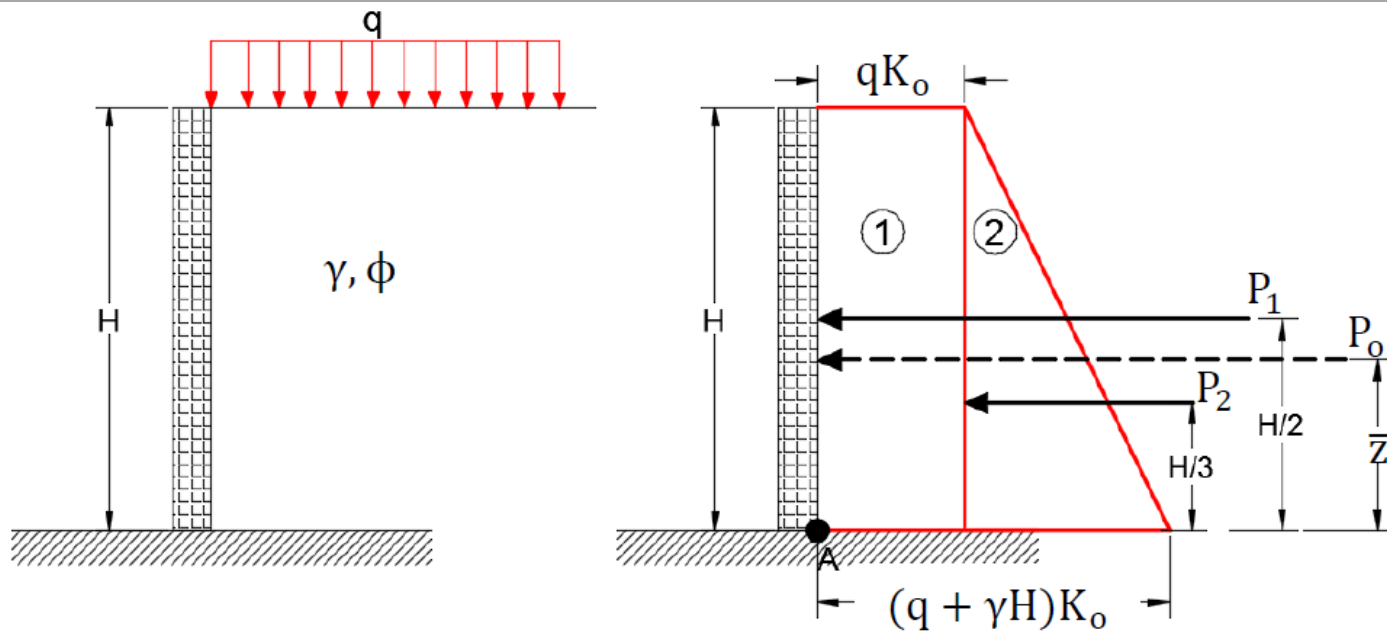


**Active**



**Passive**

# Lateral Earth Pressure at Rest



$$P_o = P_1 + P_2 = qK_o H + \frac{1}{2}\gamma H^2 K_o$$

where

$P_1$  = area of rectangle 1

$P_2$  = area of triangle 2

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

# Coefficient of Lateral Earth Pressure $K_0$

## Jaky formula

For normally consolidated clays and loose sand.

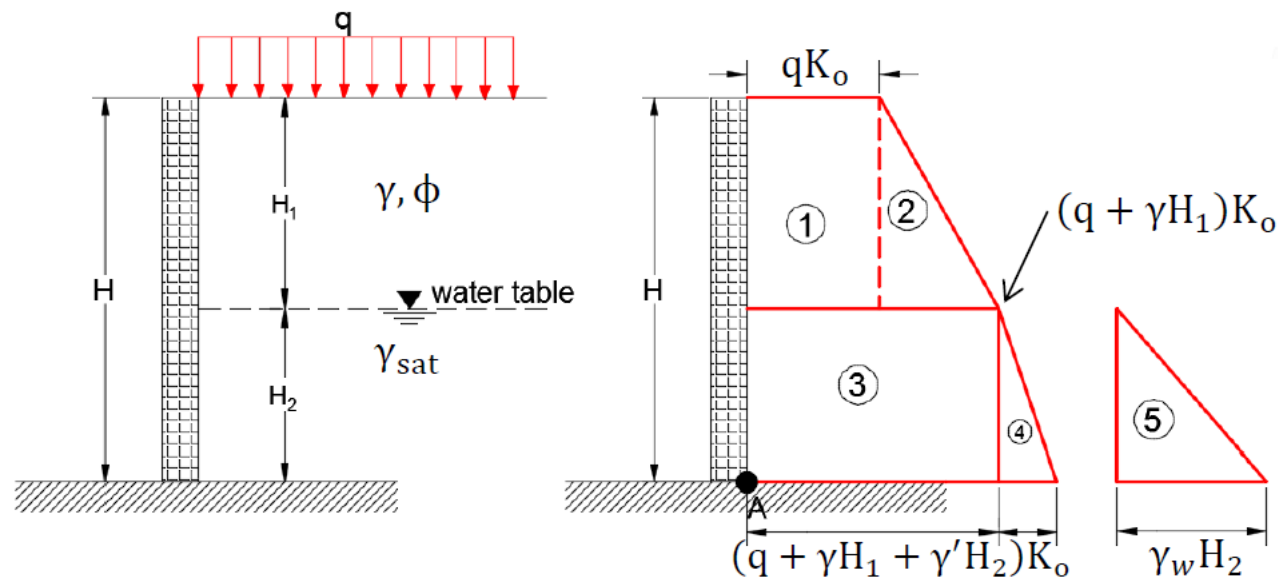
$$K_0 \approx 1 - \sin \phi'$$

## Mayne and Kulhawy

For Overconsolidated clays

$$K_0 = (1 - \sin \phi') \text{OCR}^{\sin \phi'}$$

# Lateral Earth Pressure at Rest with Water



$$\text{at } z = 0, \quad \sigma'_h = K_o \sigma'_o = K_o q$$

$$\text{at } z = H_1, \quad \sigma'_h = K_o \sigma'_o = K_o (q + \gamma H_1)$$

and

$$\text{at } z = H_2, \quad \sigma'_h = K_o \sigma'_o = K_o (q + \gamma H_1 + \gamma' H_2)$$

$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

where  $A$  = area of the pressure diagram.

So,

$$P_o = K_o q H_1 + \frac{1}{2} K_o \gamma H_1^2 + K_o (q + \gamma H_1) H_2 + \frac{1}{2} K_o \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

# EXAMPLE 16.1

For the retaining wall shown in Figure 16.6a, determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force. Assume  $\text{OCR} = 1$ .

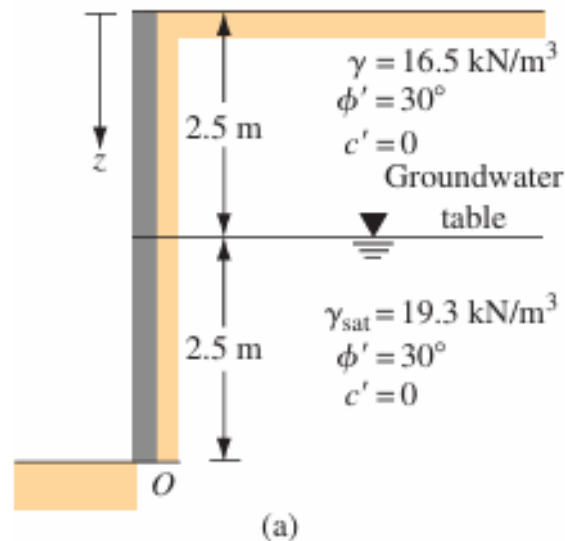


FIGURE 16.6

# EXAMPLE 16.1

## SOLUTION

$$K_o = 1 - \sin \phi' = 1 - \sin 30^\circ = 0.5$$

$$\text{At } z = 0, \sigma_o' = 0; \sigma_h' = 0$$

$$\text{At } z = 2.5 \text{ m, } \sigma_o' = (16.5)(2.5) = 41.25 \text{ kN/m}^2;$$

$$\sigma_h' = K_o \sigma_o' = (0.5)(41.25) = 20.63 \text{ kN/m}^2$$

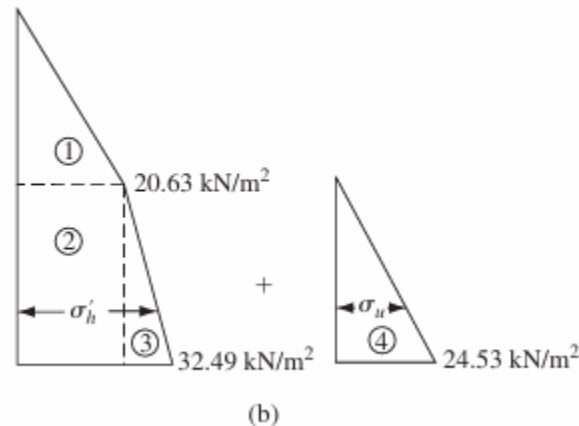
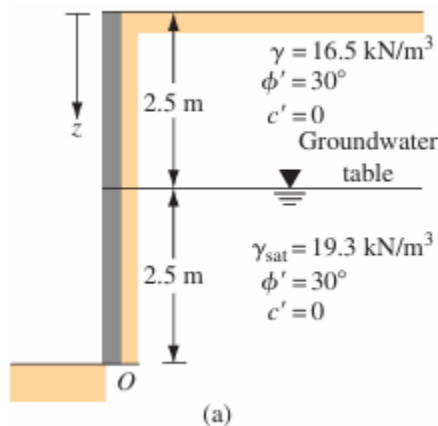
$$\text{At } z = 5 \text{ m, } \sigma_o' = (16.5)(2.5) + (19.3 - 9.81)2.5 = 64.98 \text{ kN/m}^2;$$

$$\sigma_h' = K_o \sigma_o' = (0.5)(64.98) = 32.49 \text{ kN/m}^2$$

The hydrostatic pressure distribution is as follows:

From  $z = 0$  to  $z = 2.5$  m,  $u = 0$ . At  $z = 5$  m,  $u = \gamma_w(2.5) = (9.81)(2.5) = 24.53 \text{ kN/m}^2$ .

The pressure distribution for the wall is shown in Figure 16.6b.



# EXAMPLE 16.1

The total force per unit length of the wall can be determined from the area of the pressure diagram, or

$$\begin{aligned}P_o &= \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4} \\&= \frac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \frac{1}{2}(2.5)(32.49 - 20.63) \\&\quad + \frac{1}{2}(2.5)(24.53) = \mathbf{122.85 \text{ kN/m}}\end{aligned}$$

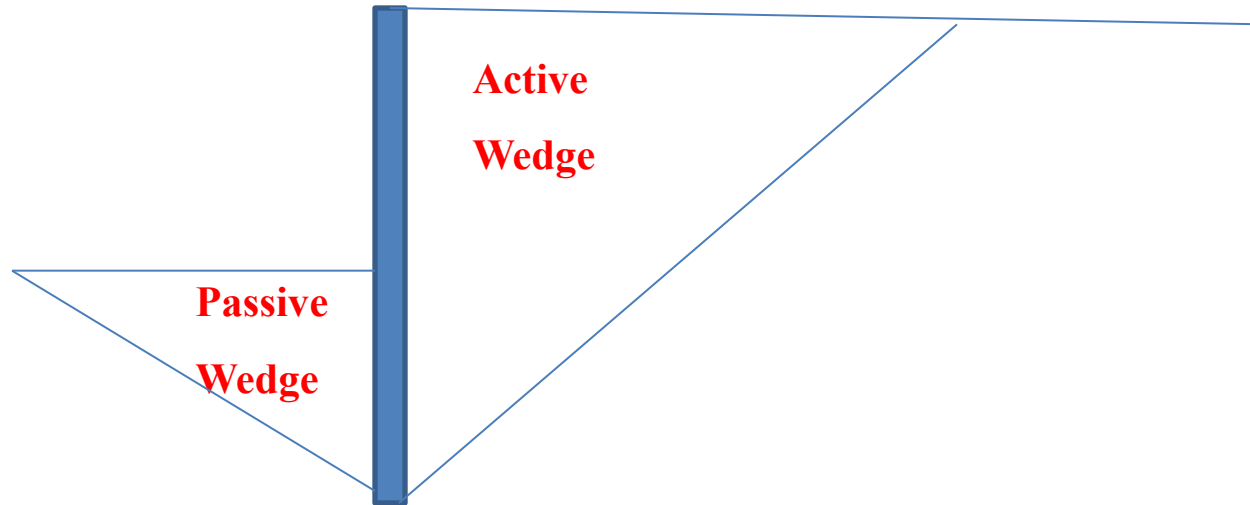
The location of the center of pressure measured from the bottom of the wall (point  $O$ ) =

$$\begin{aligned}\bar{z} &= \frac{(\text{Area 1})\left(2.5 + \frac{2.5}{3}\right) + (\text{Area 2})\left(\frac{2.5}{2}\right) + (\text{Area 3} + \text{Area 4})\left(\frac{2.5}{3}\right)}{P_o} \\&= \frac{(25.788)(3.33) + (51.575)(1.25) + (14.825 + 30.663)(0.833)}{122.85} \\&= \frac{85.87 + 64.47 + 37.89}{122.85} = \mathbf{1.53 \text{ m}}\end{aligned}$$

# NOTES

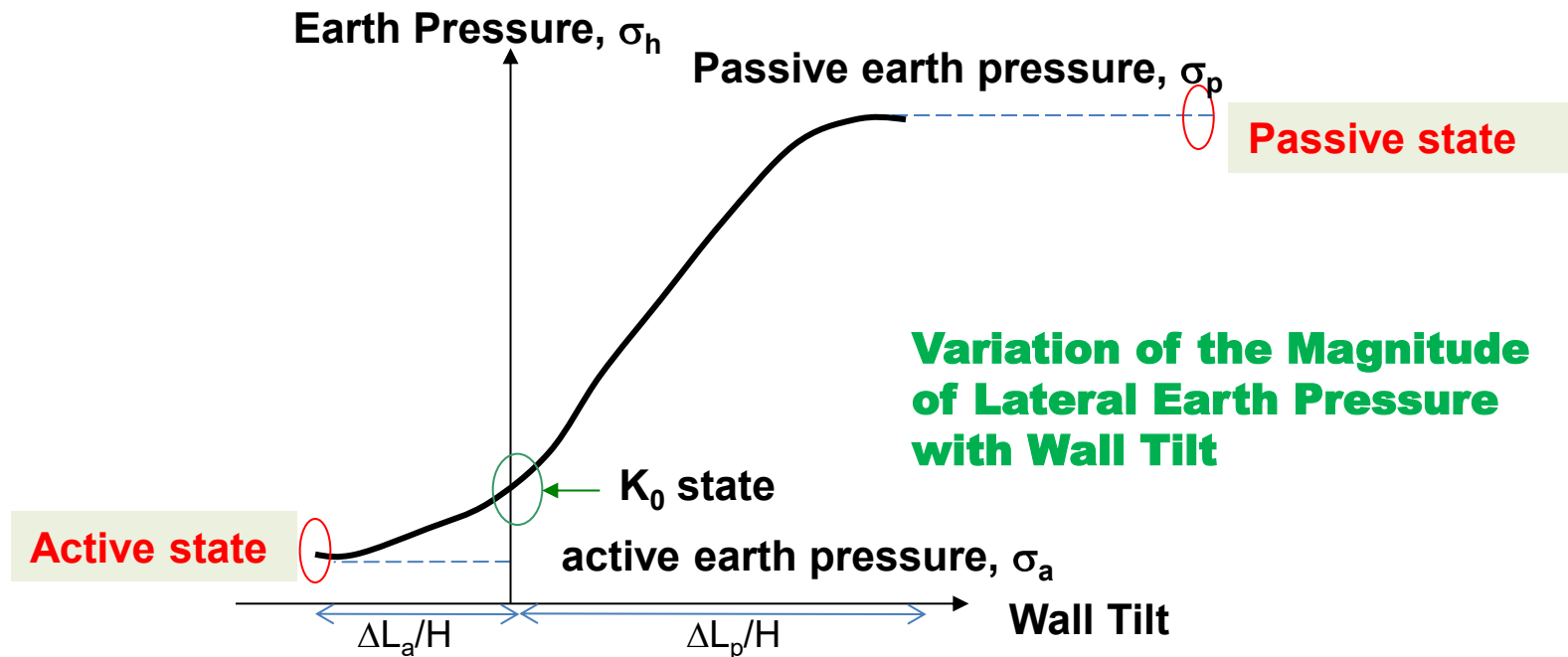
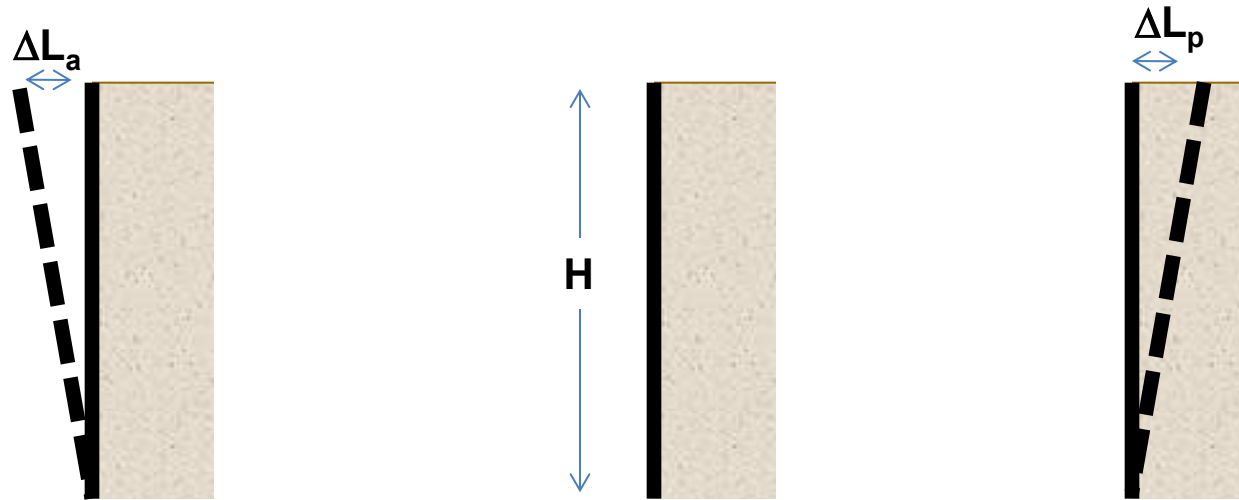
- If the lateral strain in the soil is **ZERO** the corresponding lateral pressure is called the **earth pressure at-rest**. This is the case **before construction**.
- In the case of **active** case the **soil** is the **actuating element** and in the case of **passive** the **wall** is the **actuating element**.
- For either the active or passive states to develop, the wall must **MOVE**. If the wall does not move, an intermediate stress state exists called earth pressure at **rest**. (i.e. zero lateral strain).
- For greatest economy, retaining structures are designed only sufficiently strong to resist **ACTIVE PRESSURE**. They therefore must be allowed to move.
- It may at first seem unlikely that a wall ever would be built to **PUSH** into the soil and mobilize passive earth pressure.

# NOTES

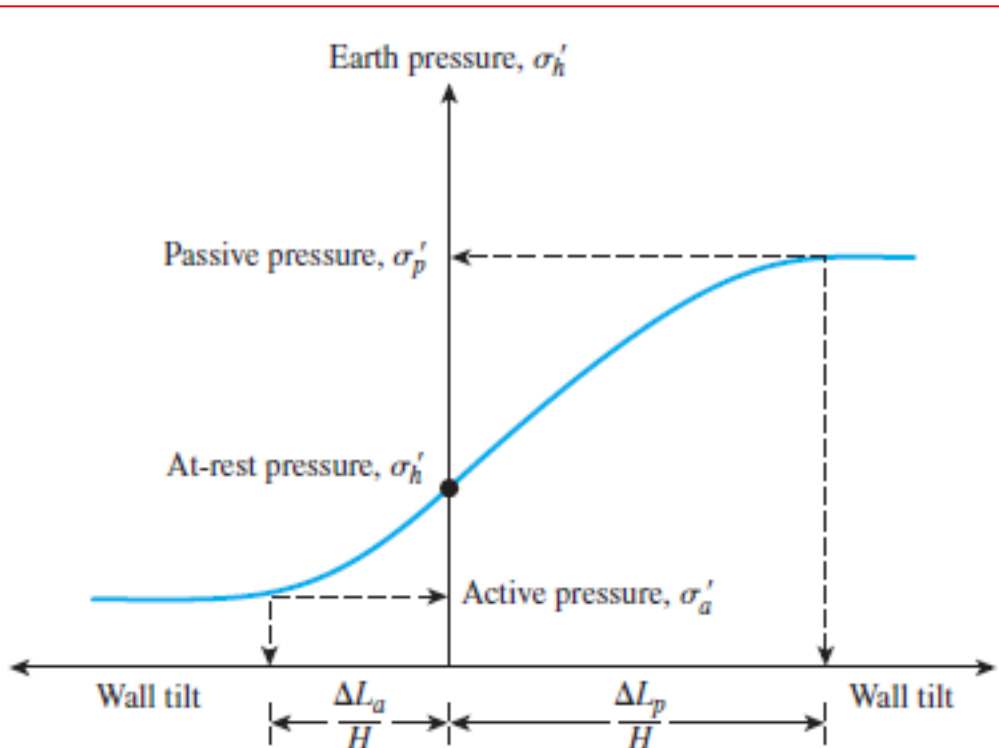


- Typically **passive** earth pressure is developed by **anchor plates** or blocks, embedded in the soil and where the anchor rod or cable tension pulls the anchor into/against the soil to develop passive resistance. Walls are **seldom** designed for **passive** pressure.
- In most retaining walls of limited height, movement may occur by **simple translation** or, more frequently, by **rotation** about the bottom.

# NOTES



# NOTES



Active or passive condition will only be reached if the wall is allowed to yield sufficiently. The amount of wall necessary depends on:-

- Soil type (sand vs. clay)
- Soil density (Loose vs. dense)
- Pressure (Active vs. passive)

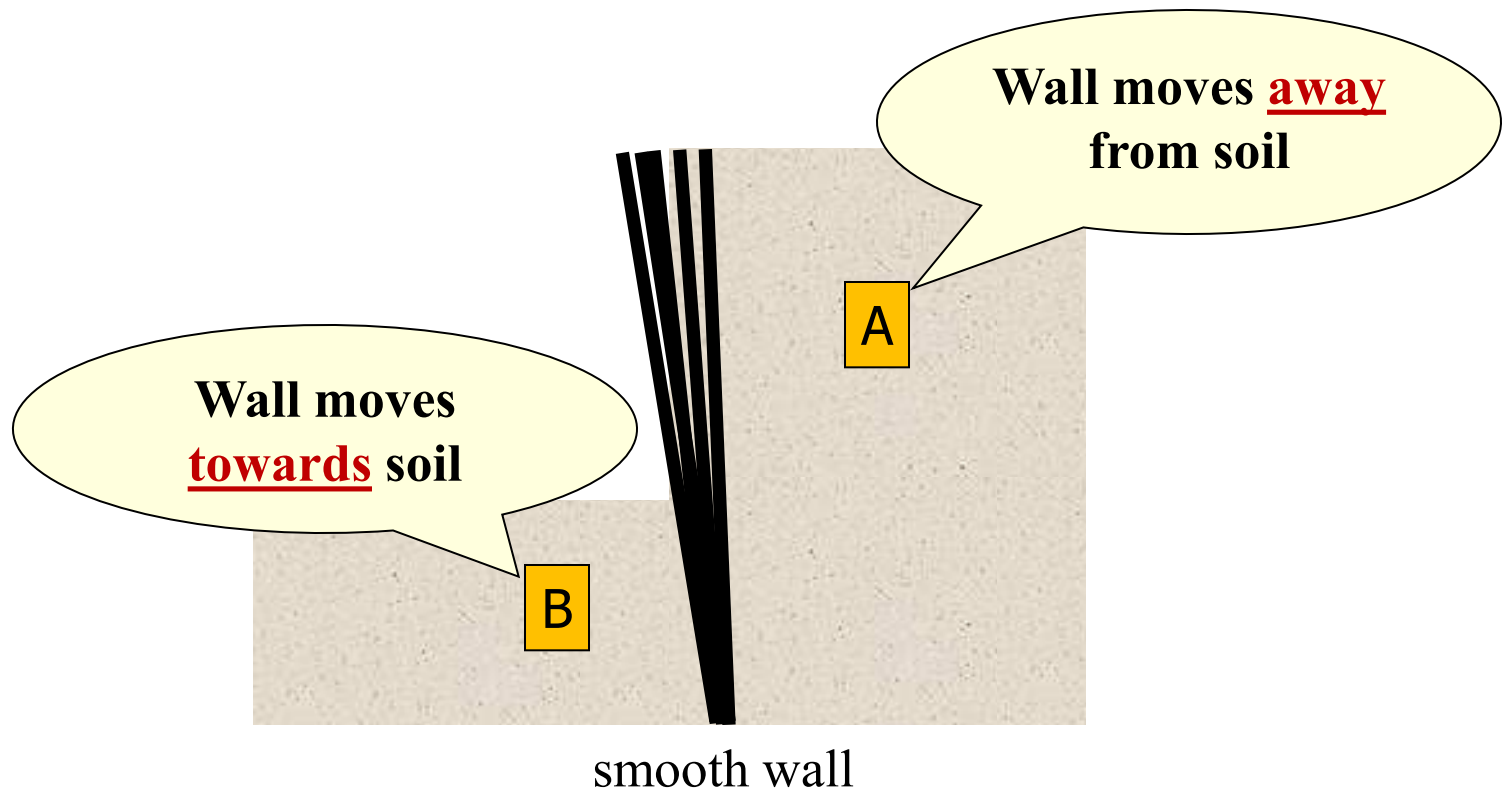
Typical Values of  $\Delta L_a/H$  and  $\Delta L_p/H$

| Soil type  | $\Delta L_a/H$ | $\Delta L_p/H$ |
|------------|----------------|----------------|
| Loose sand | 0.001–0.002    | 0.01           |
| Dense sand | 0.0005–0.001   | 0.005          |
| Soft clay  | 0.02           | 0.04           |
| Stiff clay | 0.01           | 0.02           |

# Lateral Earth Pressure Theories

- Since late 17<sup>th</sup> century many theories of earth of earth pressure have been proposed by various investigators. Of the theories the following **two** are the most popular and used for computation of **active** and **passive** earth pressures:.
  1. Rankine's Theory (**No wall friction**)
  2. Coulomb's Theory (**With wall friction**)
- Those are usually called the **classical lateral earth pressure theories**.
- In both theories it is required that the soil mass, or at least certain parts of the mass, is in a state of **PLASTIC EQUILIBRIUM**. The soil mass is on verge of failure. Failure here is defined to be the state of stress which satisfies the Mohr-Coulomb criterion.

# Active vs. Passive Earth Pressures



Let's look at the soil elements A and B during the wall movement.

- ❑ In most retaining walls of limited height, movement may occur by simple **translation** or, more frequently, by **rotation** about the bottom.

# Rankine's Earth Pressure Theory

- ❑ Rankine (1857) investigated the **stress** condition in a soil at a state of **PLASTIC EQUILIBRIUM**.
- ❑ Developed based on semi infinite “**loose granular**” soil mass for which the soil movement is uniform.
- ❑ Used stress states of soil mass to determine lateral pressures on a frictionless wall

## Assumptions:

- Vertical wall
- Smooth retaining wall
- Horizontal ground surface
- Homogeneous soil

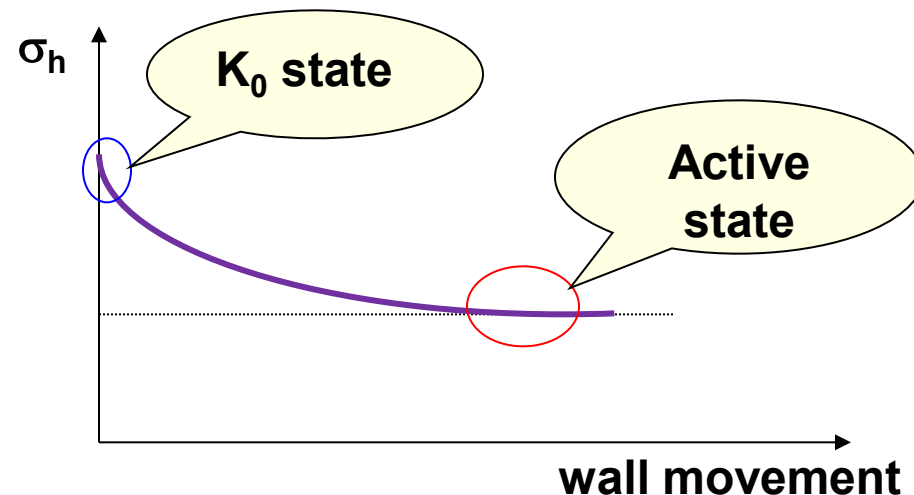
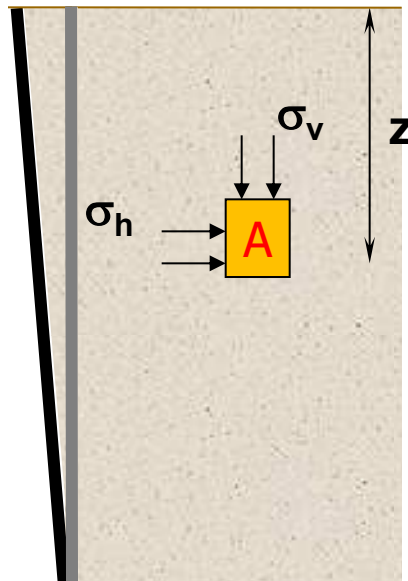
# Active Earth Pressure

## Active earth pressure

- $\sigma_v = \gamma z$
- Initially, there is no lateral movement.  
 $\therefore \sigma_h = K_0 \sigma_v = K_0 \gamma z$
- As the wall moves away from the soil,
- $\sigma_v$  remains the same; and
- $\sigma_h$  **decreases** till failure occurs.

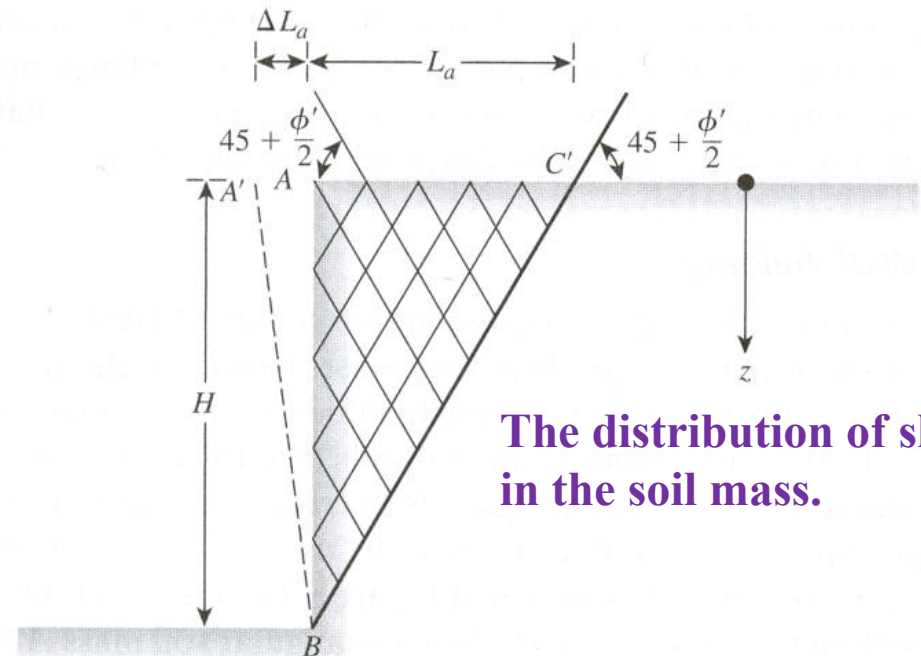
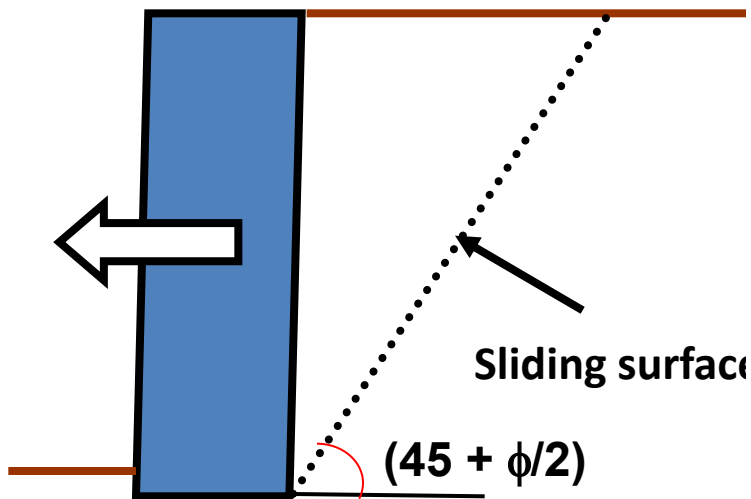
Active state

$\sigma_h \longrightarrow \sigma_a$



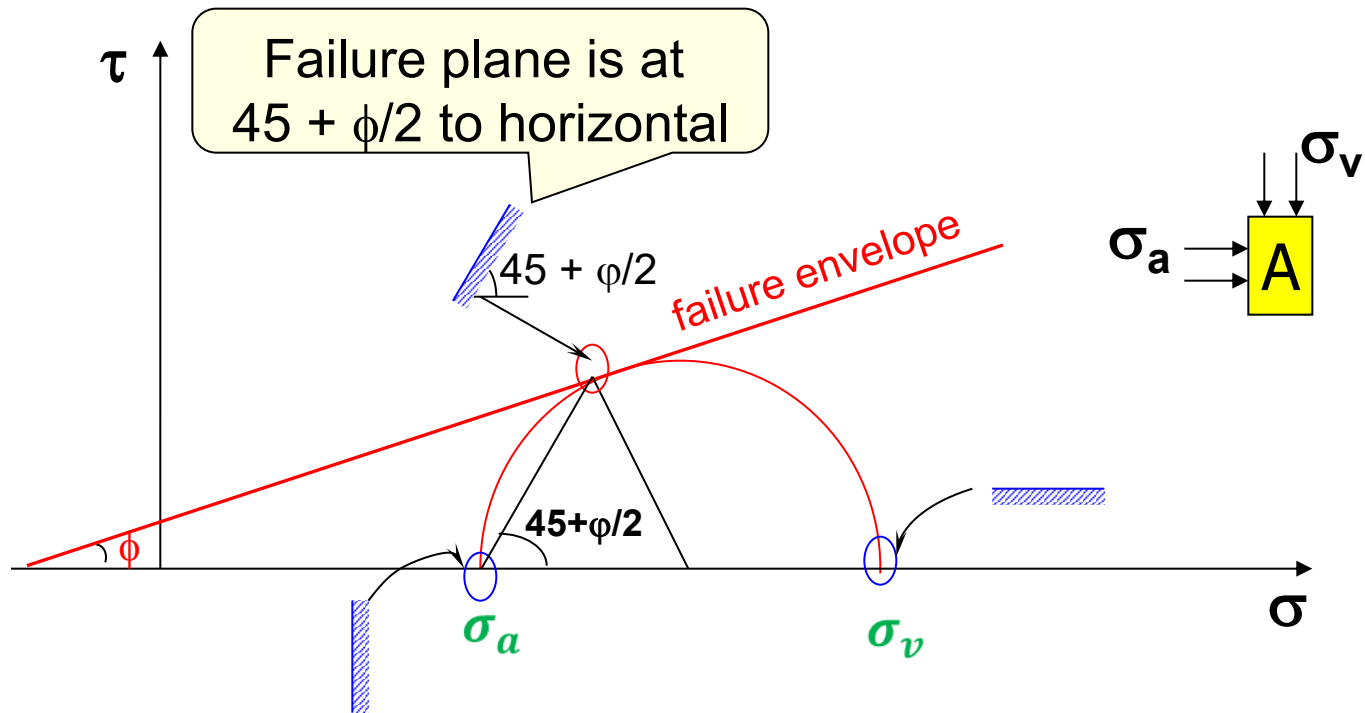
# Orientation of Failure Planes

- From Mohr Circle the failure planes in the soil make  $\pm (45 + \phi/2)$ -degree angles with the direction of the major principal plane—that is, the horizontal.
- These are called potential *slip planes*.



- Because the slip planes make angles of  $(45 + \phi/2)$  degrees with the major principal plane, the soil mass in the state of plastic equilibrium is bounded by the plane  $BC'$ . The soil inside the zone  $ABC'$  undergoes the same unit deformation in the horizontal direction everywhere, which is equal to  $\Delta L_a/L_a$ .

# Active Earth Pressure



# Active Earth Pressure

## Retaining wall with a vertical back and a horizontal backfill

Mohr–Coulomb failure envelope defined by the equation

$$s = c' + \sigma' \tan \phi'$$

The principal stresses for a Mohr's circle that touches the Mohr–Coulomb failure envelope:

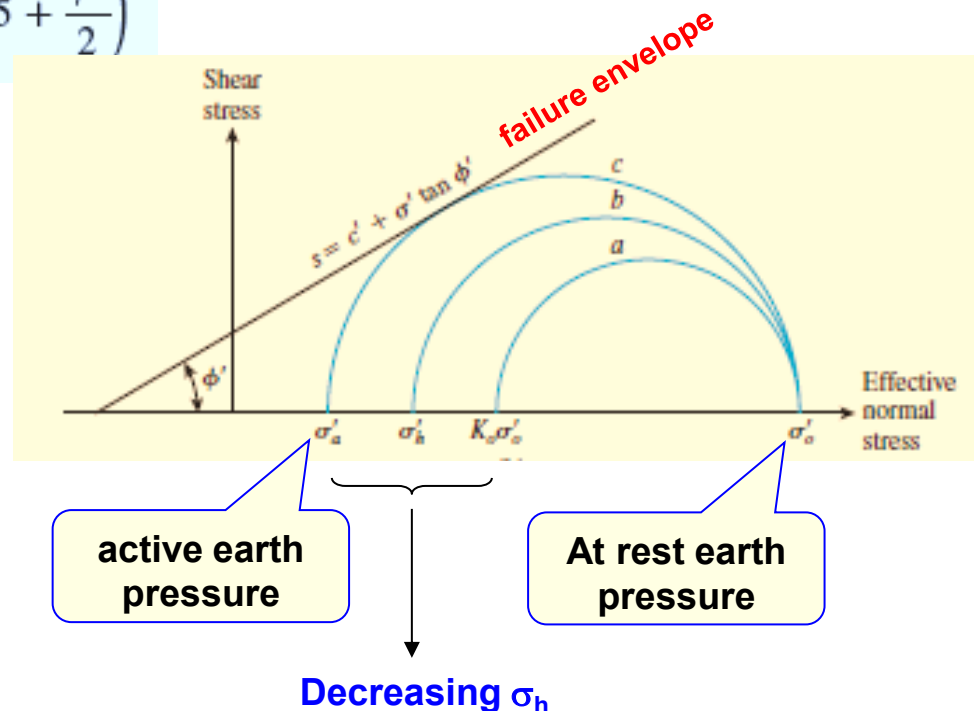
$$\sigma'_1 = \sigma'_3 \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

$$\sigma'_o = \sigma'_a \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

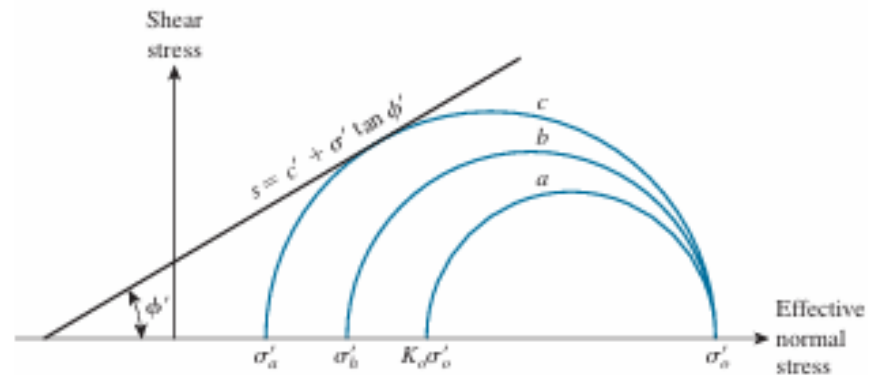
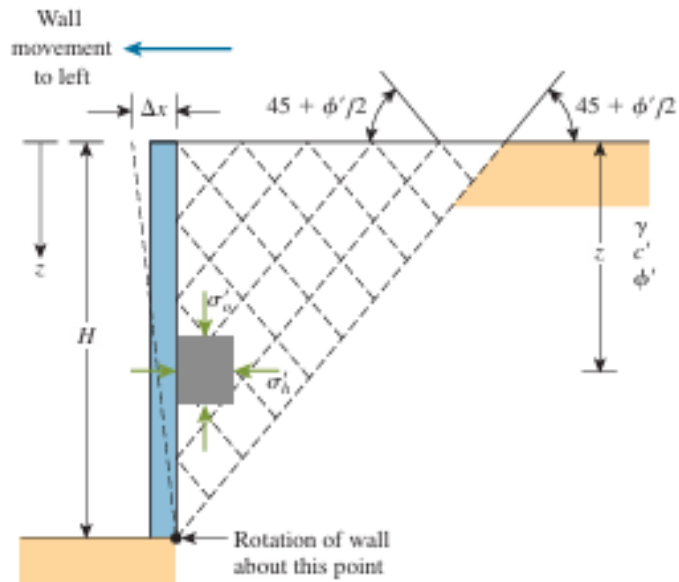
$$\sigma'_a = \frac{\sigma'_o}{\tan^2 \left( 45 + \frac{\phi'}{2} \right)} - \frac{2c'}{\tan \left( 45 + \frac{\phi'}{2} \right)}$$

$$\begin{aligned} \sigma'_a &= \sigma'_o \tan^2 \left( 45 - \frac{\phi'}{2} \right) - 2c' \tan \left( 45 - \frac{\phi'}{2} \right) \\ &= \sigma'_o K_a - 2c' \sqrt{K_a} \end{aligned}$$

$$K_a = \tan^2(45 - \phi'/2) = \text{Rankine active-pressure coefficient.}$$

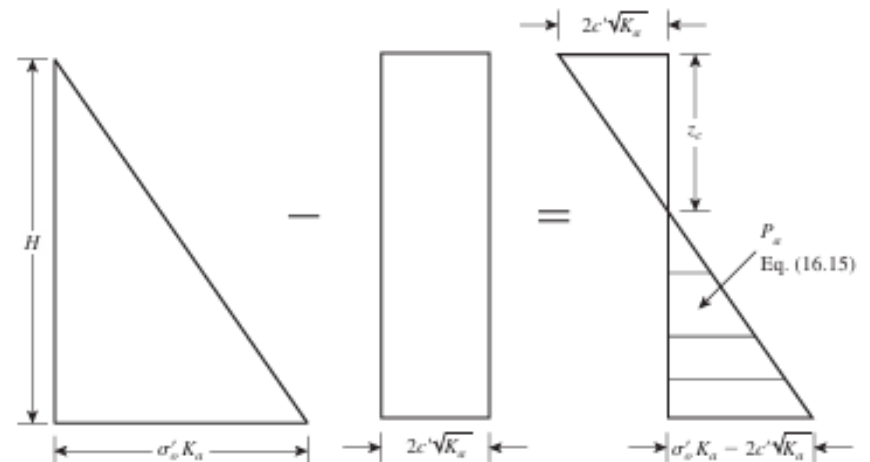


# Active Earth Pressure



$$\sigma'_a = \sigma'_o \tan^2 \left( 45 - \frac{\phi'}{2} \right) - 2c' \tan \left( 45 - \frac{\phi'}{2} \right)$$

$$= \sigma'_o K_a - 2c' \sqrt{K_a}$$



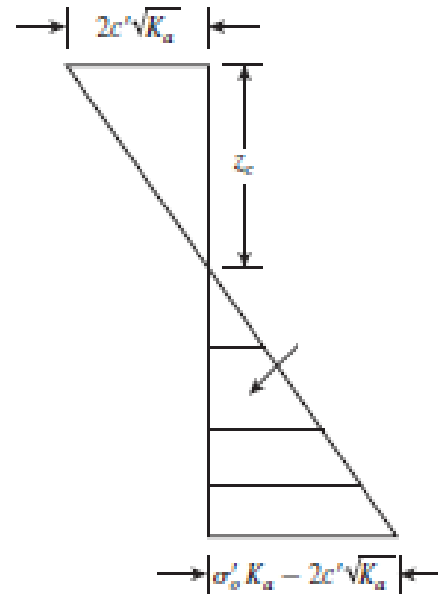
# Active Earth Pressure

Tensile stress in the soil will cause a crack along the soil–wall interface

The *depth of tensile crack*

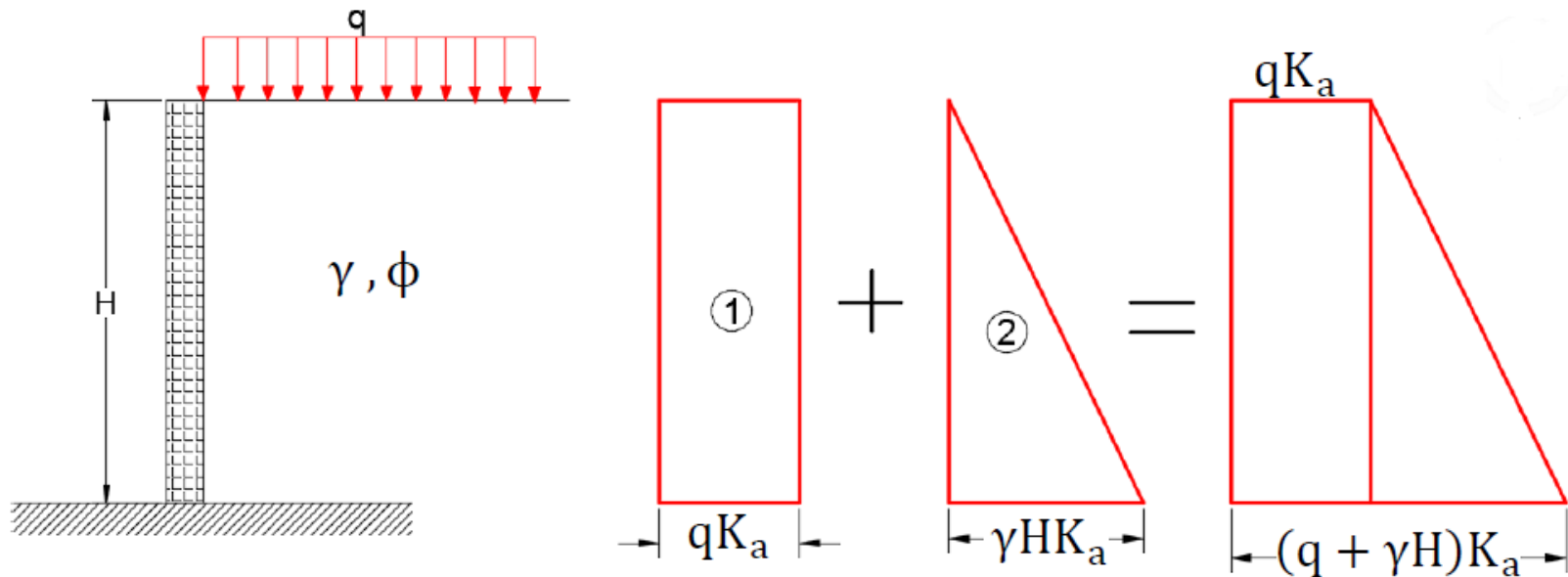
$$\gamma z_c K_a - 2c'\sqrt{K_a} = 0$$

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}}$$



# Active Earth Pressure

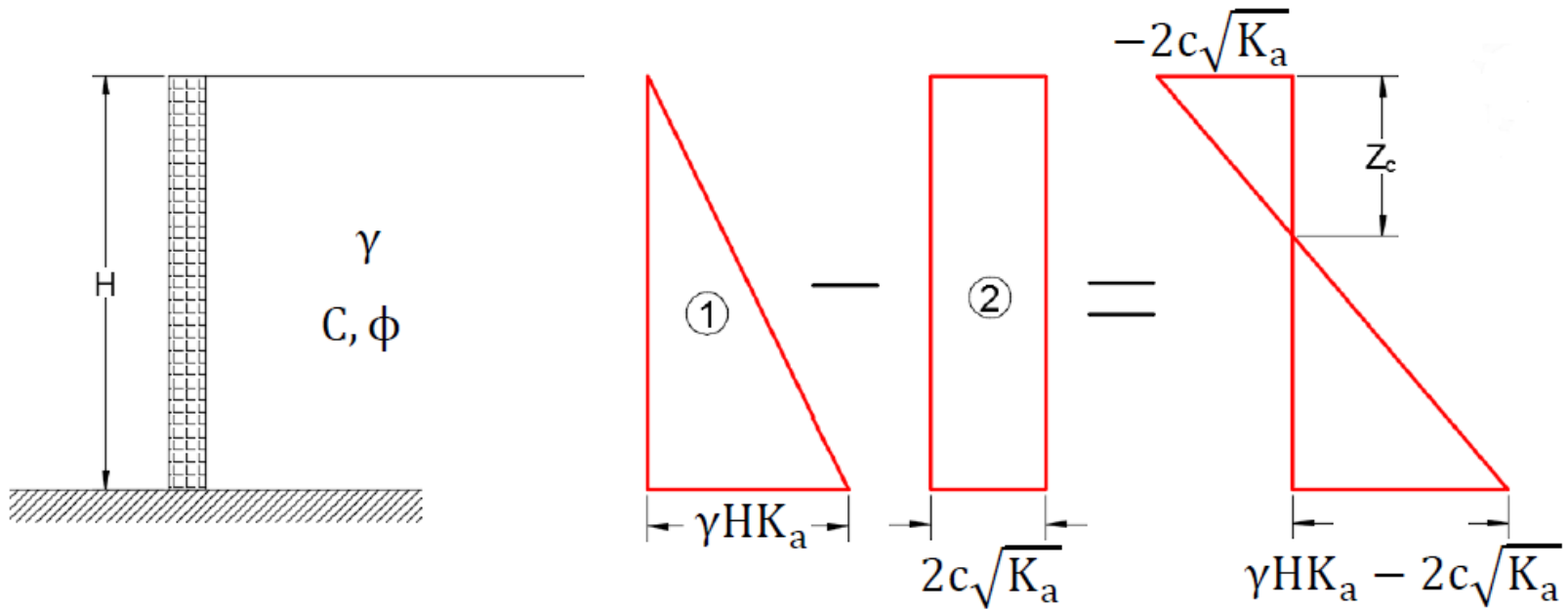
In case of granular soil (pure sand):



$$P_a = P_1 + P_2$$

# Active Earth Pressure

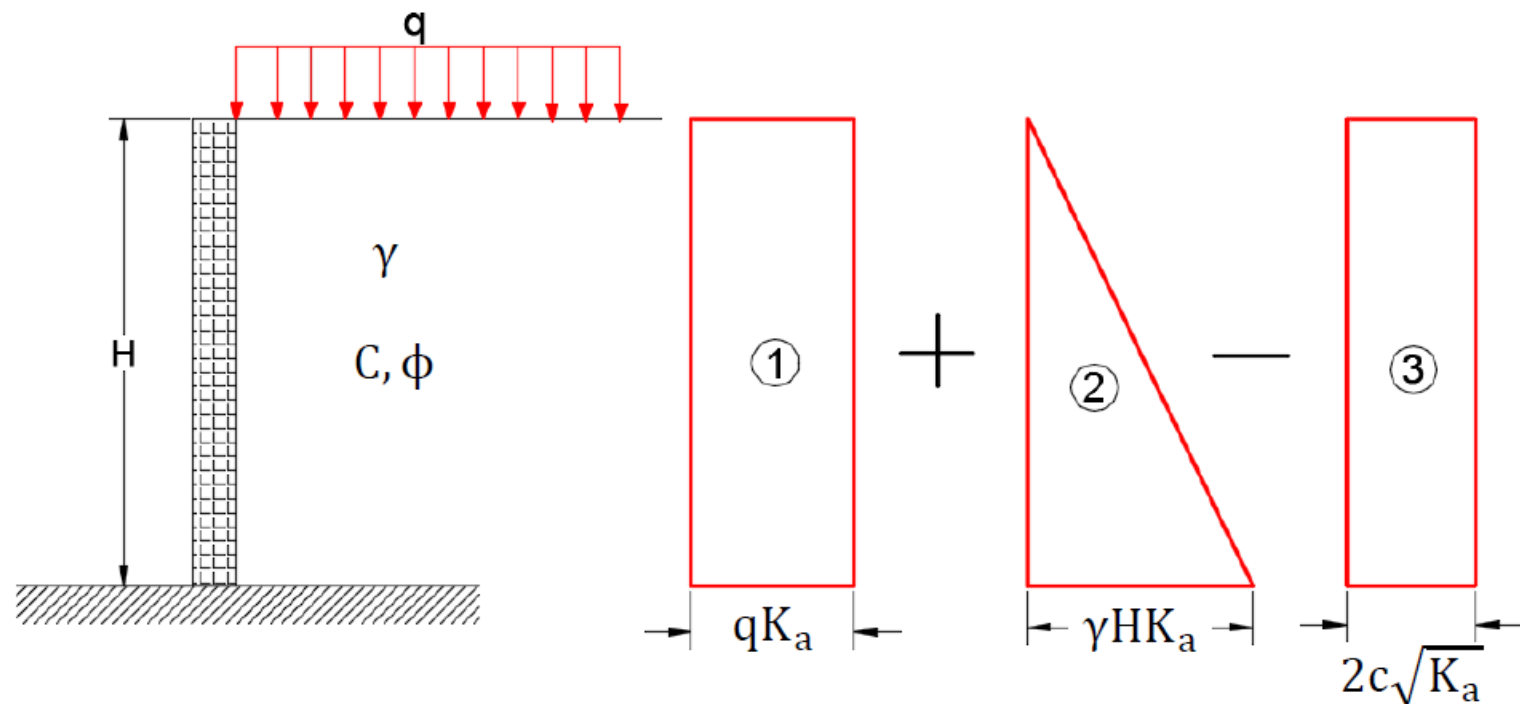
If the soil is  $C - \phi$  soil:



# Active Earth Pressure

If the soil is **C –  $\phi$**  soil:

If there exist surcharge:



$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$

## EXAMPLE 16.2

### EXAMPLE 16.2

A 6 m high retaining wall is to support a soil with unit weight  $\gamma = 17.4 \text{ kN/m}^3$ , soil friction angle  $\phi' = 26^\circ$ , and cohesion  $c' = 5 \text{ kN/m}^2$ . Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

#### SOLUTION

For  $\phi' = 26^\circ$ ,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$

From Figure 16.7c, at  $z = 0$ ,

$$\sigma'_a = -2c'\sqrt{K_a} = -2(5)(0.625) = -6.25 \text{ kN/m}^2$$

and at  $z = 6 \text{ m}$ ,

$$\begin{aligned}\sigma'_a &= (17.4)(6)(0.39) - 2(5)(0.625) \\ &= 40.72 - 6.25 = 34.47 \text{ kN/m}^2\end{aligned}$$

## EXAMPLE 16.2

Active Force Before the Tensile Crack Appeared: Eq. (16.13)

$$\begin{aligned} P_a &= \frac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a} \\ &= \frac{1}{2}(6)(40.72) - (6)(6.25) = 122.16 - 37.5 = \mathbf{84.66 \text{ kN/m}} \end{aligned}$$

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a \bar{z} = (122.16)\left(\frac{6}{3}\right) - (37.5)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 112.5}{84.66} = \mathbf{1.56 \text{ m}}$$

## EXAMPLE 16.2

**Active Force After the Tensile Crack Appeared: Eq. (16.12)**

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(5.0)}{(17.4)(0.625)} = 0.92 \text{ m}$$

Using Eq. (16.14) gives

$$P_a = \frac{1}{2}(H - z_c)(\gamma HK_a - 2c'\sqrt{K_a}) = \frac{1}{2}(6 - 0.92)(34.47) = \mathbf{87.55 \text{ kN/m}}$$

Figure 16.7c indicates that the force  $P_a = 87.55 \text{ kN/m}$  is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height  $\bar{z} = (H - z_c)/3$  above the bottom of the wall, or

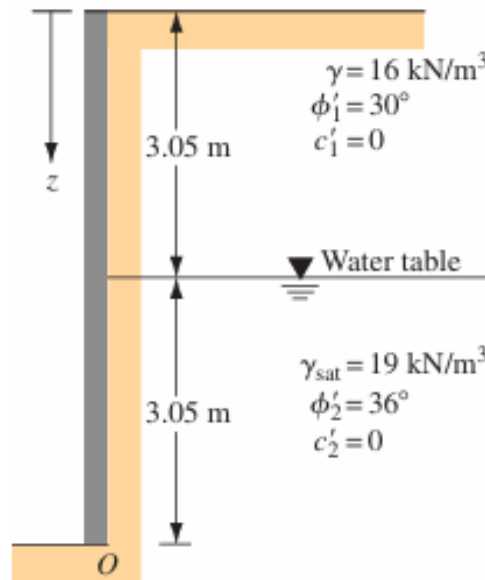
$$\bar{z} = \frac{6 - 0.92}{3} = \mathbf{1.69 \text{ m}}$$



## EXAMPLE 16.3

### EXAMPLE 16.3

Assume that the retaining wall shown in Figure 16.8a can yield sufficiently to develop an active state. Determine the Rankine active force per unit length of the wall and the location of the resultant line of action.



(a)

# EXAMPLE 16.3

## SOLUTION

If the cohesion,  $c'$ , is zero, then

$$\sigma'_a = \sigma'_o K_a$$

For the top layer of soil,  $\phi'_1 = 30^\circ$ , so

$$K_{a(1)} = \tan^2\left(45 - \frac{\phi'_1}{2}\right) = \tan^2(45 - 15) = \frac{1}{3}$$

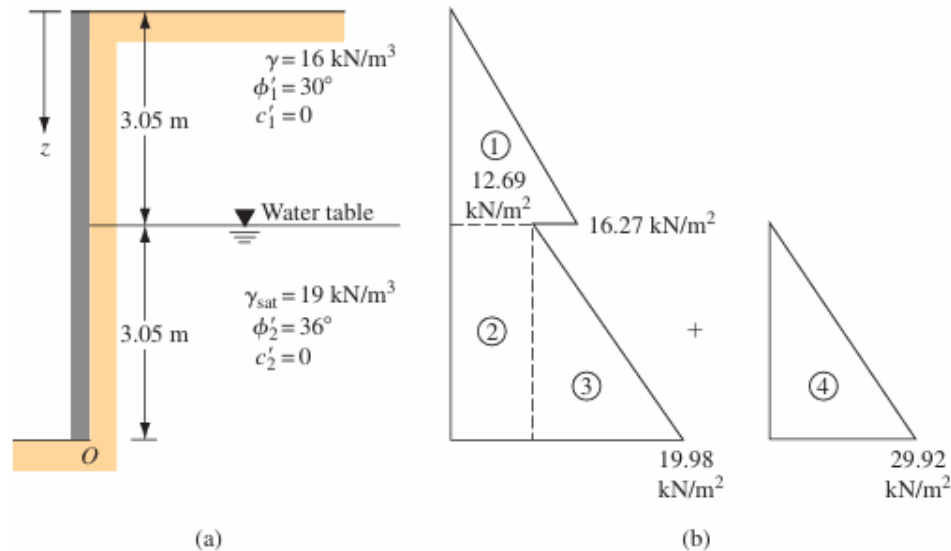
Similarly, for the bottom layer of soil,  $\phi'_2 = 36^\circ$ , and it follows that

$$K_{a(2)} = \tan^2\left(45 - \frac{36}{2}\right) = 0.26$$

The following table shows the calculation of  $\sigma'_a$  and  $u$  at various depths below the ground surface.

| Depth, $z$<br>(m) |                                        | $\sigma'_o$<br>(kN/m <sup>2</sup> ) | $K_a$ | $\sigma'_a = K_a \sigma'_o$<br>(kN/m <sup>2</sup> ) | $u$<br>(kN/m <sup>2</sup> ) |
|-------------------|----------------------------------------|-------------------------------------|-------|-----------------------------------------------------|-----------------------------|
| 0                 | 0                                      |                                     | 1/3   | 0                                                   | 0                           |
| 3.05 <sup>-</sup> | (16)(3.05) = 48.8                      |                                     | 1/3   | 16.27                                               | 0                           |
| 3.05 <sup>+</sup> | 48.8                                   |                                     | 0.26  | 12.69                                               | 0                           |
| 6.1               | (16)(3.05) + (19 - 9.81)(3.05) = 76.83 |                                     | 0.26  | 19.98                                               | (9.81)(3.05) = 29.92        |

# EXAMPLE 16.3



The pressure distribution diagram is plotted in Figure 16.8b. The force per unit length is

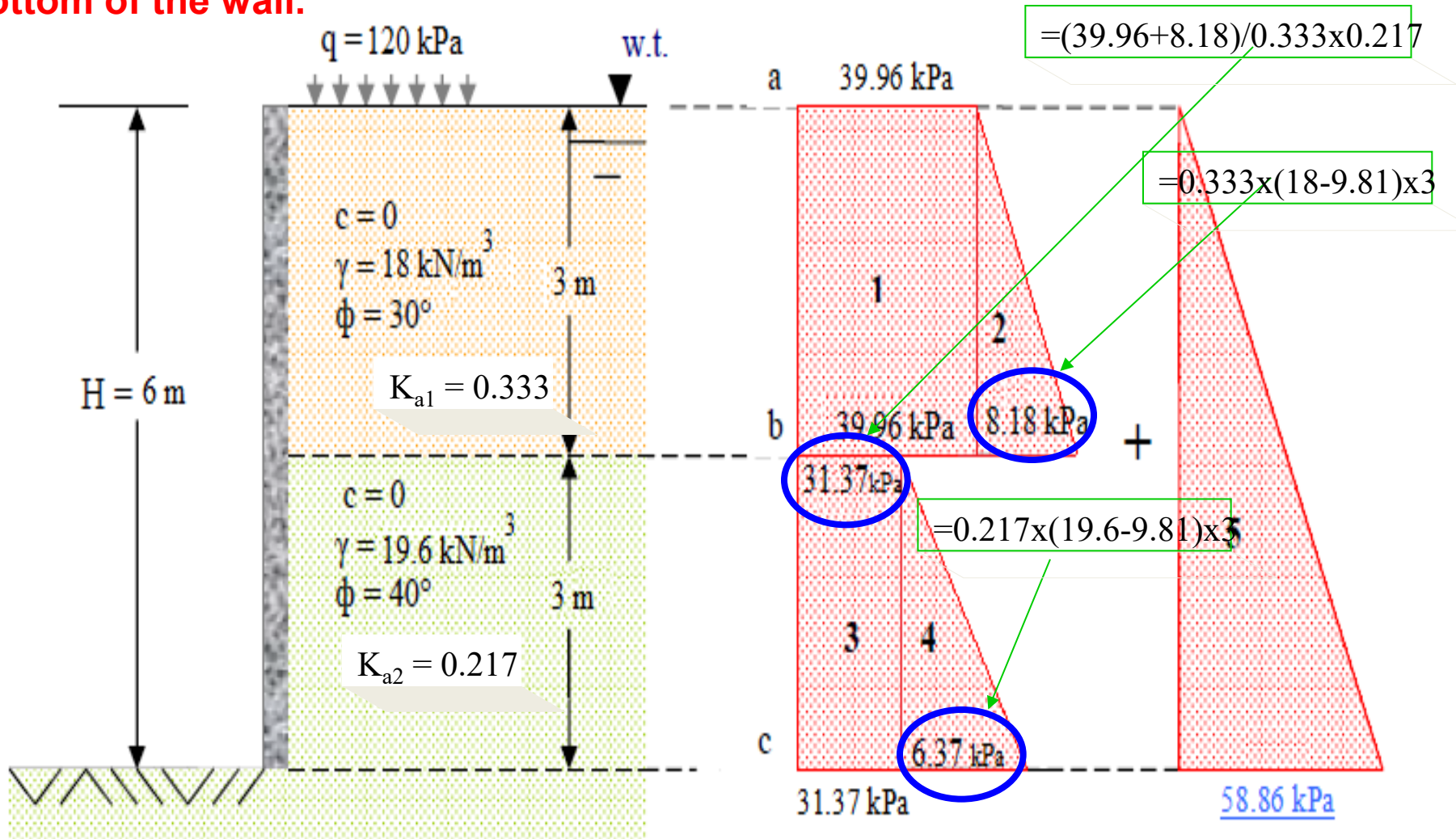
$$\begin{aligned}
 P_a &= \text{area 1} + \text{area 2} + \text{area 3} + \text{area 4} \\
 &= \frac{1}{2}(3.05)(16.27) + (12.69)(3.05) + \frac{1}{2}(19.98 - 12.69)(3.05) + \frac{1}{2}(29.92)(3.05) \\
 &= 24.81 + 38.70 + 11.12 + 45.63 = \mathbf{120.26 \text{ kN/m}}
 \end{aligned}$$

The distance of the line of action of the resultant force from the bottom of the wall can be determined by taking the moments about the bottom of the wall (point O in Figure 16.8a) and is

$$\bar{z} = \frac{(24.81)\left(3.05 + \frac{3.05}{3}\right) + (38.7)\left(\frac{3.05}{2}\right) + (11.12 + 45.63)\left(\frac{10}{3}\right)}{120.26} = \mathbf{1.81 \text{ m}}$$

# EXAMPLE

Draw the pressure diagram on the wall in an **active** pressure condition, and find the total resultant **F** on the wall and its **location** with respect to the bottom of the wall.



# SOLUTION

## Step 1

$$K_{a1} = \tan^2 (45^\circ - 30^\circ/2) = 0.333$$

$$K_{a2} = \tan^2 (45^\circ - 40^\circ/2) = 0.217$$

## Step 2

The stress on the wall at point  $a$  is:

$$p_a = q K_{a1} = (120) (0.333) = \underline{39.96 \text{ kPa}}$$

The stress at  $b$  (within the top stratum) is:

$$\begin{aligned} p_{b-} &= (q + \gamma' h) K_{a1} \\ &= [120 + (18 - 9.81) (3)] [0.333] = \underline{48.14 \text{ kPa}} \end{aligned}$$

The stress at  $b$  (within bottom stratum) is:

$$\begin{aligned} p_{b+} &= (q + \gamma' h) K_{a2} \\ &= [120 + (18 - 9.81) (3)] [0.217] = \underline{31.37 \text{ kPa}} \end{aligned}$$

The stress at point  $c$  is:

$$\begin{aligned} p_c &= [q + (\gamma' h)_1 + (\gamma' h)_2] K_{a2} \\ &= [120 + (18 - 9.81) (3) + (19.6 - 9.81) (3)] [0.217] = \underline{37.75 \text{ kPa}} \end{aligned}$$

The pressure of the water upon the wall is:

$$p_w = \gamma_w h = (9.81) (6) = \underline{58.86 \text{ kPa}}$$

# SOLUTION

## Step 3

The forces from each area:

$$F_1 = (3) (39.96) = 119.88 \text{ kN/m}$$

$$F_2 = \frac{1}{2} (3) (8.18) = 12.27 \text{ kN/m}$$

$$F_3 = (3) (31.37) = 94.11 \text{ kN/m}$$

$$F_4 = \frac{1}{2} (3) (6.37) = 9.555 \text{ kN/m}$$

$$F_5 = \frac{1}{2} (58.86) (6) = 176.58 \text{ kN/m}$$

$$F_{\text{total}} = \underline{412.395 \text{ kN/m}}$$

## Step 4

The location of forces  $\hat{y}$  is at:

$$\begin{aligned} \hat{y} \ 412.395 &= 119.88 (4.5) + 12.27 (4) + 94.11 (1.5) + 9.555 (1) + 176.58 (2) \\ &= 539.46 + 49.08 + 141.165 + 9.555 + 353.16 = 1092.42 \text{ kN} \end{aligned}$$

$$\hat{y} = \underline{2.65 \text{ m from bottom of wall}}$$

# General Case: Inclined Wall with Inclined Backfill

## Granular backfill ( $c' = 0$ )

$\alpha$  = inclination of backfill with horizontal

$\theta$  = inclination of wall with vertical

$\beta$  = inclination of  $P_a$  with the normal to the wall

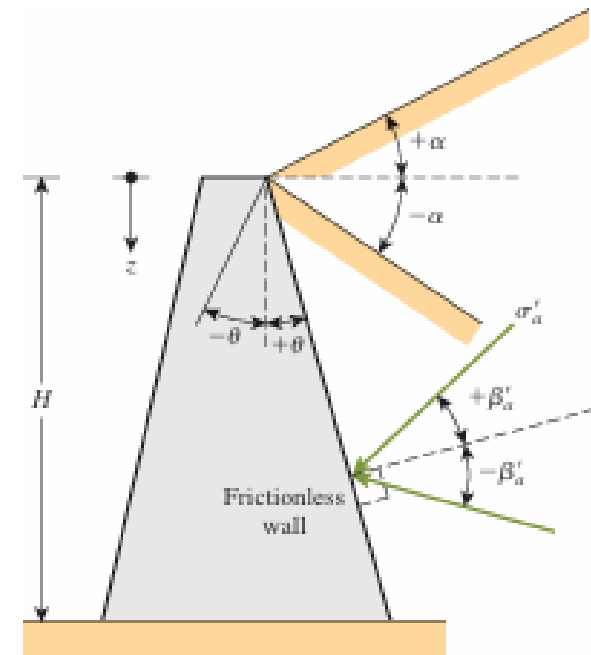
$$\sigma'_a = \frac{\gamma z \cos \alpha \sqrt{1 + \sin^2 \phi' - 2 \sin \phi' \cos \psi_a}}{\cos \alpha + \sqrt{\sin^2 \phi' - \sin^2 \alpha}}$$

$$\psi_a = \sin^{-1} \left( \frac{\sin \alpha}{\sin \phi'} \right) - \alpha + 2\theta.$$

$$\beta'_a = \tan^{-1} \left( \frac{\sin \phi' \sin \psi_a}{1 - \sin \phi' \cos \psi_a} \right)$$

$$P_a = \frac{1}{2} \gamma H^2 K_a$$

$$K_{a(R)} = \frac{\cos(\alpha - \theta) \sqrt{1 + \sin^2 \phi' - 2 \sin \phi' \cos \psi_a}}{\cos^2 \theta (\cos \alpha + \sqrt{\sin^2 \phi' - \sin^2 \alpha})}$$



# General Case : Inclined Wall with Inclined Backfill

TABLE 16.1 Variation of  $K_{a(R)}$  [Eq. (16.20)]

| $\alpha$ (deg) | $\theta$ (deg) | $K_{a(R)}$    |       |       |       |       |       |       |
|----------------|----------------|---------------|-------|-------|-------|-------|-------|-------|
|                |                | $\phi'$ (deg) |       |       |       |       |       |       |
|                |                | 28            | 30    | 32    | 34    | 36    | 38    | 40    |
| 0              | 0              | 0.361         | 0.333 | 0.307 | 0.283 | 0.260 | 0.238 | 0.217 |
|                | 2              | 0.363         | 0.335 | 0.309 | 0.285 | 0.262 | 0.240 | 0.220 |
|                | 4              | 0.368         | 0.341 | 0.315 | 0.291 | 0.269 | 0.248 | 0.228 |
|                | 6              | 0.376         | 0.350 | 0.325 | 0.302 | 0.280 | 0.260 | 0.242 |
|                | 8              | 0.387         | 0.362 | 0.338 | 0.316 | 0.295 | 0.276 | 0.259 |
|                | 10             | 0.402         | 0.377 | 0.354 | 0.333 | 0.314 | 0.296 | 0.280 |
|                | 15             | 0.450         | 0.428 | 0.408 | 0.390 | 0.373 | 0.358 | 0.345 |
| 5              | 0              | 0.366         | 0.337 | 0.311 | 0.286 | 0.262 | 0.240 | 0.219 |
|                | 2              | 0.373         | 0.344 | 0.317 | 0.292 | 0.269 | 0.247 | 0.226 |
|                | 4              | 0.383         | 0.354 | 0.328 | 0.303 | 0.280 | 0.259 | 0.239 |
|                | 6              | 0.396         | 0.368 | 0.342 | 0.318 | 0.296 | 0.275 | 0.255 |
|                | 8              | 0.412         | 0.385 | 0.360 | 0.336 | 0.315 | 0.295 | 0.276 |
|                | 10             | 0.431         | 0.405 | 0.380 | 0.358 | 0.337 | 0.318 | 0.300 |
|                | 15             | 0.490         | 0.466 | 0.443 | 0.423 | 0.405 | 0.388 | 0.373 |
| 10             | 0              | 0.380         | 0.350 | 0.321 | 0.294 | 0.270 | 0.246 | 0.225 |
|                | 2              | 0.393         | 0.362 | 0.333 | 0.306 | 0.281 | 0.258 | 0.236 |
|                | 4              | 0.408         | 0.377 | 0.348 | 0.322 | 0.297 | 0.274 | 0.252 |
|                | 6              | 0.426         | 0.395 | 0.367 | 0.341 | 0.316 | 0.294 | 0.273 |
|                | 8              | 0.447         | 0.417 | 0.389 | 0.363 | 0.339 | 0.317 | 0.297 |
|                | 10             | 0.471         | 0.441 | 0.414 | 0.388 | 0.365 | 0.344 | 0.324 |
|                | 15             | 0.542         | 0.513 | 0.487 | 0.463 | 0.442 | 0.422 | 0.404 |

# General Case : Inclined Wall with Inclined Backfill

TABLE 16.2 Variation of  $\beta'_a$  [Eq. (16.18)]

| $\alpha$ (deg) | $\theta$ (deg) | $\beta'_a$    |        |        |        |        |        |        |
|----------------|----------------|---------------|--------|--------|--------|--------|--------|--------|
|                |                | $\phi'$ (deg) |        |        |        |        |        |        |
|                |                | 28            | 30     | 32     | 34     | 36     | 38     | 40     |
| 0              | 0              | 0.000         | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  | 0.000  |
|                | 2              | 3.525         | 3.981  | 4.484  | 5.041  | 5.661  | 6.351  | 7.124  |
|                | 4              | 6.962         | 7.848  | 8.821  | 9.893  | 11.075 | 12.381 | 13.827 |
|                | 6              | 10.231        | 11.501 | 12.884 | 14.394 | 16.040 | 17.837 | 19.797 |
|                | 8              | 13.270        | 14.861 | 16.579 | 18.432 | 20.428 | 22.575 | 24.876 |
|                | 10             | 16.031        | 17.878 | 19.850 | 21.951 | 24.184 | 26.547 | 29.039 |
|                | 15             | 21.582        | 23.794 | 26.091 | 28.464 | 30.905 | 33.402 | 35.940 |
| 5              | 0              | 5.000         | 5.000  | 5.000  | 5.000  | 5.000  | 5.000  | 5.000  |
|                | 2              | 8.375         | 8.820  | 9.311  | 9.854  | 10.455 | 11.123 | 11.870 |
|                | 4              | 11.553        | 12.404 | 13.336 | 14.358 | 15.482 | 16.719 | 18.085 |
|                | 6              | 14.478        | 15.679 | 16.983 | 18.401 | 19.942 | 21.618 | 23.441 |
|                | 8              | 17.112        | 18.601 | 20.203 | 21.924 | 23.773 | 25.755 | 27.876 |
|                | 10             | 19.435        | 21.150 | 22.975 | 24.915 | 26.971 | 29.144 | 31.434 |
|                | 15             | 23.881        | 25.922 | 28.039 | 30.227 | 32.479 | 34.787 | 37.140 |
| 10             | 0              | 10.000        | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 |
|                | 2              | 13.057        | 13.491 | 13.967 | 14.491 | 15.070 | 15.712 | 16.426 |
|                | 4              | 15.839        | 16.657 | 17.547 | 18.519 | 19.583 | 20.751 | 22.034 |
|                | 6              | 18.319        | 19.460 | 20.693 | 22.026 | 23.469 | 25.032 | 26.726 |
|                | 8              | 20.483        | 21.888 | 23.391 | 24.999 | 26.720 | 28.559 | 30.522 |
|                | 10             | 22.335        | 23.946 | 25.653 | 27.460 | 29.370 | 31.385 | 33.504 |
|                | 15             | 25.683        | 27.603 | 29.589 | 31.639 | 33.747 | 35.908 | 38.114 |

# Vertical Wall with Inclined Backfill

Granular backfill ( $c' = 0$ )

$$P_a = \frac{1}{2} \gamma H^2 K_a$$

$$K_a = \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi'}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi'}}$$

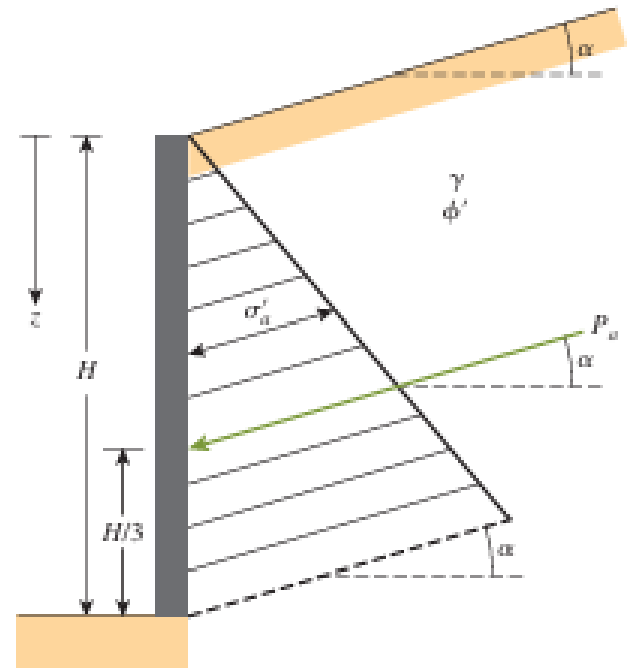


Table 16.3 gives the variation of  $K_a$  for various values of  $\alpha$ , and  $\phi$

# Vertical Wall with Inclined Backfill

TABLE 16.3 Values of  $K_a$  for Wall with Vertical Backface and Inclined Backfill [Eq. (16.22)]

| $\alpha$<br>(deg) | $\phi'$ (deg) $\rightarrow$ |        |        |        |        |        |        |        |        |        |        |        |        |
|-------------------|-----------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $\downarrow$      | 28                          | 29     | 30     | 31     | 32     | 33     | 34     | 35     | 36     | 37     | 38     | 39     | 40     |
| 0                 | 0.3610                      | 0.3470 | 0.3333 | 0.3201 | 0.3073 | 0.2948 | 0.2827 | 0.2710 | 0.2596 | 0.2486 | 0.2379 | 0.2275 | 0.2174 |
| 1                 | 0.3612                      | 0.3471 | 0.3335 | 0.3202 | 0.3074 | 0.2949 | 0.2828 | 0.2711 | 0.2597 | 0.2487 | 0.2380 | 0.2276 | 0.2175 |
| 2                 | 0.3618                      | 0.3476 | 0.3339 | 0.3207 | 0.3078 | 0.2953 | 0.2832 | 0.2714 | 0.2600 | 0.2489 | 0.2382 | 0.2278 | 0.2177 |
| 3                 | 0.3627                      | 0.3485 | 0.3347 | 0.3214 | 0.3084 | 0.2959 | 0.2837 | 0.2719 | 0.2605 | 0.2494 | 0.2386 | 0.2282 | 0.2181 |
| 4                 | 0.3639                      | 0.3496 | 0.3358 | 0.3224 | 0.3094 | 0.2967 | 0.2845 | 0.2726 | 0.2611 | 0.2500 | 0.2392 | 0.2287 | 0.2186 |
| 5                 | 0.3656                      | 0.3512 | 0.3372 | 0.3237 | 0.3105 | 0.2978 | 0.2855 | 0.2736 | 0.2620 | 0.2508 | 0.2399 | 0.2294 | 0.2192 |
| 6                 | 0.3676                      | 0.3531 | 0.3389 | 0.3253 | 0.3120 | 0.2992 | 0.2868 | 0.2747 | 0.2631 | 0.2518 | 0.2409 | 0.2303 | 0.2200 |
| 7                 | 0.3701                      | 0.3553 | 0.3410 | 0.3272 | 0.3138 | 0.3008 | 0.2883 | 0.2761 | 0.2644 | 0.2530 | 0.2420 | 0.2313 | 0.2209 |
| 8                 | 0.3730                      | 0.3580 | 0.3435 | 0.3294 | 0.3159 | 0.3027 | 0.2900 | 0.2778 | 0.2659 | 0.2544 | 0.2432 | 0.2325 | 0.2220 |
| 9                 | 0.3764                      | 0.3611 | 0.3463 | 0.3320 | 0.3182 | 0.3049 | 0.2921 | 0.2796 | 0.2676 | 0.2560 | 0.2447 | 0.2338 | 0.2233 |
| 10                | 0.3802                      | 0.3646 | 0.3495 | 0.3350 | 0.3210 | 0.3074 | 0.2944 | 0.2818 | 0.2696 | 0.2578 | 0.2464 | 0.2354 | 0.2247 |
| 11                | 0.3846                      | 0.3686 | 0.3532 | 0.3383 | 0.3241 | 0.3103 | 0.2970 | 0.2841 | 0.2718 | 0.2598 | 0.2482 | 0.2371 | 0.2263 |
| 12                | 0.3896                      | 0.3731 | 0.3573 | 0.3421 | 0.3275 | 0.3134 | 0.2999 | 0.2868 | 0.2742 | 0.2621 | 0.2503 | 0.2390 | 0.2281 |
| 13                | 0.3952                      | 0.3782 | 0.3620 | 0.3464 | 0.3314 | 0.3170 | 0.3031 | 0.2898 | 0.2770 | 0.2646 | 0.2527 | 0.2412 | 0.2301 |
| 14                | 0.4015                      | 0.3839 | 0.3671 | 0.3511 | 0.3357 | 0.3209 | 0.3068 | 0.2931 | 0.2800 | 0.2674 | 0.2552 | 0.2435 | 0.2322 |
| 15                | 0.4086                      | 0.3903 | 0.3729 | 0.3564 | 0.3405 | 0.3253 | 0.3108 | 0.2968 | 0.2834 | 0.2705 | 0.2581 | 0.2461 | 0.2346 |
| 16                | 0.4165                      | 0.3975 | 0.3794 | 0.3622 | 0.3458 | 0.3302 | 0.3152 | 0.3008 | 0.2871 | 0.2739 | 0.2612 | 0.2490 | 0.2373 |
| 17                | 0.4255                      | 0.4056 | 0.3867 | 0.3688 | 0.3518 | 0.3356 | 0.3201 | 0.3053 | 0.2911 | 0.2776 | 0.2646 | 0.2521 | 0.2401 |
| 18                | 0.4357                      | 0.4146 | 0.3948 | 0.3761 | 0.3584 | 0.3415 | 0.3255 | 0.3102 | 0.2956 | 0.2817 | 0.2683 | 0.2555 | 0.2433 |
| 19                | 0.4473                      | 0.4249 | 0.4039 | 0.3842 | 0.3657 | 0.3481 | 0.3315 | 0.3156 | 0.3006 | 0.2862 | 0.2724 | 0.2593 | 0.2467 |
| 20                | 0.4605                      | 0.4365 | 0.4142 | 0.3934 | 0.3739 | 0.3555 | 0.3381 | 0.3216 | 0.3060 | 0.2911 | 0.2769 | 0.2634 | 0.2504 |
| 21                | 0.4758                      | 0.4498 | 0.4259 | 0.4037 | 0.3830 | 0.3637 | 0.3455 | 0.3283 | 0.3120 | 0.2965 | 0.2818 | 0.2678 | 0.2545 |
| 22                | 0.4936                      | 0.4651 | 0.4392 | 0.4154 | 0.3934 | 0.3729 | 0.3537 | 0.3356 | 0.3186 | 0.3025 | 0.2872 | 0.2727 | 0.2590 |
| 23                | 0.5147                      | 0.4829 | 0.4545 | 0.4287 | 0.4050 | 0.3832 | 0.3628 | 0.3438 | 0.3259 | 0.3091 | 0.2932 | 0.2781 | 0.2638 |
| 24                | 0.5404                      | 0.5041 | 0.4724 | 0.4440 | 0.4183 | 0.3948 | 0.3731 | 0.3529 | 0.3341 | 0.3164 | 0.2997 | 0.2840 | 0.2692 |
| 25                | 0.5727                      | 0.5299 | 0.4936 | 0.4619 | 0.4336 | 0.4081 | 0.3847 | 0.3631 | 0.3431 | 0.3245 | 0.3070 | 0.2905 | 0.2750 |

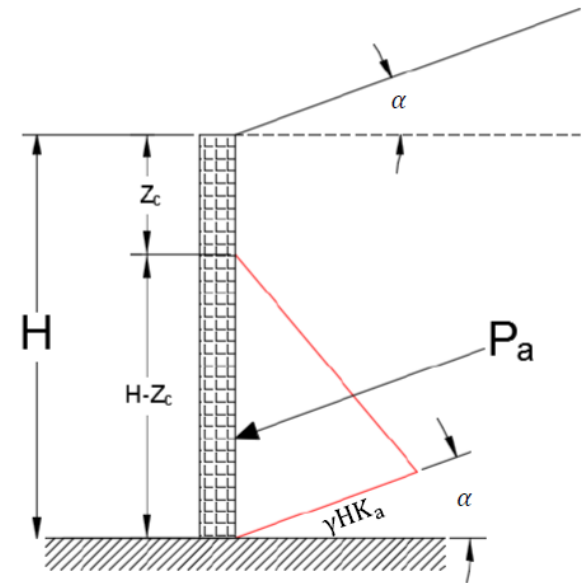
# Vertical Wall with Inclined Backfill

**$(c' - \phi')$  backfill**

$$\sigma'_a = \gamma z K'_a = \gamma z K'_a \cos \alpha$$

$$Z_c = \frac{2c}{\gamma} \sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}$$

$$P_a = \frac{1}{2} \times (\gamma H K'_a \cos \alpha) \times (H - Z_c)$$



$$K'_a = \frac{1}{\cos^2 \phi'} \left\{ \frac{2 \cos^2 \alpha + 2 \left( \frac{c'}{\gamma z} \right) \cos \phi' \sin \phi'}{-\sqrt{\left[ 4 \cos^2 \alpha (\cos^2 \alpha - \cos^2 \phi') + 4 \left( \frac{c'}{\gamma z} \right)^2 \cos^2 \phi' + 8 \left( \frac{c'}{\gamma z} \right) \cos^2 \alpha \sin \phi' \cos \phi' \right]}} \right\} - 1$$

Table 16.4 gives the variation of  $K'_a$

# Vertical Wall with Inclined Backfill

TABLE 16.4 Values of  $K_a$

| $\alpha$ (deg) | $\phi'$ (deg) | $c'/\gamma z$ |        |        |        |        |        |         |
|----------------|---------------|---------------|--------|--------|--------|--------|--------|---------|
|                |               | 0             | 0.025  | 0.05   | 0.075  | 0.1    | 0.2    | 0.5     |
| 0              | 15            | 0.5888        | 0.5504 | 0.5121 | 0.4737 | 0.4353 | 0.2819 | -0.1785 |
| 0              | 20            | 0.4903        | 0.4553 | 0.4203 | 0.3853 | 0.3502 | 0.2102 | -0.2099 |
| 0              | 25            | 0.4059        | 0.3740 | 0.3422 | 0.3103 | 0.2784 | 0.1510 | -0.2312 |
| 0              | 30            | 0.3333        | 0.3045 | 0.2756 | 0.2467 | 0.2179 | 0.1024 | -0.2440 |
| 0              | 35            | 0.2710        | 0.2450 | 0.2189 | 0.1929 | 0.1669 | 0.0628 | -0.2496 |
| 0              | 40            | 0.2174        | 0.1941 | 0.1708 | 0.1475 | 0.1242 | 0.0309 | -0.2489 |
| 5              | 15            | 0.6069        | 0.5658 | 0.5252 | 0.4849 | 0.4449 | 0.2867 | -0.1804 |
| 5              | 20            | 0.5015        | 0.4650 | 0.4287 | 0.3925 | 0.3565 | 0.2133 | -0.2119 |
| 5              | 25            | 0.4133        | 0.3805 | 0.3478 | 0.3152 | 0.2826 | 0.1530 | -0.2332 |
| 5              | 30            | 0.3385        | 0.3090 | 0.2795 | 0.2501 | 0.2207 | 0.1036 | -0.2460 |
| 5              | 35            | 0.2746        | 0.2481 | 0.2217 | 0.1952 | 0.1688 | 0.0634 | -0.2515 |
| 5              | 40            | 0.2200        | 0.1964 | 0.1727 | 0.1491 | 0.1255 | 0.0312 | -0.2507 |
| 10             | 15            | 0.6738        | 0.6206 | 0.5707 | 0.5230 | 0.4769 | 0.3022 | -0.1861 |
| 10             | 20            | 0.5394        | 0.4974 | 0.4564 | 0.4162 | 0.3767 | 0.2230 | -0.2180 |
| 10             | 25            | 0.4376        | 0.4015 | 0.3660 | 0.3308 | 0.2960 | 0.1590 | -0.2394 |
| 10             | 30            | 0.3549        | 0.3233 | 0.2919 | 0.2607 | 0.2297 | 0.1072 | -0.2522 |
| 10             | 35            | 0.2861        | 0.2581 | 0.2303 | 0.2025 | 0.1749 | 0.0654 | -0.2575 |
| 10             | 40            | 0.2282        | 0.2034 | 0.1787 | 0.1541 | 0.1296 | 0.0321 | -0.2564 |
| 15             | 15            | 1.0000        | 0.7762 | 0.6834 | 0.6102 | 0.5464 | 0.3324 | -0.1962 |
| 15             | 20            | 0.6241        | 0.5666 | 0.5137 | 0.4639 | 0.4165 | 0.2411 | -0.2287 |
| 15             | 25            | 0.4860        | 0.4428 | 0.4011 | 0.3606 | 0.3211 | 0.1700 | -0.2503 |
| 15             | 30            | 0.3861        | 0.3502 | 0.3150 | 0.2803 | 0.2462 | 0.1137 | -0.2628 |
| 15             | 35            | 0.3073        | 0.2764 | 0.2459 | 0.2158 | 0.1860 | 0.0690 | -0.2678 |
| 15             | 40            | 0.2429        | 0.2161 | 0.1895 | 0.1632 | 0.1370 | 0.0338 | -0.2662 |
| 20             | 20            | 1.0000        | 0.7432 | 0.6403 | 0.5608 | 0.4927 | 0.2718 | -0.2449 |
| 20             | 25            | 0.5820        | 0.5207 | 0.4650 | 0.4133 | 0.3645 | 0.1879 | -0.2665 |
| 20             | 30            | 0.4408        | 0.3964 | 0.3539 | 0.3130 | 0.2734 | 0.1241 | -0.2787 |
| 20             | 35            | 0.3423        | 0.3064 | 0.2713 | 0.2371 | 0.2035 | 0.0746 | -0.2831 |
| 20             | 40            | 0.2665        | 0.2363 | 0.2066 | 0.1773 | 0.1484 | 0.0363 | -0.2807 |

# EXAMPLE 16.4

## EXAMPLE 16.4

Refer to the retaining wall in Figure 16.10. The backfill is granular soil. Given:

$$\begin{array}{ll}\text{Wall:} & H = 3.05 \text{ m} \\ & \theta = +10^\circ \\ \text{Backfill:} & \alpha = 15^\circ \\ & \phi' = 35^\circ \\ & c' = 0 \\ & \gamma = 17.29 \text{ kN/m}^3\end{array}$$

Determine the Rankine active force,  $P_a$ , and its location and direction.

### SOLUTION

From Table 16.1, for  $\alpha = 15^\circ$  and  $\theta = +10^\circ$ , the value of  $K_{a(R)} \approx 0.42$ . From Eq. (12.16),

$$P_a = \frac{1}{2} \gamma H^2 K_{a(R)} = \left( \frac{1}{2} \right) (17.29)(3.05)^2 (0.42) = 33.78 \text{ kN/m}$$

Again, from Table 16.2, for  $\alpha = 15^\circ$  and  $\theta = +10^\circ$ ,  $\beta'_a \approx 30.5^\circ$ .

The force  $P_a$  will act at a distance of  $3.05/3 = 1.02$  m above the bottom of the wall and will be inclined at an angle of  $+30.5^\circ$  to the normal drawn to the backface of the wall.

# EXAMPLE 16.5

## EXAMPLE 16.5

For the retaining wall shown in Figure 16.11,  $H = 7.5$  m,  $\gamma = 18$  kN/m<sup>3</sup>,  $\phi' = 20^\circ$ ,  $c' = 13.5$  kN/m<sup>2</sup>, and  $\alpha = 10^\circ$ . Calculate the Rankine active force,  $P_a$ , per unit length of the wall and the location of the resultant force after the occurrence of the tensile crack.

### SOLUTION

From Eq. (16.34).

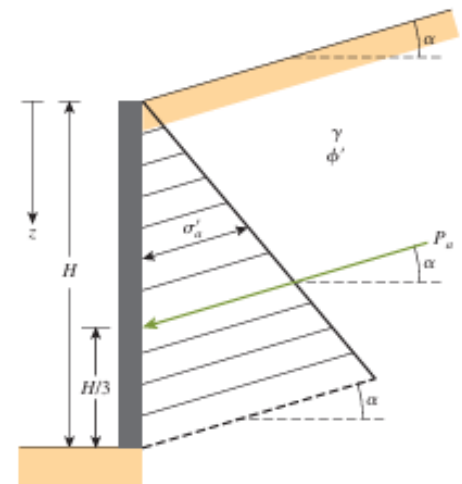
$$z_r = \frac{2c'}{\gamma} \sqrt{\frac{1 + \sin \phi'}{1 - \sin \phi'}} = \frac{(2)(13.5)}{18} \sqrt{\frac{1 + \sin 20}{1 - \sin 20}} = 2.14 \text{ m}$$

At  $z = 7.5$  m,

$$\frac{c'}{\gamma z} = \frac{13.5}{(18)(7.5)} = 0.1$$

From Table 16.4, for  $\phi' = 20^\circ$ ,  $c'/\gamma z = 0.1$ , and  $\alpha = 10^\circ$ , the value of  $K_a' \approx 0.377$ , so at  $z = 7.5$  m,

$$\sigma'_a = \gamma z K_a' \cos \alpha = (18)(7.5)(0.377)(\cos 10) = 50.1 \text{ kN/m}^2$$



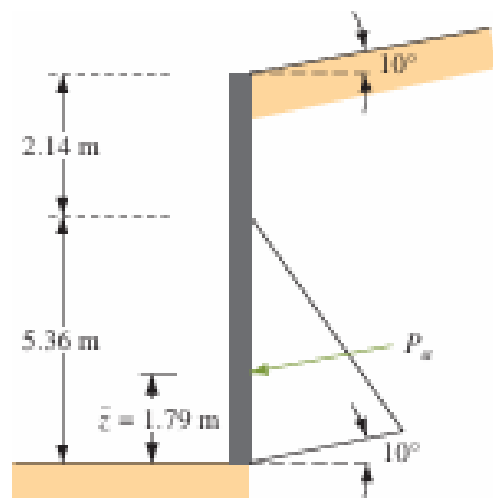
## EXAMPLE 16.5

After the occurrence of the tensile crack, the pressure distribution on the wall will be as shown in Figure 16.13, so

$$P_a = \left(\frac{1}{2}\right)(50.1)(7.5 - 2.14) = 134.3 \text{ kN/m}$$

and

$$\bar{z} = \frac{7.5 - 2.14}{3} = 1.79 \text{ m}$$

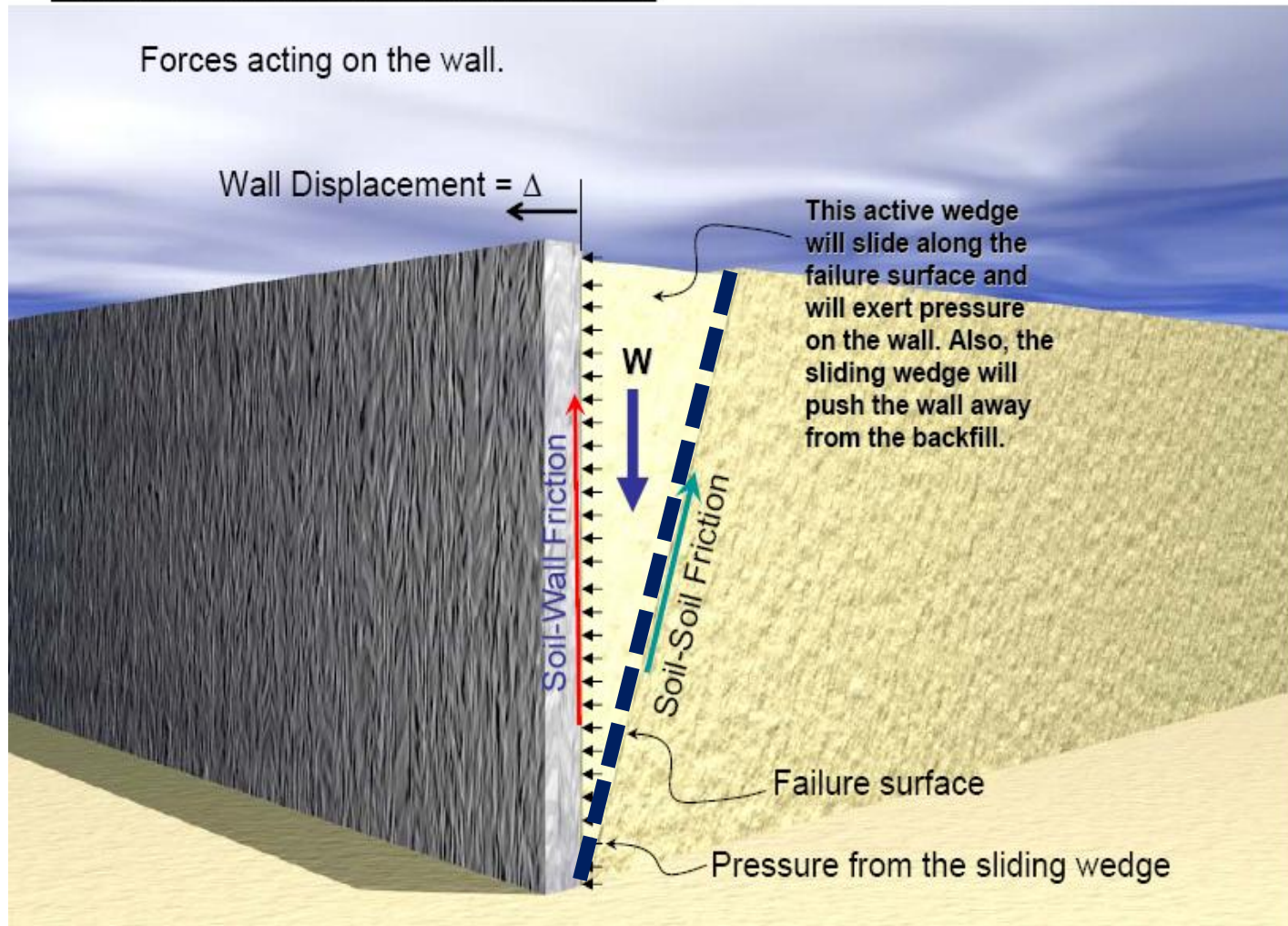


**FIGURE 16.13** Calculation of Rankine active force,  $c' = \phi'$  soil

# COULOMB'S EARTH PRESSURE

## Coulomb Earth Pressure Method (1776)

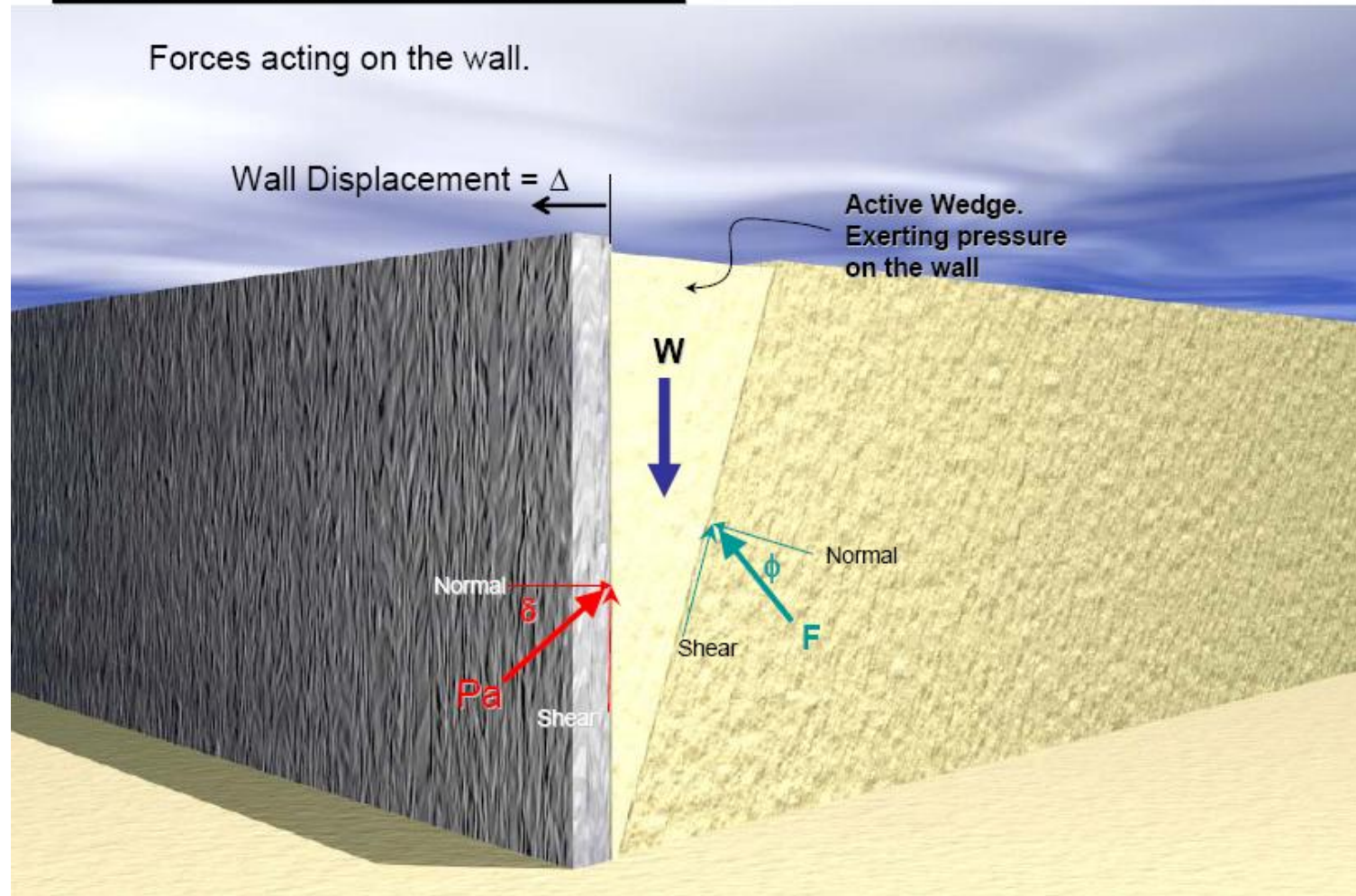
Forces acting on the wall.



# COULOMB'S EARTH PRESSURE

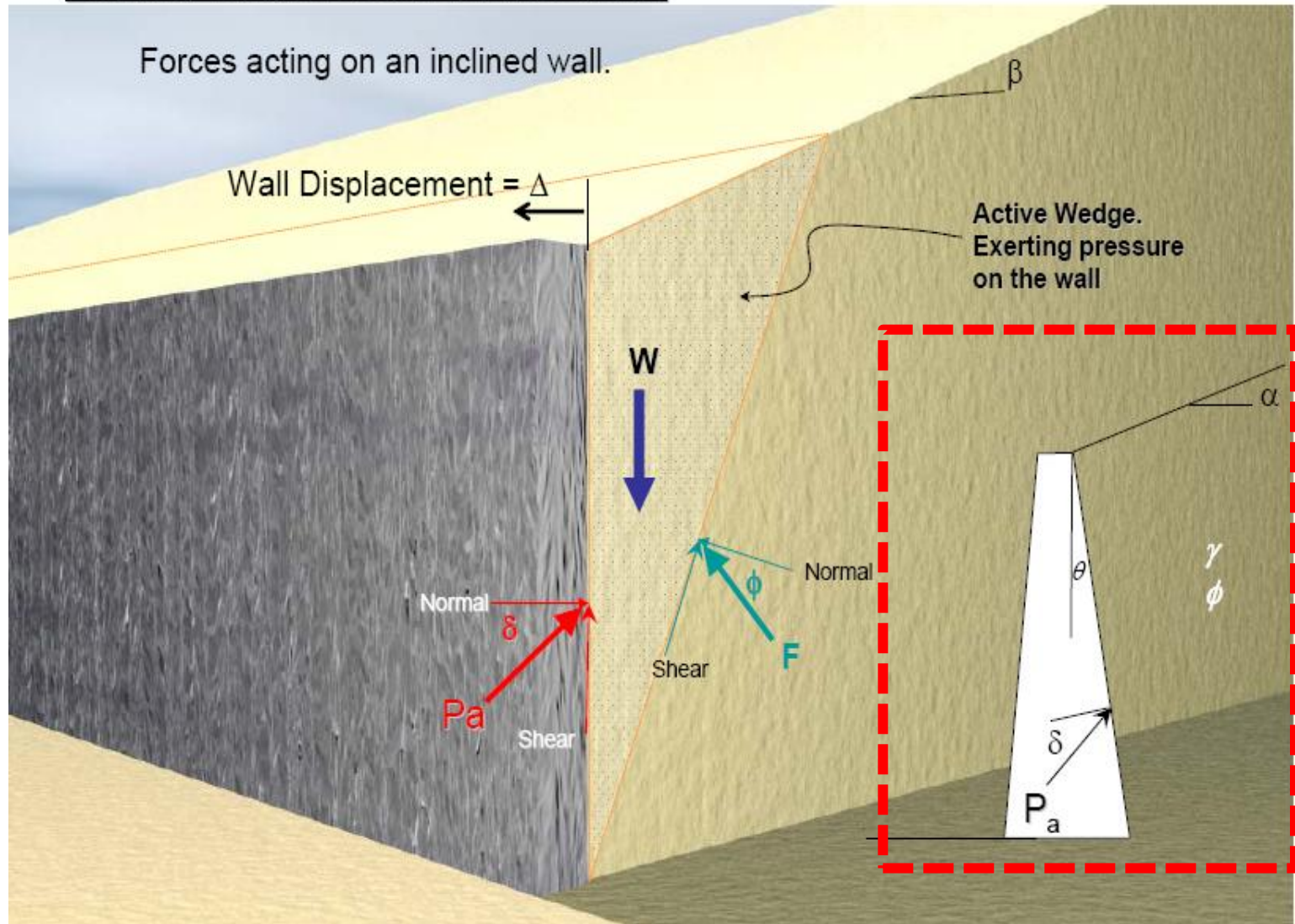
## Coulomb Earth Pressure Method (1776)

Forces acting on the wall.



# COULOMB'S EARTH PRESSURE

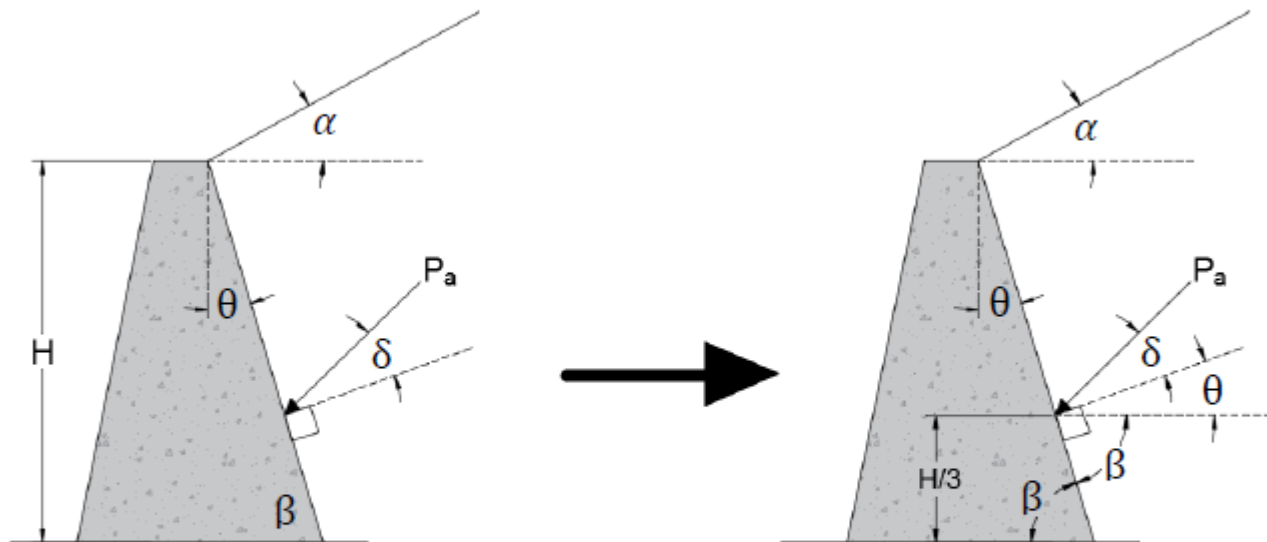
## Coulomb Earth Pressure Method (1776)



# Coulomb's Active Earth Pressure

## Granular backfill ( $c' = 0$ )

General case (inclined wall and inclined backfill):



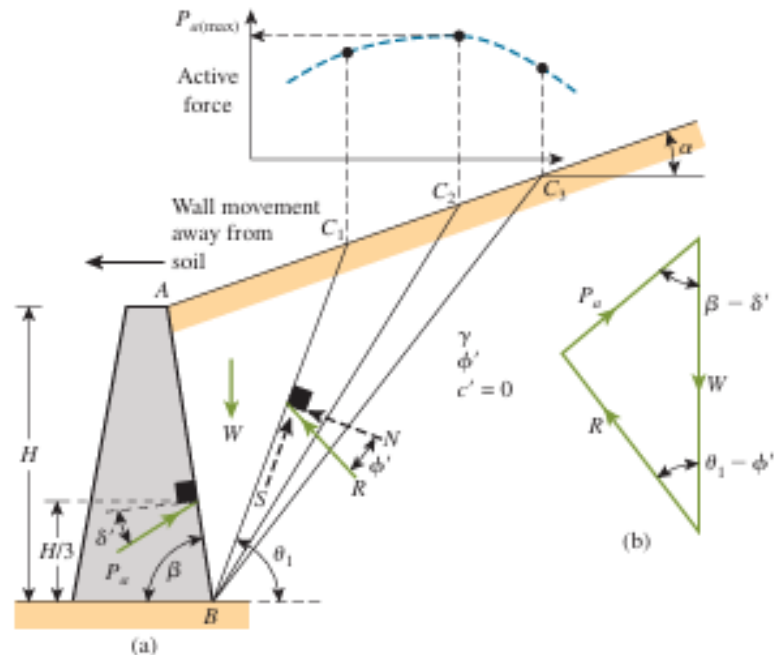
$\alpha$  = inclination of backfill with horizontal

$\theta$  = inclination of wall with vertical

$\beta$  = inclination of wall with the horizontal

$\delta$  = friction angle between soil and wall

# Coulomb's Active Earth Pressure



The forces acting on this wedge (per unit length at right angles to the cross section shown) are as follows:

1. The weight of the wedge,  $W$ .
2. The resultant,  $R$ , of the normal and resisting shear forces along the surface,  $BC_1$ . The force  $R$  will be inclined at an angle  $\phi'$  to the normal drawn to  $BC_1$ .
3. The active force per unit length of the wall,  $P_a$ , which will be inclined at an angle  $\delta'$  to the normal drawn to the backface of the wall.

# Coulomb's Active Earth Pressure

$$P_a = \frac{1}{2} K_a \gamma H^2$$

$K_a$  = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2 (\beta + \phi')}{\sin^2 \beta \sin (\beta - \delta') \left[ 1 + \sqrt{\frac{\sin (\phi' + \delta') \sin (\phi' - \alpha)}{\sin (\beta - \delta') \sin (\alpha + \beta)}} \right]^2}$$

**When  $\alpha = 0^\circ$ ,  $\beta = 90^\circ$ ,  $\delta' = 0^\circ$ , Coulomb's active earth pressure coefficient becomes equal to  $(1 - \sin \phi') / (1 + \sin \phi')$ , which is the same as Rankine's active earth pressure coefficient.**

# Coulomb's Active Earth Pressure

$K_a$  = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2 (\beta + \phi')}{\sin^2 \beta \sin (\beta - \delta') \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' - \alpha)}{\sin(\beta - \delta') \sin(\alpha + \beta)}} \right]^2}$$

The values of the active earth pressure coefficient,  $K_a$  for a vertical retaining wall,  $\beta = 90^\circ$  with horizontal backfill  $\alpha = 0^\circ$  are given in Table 16.5

# Coulomb's Active Earth Pressure

TABLE 16.5 Values of  $K_a$  [Eq. (16.36)] for  $\beta = 90^\circ$  and  $\alpha = 0^\circ$

| $\phi'$ (deg) | $\delta'$ (deg) |        |        |        |        |        |
|---------------|-----------------|--------|--------|--------|--------|--------|
|               | 0               | 5      | 10     | 15     | 20     | 25     |
| 28            | 0.3610          | 0.3448 | 0.3330 | 0.3251 | 0.3203 | 0.3186 |
| 30            | 0.3333          | 0.3189 | 0.3085 | 0.3014 | 0.2973 | 0.2956 |
| 32            | 0.3073          | 0.2945 | 0.2853 | 0.2791 | 0.2755 | 0.2745 |
| 34            | 0.2827          | 0.2714 | 0.2633 | 0.2579 | 0.2549 | 0.2542 |
| 36            | 0.2596          | 0.2497 | 0.2426 | 0.2379 | 0.2354 | 0.2350 |
| 38            | 0.2379          | 0.2292 | 0.2230 | 0.2190 | 0.2169 | 0.2167 |
| 40            | 0.2174          | 0.2098 | 0.2045 | 0.2011 | 0.1994 | 0.1995 |
| 42            | 0.1982          | 0.1916 | 0.1870 | 0.1841 | 0.1828 | 0.1831 |

# Coulomb's Active Earth Pressure

$K_a$  = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2 (\beta + \phi')}{\sin^2 \beta \sin (\beta - \delta') \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' - \alpha)}{\sin(\beta - \delta') \sin(\alpha + \beta)}} \right]^2}$$

The wall friction angle  $\delta'$  can be determined in the laboratory by means of **direct** shear test. It is assumed to be between  $\phi'/2$  and  $2\phi'/3$

angle  $\delta' = 2\phi'/3$  Table 16.6

angle  $\delta' = \phi'/2$  Table 16.7

# Coulomb's Active Earth Pressure

**TABLE 16.6** Values of  $K_a$  [from Eq. (16.36)] for  $\delta' = \frac{2}{3} \phi'$

| $\alpha$ (deg) | $\phi'$ (deg) | $\beta$ (deg) |        |        |        |        |        |
|----------------|---------------|---------------|--------|--------|--------|--------|--------|
|                |               | 90            | 85     | 80     | 75     | 70     | 65     |
| 0              | 28            | 0.3213        | 0.3588 | 0.4007 | 0.4481 | 0.5026 | 0.5662 |
|                | 29            | 0.3091        | 0.3467 | 0.3886 | 0.4362 | 0.4908 | 0.5547 |
|                | 30            | 0.2973        | 0.3349 | 0.3769 | 0.4245 | 0.4794 | 0.5435 |
|                | 31            | 0.2860        | 0.3235 | 0.3655 | 0.4133 | 0.4682 | 0.5326 |
|                | 32            | 0.2750        | 0.3125 | 0.3545 | 0.4023 | 0.4574 | 0.5220 |
|                | 33            | 0.2645        | 0.3019 | 0.3439 | 0.3917 | 0.4469 | 0.5117 |
|                | 34            | 0.2543        | 0.2916 | 0.3335 | 0.3813 | 0.4367 | 0.5017 |
|                | 35            | 0.2444        | 0.2816 | 0.3235 | 0.3713 | 0.4267 | 0.4919 |
|                | 36            | 0.2349        | 0.2719 | 0.3137 | 0.3615 | 0.4170 | 0.4824 |
|                | 37            | 0.2257        | 0.2626 | 0.3042 | 0.3520 | 0.4075 | 0.4732 |
|                | 38            | 0.2168        | 0.2535 | 0.2950 | 0.3427 | 0.3983 | 0.4641 |
|                | 39            | 0.2082        | 0.2447 | 0.2861 | 0.3337 | 0.3894 | 0.4553 |
|                | 40            | 0.1998        | 0.2361 | 0.2774 | 0.3249 | 0.3806 | 0.4468 |
|                | 41            | 0.1918        | 0.2278 | 0.2689 | 0.3164 | 0.3721 | 0.4384 |
|                | 42            | 0.1840        | 0.2197 | 0.2606 | 0.3080 | 0.3637 | 0.4302 |
| 5              | 28            | 0.3431        | 0.3845 | 0.4311 | 0.4843 | 0.5461 | 0.6190 |
|                | 29            | 0.3295        | 0.3709 | 0.4175 | 0.4707 | 0.5325 | 0.6056 |
|                | 30            | 0.3165        | 0.3578 | 0.4043 | 0.4575 | 0.5194 | 0.5926 |
|                | 31            | 0.3039        | 0.3451 | 0.3916 | 0.4447 | 0.5067 | 0.5800 |
|                | 32            | 0.2919        | 0.3329 | 0.3792 | 0.4324 | 0.4943 | 0.5677 |
|                | 33            | 0.2803        | 0.3211 | 0.3673 | 0.4204 | 0.4823 | 0.5558 |
|                | 34            | 0.2691        | 0.3097 | 0.3558 | 0.4088 | 0.4707 | 0.5443 |
|                | 35            | 0.2583        | 0.2987 | 0.3446 | 0.3975 | 0.4594 | 0.5330 |
|                | 36            | 0.2479        | 0.2881 | 0.3338 | 0.3866 | 0.4484 | 0.5221 |
|                | 37            | 0.2379        | 0.2778 | 0.3233 | 0.3759 | 0.4377 | 0.5115 |
|                | 38            | 0.2282        | 0.2679 | 0.3131 | 0.3656 | 0.4273 | 0.5012 |
|                | 39            | 0.2188        | 0.2582 | 0.3033 | 0.3556 | 0.4172 | 0.4911 |
|                | 40            | 0.2098        | 0.2489 | 0.2937 | 0.3458 | 0.4074 | 0.4813 |
|                | 41            | 0.2011        | 0.2398 | 0.2844 | 0.3363 | 0.3978 | 0.4718 |
|                | 42            | 0.1927        | 0.2311 | 0.2753 | 0.3271 | 0.3884 | 0.4625 |

# Coulomb's Active Earth Pressure

**TABLE 16.7** Values of  $K_a$  [from Eq. (16.36)] for  $\delta' = \frac{1}{2} \phi'$

| $\alpha$ (deg) | $\phi'$ (deg) | $\beta$ (deg) |        |        |        |        |        |
|----------------|---------------|---------------|--------|--------|--------|--------|--------|
|                |               | 90            | 85     | 80     | 75     | 70     | 65     |
| 0              | 28            | 0.3264        | 0.3629 | 0.4034 | 0.4490 | 0.5011 | 0.5616 |
|                | 29            | 0.3137        | 0.3502 | 0.3907 | 0.4363 | 0.4886 | 0.5492 |
|                | 30            | 0.3014        | 0.3379 | 0.3784 | 0.4241 | 0.4764 | 0.5371 |
|                | 31            | 0.2896        | 0.3260 | 0.3665 | 0.4121 | 0.4645 | 0.5253 |
|                | 32            | 0.2782        | 0.3145 | 0.3549 | 0.4005 | 0.4529 | 0.5137 |
|                | 33            | 0.2671        | 0.3033 | 0.3436 | 0.3892 | 0.4415 | 0.5025 |
|                | 34            | 0.2564        | 0.2925 | 0.3327 | 0.3782 | 0.4305 | 0.4915 |
|                | 35            | 0.2461        | 0.2820 | 0.3221 | 0.3675 | 0.4197 | 0.4807 |
|                | 36            | 0.2362        | 0.2718 | 0.3118 | 0.3571 | 0.4092 | 0.4702 |
|                | 37            | 0.2265        | 0.2620 | 0.3017 | 0.3469 | 0.3990 | 0.4599 |
|                | 38            | 0.2172        | 0.2524 | 0.2920 | 0.3370 | 0.3890 | 0.4498 |
|                | 39            | 0.2081        | 0.2431 | 0.2825 | 0.3273 | 0.3792 | 0.4400 |
|                | 40            | 0.1994        | 0.2341 | 0.2732 | 0.3179 | 0.3696 | 0.4304 |
|                | 41            | 0.1909        | 0.2253 | 0.2642 | 0.3087 | 0.3602 | 0.4209 |
|                | 42            | 0.1828        | 0.2168 | 0.2554 | 0.2997 | 0.3511 | 0.4177 |
| 5              | 28            | 0.3477        | 0.3879 | 0.4327 | 0.4837 | 0.5425 | 0.6115 |
|                | 29            | 0.3337        | 0.3737 | 0.4185 | 0.4694 | 0.5282 | 0.5972 |
|                | 30            | 0.3202        | 0.3601 | 0.4048 | 0.4556 | 0.5144 | 0.5833 |
|                | 31            | 0.3072        | 0.3470 | 0.3915 | 0.4422 | 0.5009 | 0.5698 |
|                | 32            | 0.2946        | 0.3342 | 0.3787 | 0.4292 | 0.4878 | 0.5566 |
|                | 33            | 0.2825        | 0.3219 | 0.3662 | 0.4166 | 0.4750 | 0.5437 |
|                | 34            | 0.2709        | 0.3101 | 0.3541 | 0.4043 | 0.4626 | 0.5312 |
|                | 35            | 0.2596        | 0.2986 | 0.3424 | 0.3924 | 0.4505 | 0.5190 |
|                | 36            | 0.2488        | 0.2874 | 0.3310 | 0.3808 | 0.4387 | 0.5070 |
|                | 37            | 0.2383        | 0.2767 | 0.3199 | 0.3695 | 0.4272 | 0.4954 |
|                | 38            | 0.2282        | 0.2662 | 0.3092 | 0.3585 | 0.4160 | 0.4840 |
|                | 39            | 0.2185        | 0.2561 | 0.2988 | 0.3478 | 0.4050 | 0.4729 |
| 10             | 40            | 0.2090        | 0.2463 | 0.2887 | 0.3374 | 0.3944 | 0.4620 |
|                | 41            | 0.1999        | 0.2368 | 0.2788 | 0.3273 | 0.3840 | 0.4514 |
|                | 42            | 0.1911        | 0.2276 | 0.2693 | 0.3174 | 0.3738 | 0.4410 |

# Coulomb's Active Earth Pressure

If a uniform surcharge of intensity  $q$  is located above the backfill

$$P_a = \frac{1}{2} K_a \gamma_{eq} H^2$$

$$\gamma_{eq} = \gamma + \left[ \frac{\sin \beta}{\sin(\beta + \alpha)} \right] \left( \frac{2q}{H} \right)$$

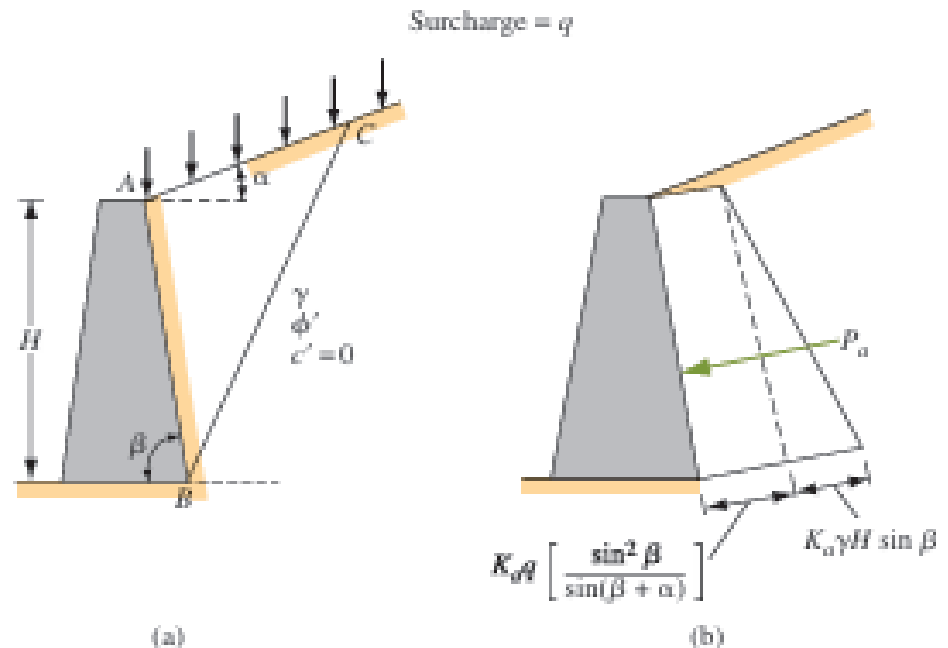


FIGURE 16.15 Coulomb's active pressure with a surcharge on the backfill

## EXAMPLE 16.6

### EXAMPLE 16.6

Consider the retaining wall shown in Figure 16.14a. Given:  $H = 5$  m; unit weight of soil =  $17.6$  kN/m<sup>3</sup>; angle of friction of soil =  $35^\circ$ ; wall friction angle,  $\delta' = \frac{2}{3}\phi'$ ; soil cohesion,  $c' = 0$ ;  $\alpha = 0$ ; and  $\beta = 90^\circ$ . Calculate the Coulomb's active force per unit length of the wall.

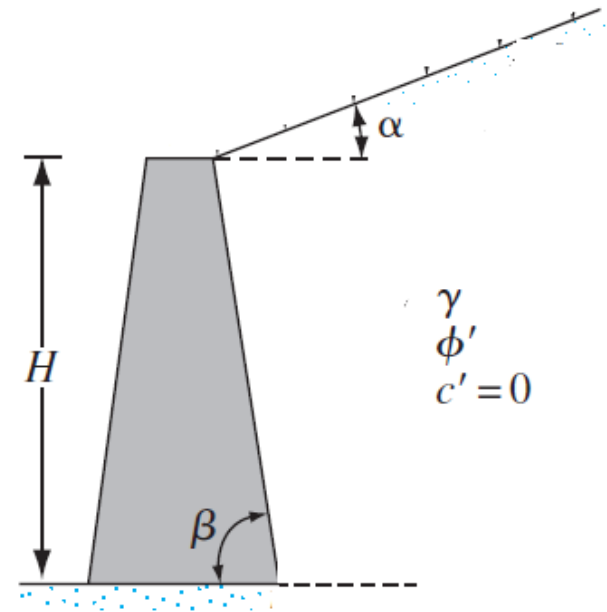
#### SOLUTION

From Eq. (16.35),

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

From Table 16.6, for  $\alpha = 0^\circ$ ,  $\beta = 90^\circ$ ,  $\phi' = 35^\circ$ , and  $\delta' = \frac{2}{3}\phi' = 23.33^\circ$ ,  $K_a = 0.2444$ . Hence,

$$P_a = \frac{1}{2}(17.6)(5)^2(0.2444) = 53.77 \text{ kN/m}$$



# EXAMPLE 16.7

## EXAMPLE 16.7

Refer to Figure 16.15a. Given:  $H = 6.1$  m,  $\phi' = 30^\circ$ ,  $\delta' = 20^\circ$ ,  $\alpha = 5^\circ$ ,  $\beta = 85^\circ$ ,  $q = 96$  kN/m<sup>2</sup>, and  $\gamma = 18$  kN/m<sup>3</sup>. Determine Coulomb's active force and the location of the line of action of the resultant  $P_a$ .

### SOLUTION

For  $\beta = 85^\circ$ ,  $\alpha = 5^\circ$ ,  $\delta' = 20^\circ$ ,  $\phi' = 30^\circ$ , and  $K_a = 0.3578$  (Table 16.6). From Eqs. (16.37) and (16.38),

$$\begin{aligned}
 P_a &= \frac{1}{2} K_a \gamma_{eq} H^2 = \frac{1}{2} K_a \left[ \gamma + \frac{2q}{H} \frac{\sin \beta}{\sin (\beta + \alpha)} \right] H^2 = \underbrace{\frac{1}{2} K_a \gamma H^2}_{P_{a(1)}} \\
 &\quad + \underbrace{K_a H q \left[ \frac{\sin \beta}{\sin (\beta + \alpha)} \right]}_{P_{a(2)}} \\
 &= (0.5)(0.3578)(18)(6.1)^2 + (0.3578)(6.1)(96) \left[ \frac{\sin 85}{\sin (85 + 5)} \right] \\
 &= 119.8 + 208.7 = \mathbf{328.5 \text{ kN/m}}
 \end{aligned}$$

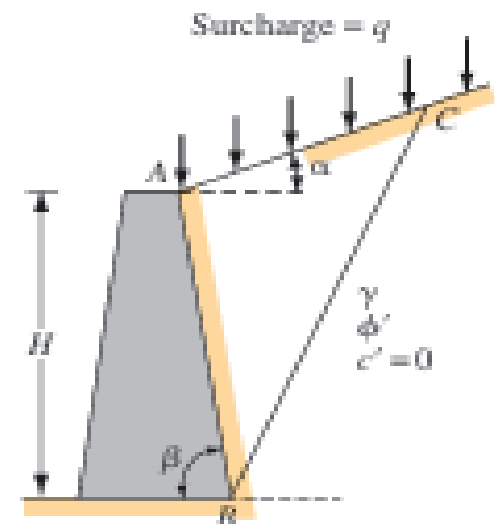
Location of the line of action of the resultant:

$$P_a \bar{z} = P_{a(1)} \frac{H}{3} + P_{a(2)} \frac{H}{2}$$

or

$$\bar{z} = \frac{(119.8) \left( \frac{6.1}{3} \right) + (208.7) \left( \frac{6.1}{2} \right)}{328.5}$$

= **2.68 m** (measured vertically from the bottom of the wall)

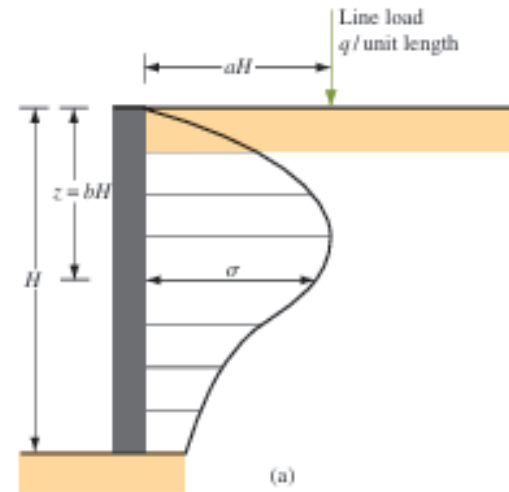


# Lateral Earth Pressure Due to Surcharge

Line load of intensity  $q$ /unit length

$$\sigma = \frac{2q}{\pi H} \frac{a^2 b}{(a^2 + b^2)^2}$$

$\sigma$  = horizontal stress at depth  $z = bH$



Because soil is not a perfectly elastic medium. The modified forms of the equation above are :

$$\sigma = \frac{4a}{\pi H} \frac{a^2 b}{(a^2 + b^2)} \quad \text{for } a > 0.4$$

$$\sigma = \frac{q}{H} \frac{0.203b}{(0.16 + b^2)^2} \quad \text{for } a \leq 0.4$$

# Lateral Earth Pressure Due to Surcharge

Strip load of intensity  $q$ /unit area

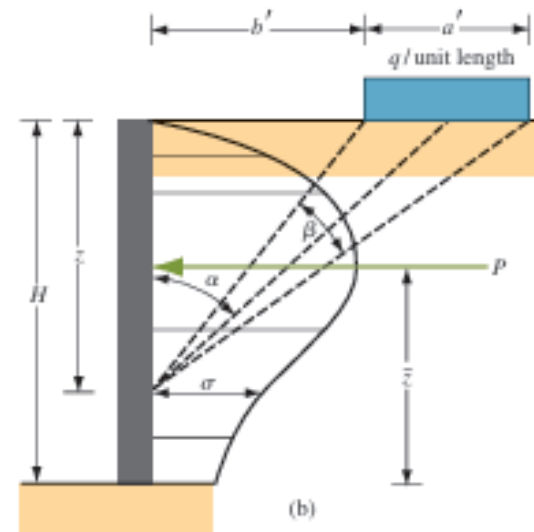
$$\sigma = \frac{2q}{\pi}(\beta - \sin \beta \cos 2\alpha)$$

The total force per unit length (P) due to the *strip loading only*

$$P = \frac{q}{90} [H(\theta_2 - \theta_1)]$$

$$\theta_1 = \tan^{-1} \left( \frac{b'}{H} \right) (\text{deg})$$

$$\theta_2 = \tan^{-1} \left( \frac{a' + b'}{H} \right) (\text{deg})$$



$$\bar{z} = H - \left[ \frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)} \right]$$

$$R = (a' + b')^2(90 - \theta_2)$$

$$Q = b'^2(90 - \theta_1)$$

# EXAMPLE 16.8

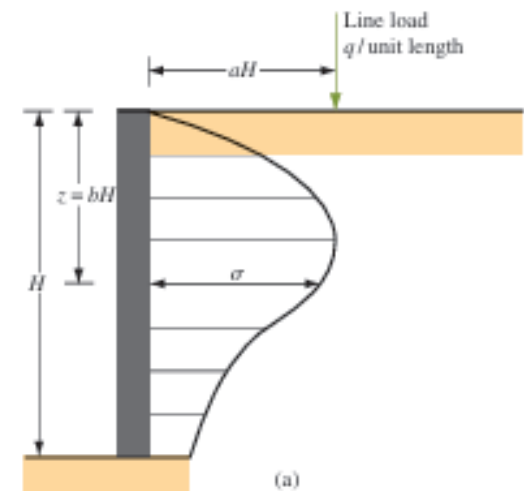
## EXAMPLE 16.8

Refer to Figure 16.16a, which shows a line load surcharge. Given:  $H = 6$  m,  $a = 0.25$ , and  $q = 3$  kN/m. Calculate the variation of the lateral stress  $\sigma$  on the retaining structure at  $z = 1, 2, 3, 4, 5$ , and  $6$  m.

### SOLUTION

For  $a = 0.25$ , which is less than  $0.4$ , we will use Eq. (16.41). Now the following table can be prepared.

| $z$ (m) | $H$ (m) | $b = z/H$ | $a$  | $\sigma$ (kN/m <sup>2</sup> ) |
|---------|---------|-----------|------|-------------------------------|
| 1       | 6       | 0.167     | 0.25 | 0.48                          |
| 2       | 6       | 0.333     | 0.25 | 0.46                          |
| 3       | 6       | 0.5       | 0.25 | 0.302                         |
| 4       | 6       | 0.667     | 0.25 | 0.185                         |
| 5       | 6       | 0.833     | 0.25 | 0.116                         |
| 6       | 6       | 1         | 0.25 | 0.073                         |



$$\sigma = \frac{q}{H} \frac{0.203b}{(0.16 + b^2)^2} \quad \text{for } a \leq 0.4$$

# EXAMPLE 16.9

## EXAMPLE 16.9

Refer to Figure 16.16b. Here,  $a' = 2$  m,  $b' = 1$  m,  $q = 40$  kN/m<sup>2</sup>, and  $H = 6$  m. Determine the total force on the wall (kN/m) caused by the strip loading only.

### SOLUTION

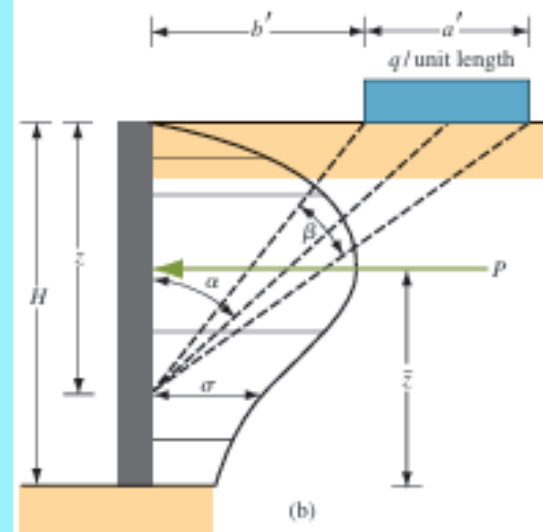
From Eqs. (16.45) and (16.46),

$$\theta_1 = \tan^{-1}\left(\frac{1}{6}\right) = 9.46^\circ$$

$$\theta_2 = \tan^{-1}\left(\frac{2+1}{6}\right) = 26.57^\circ$$

From Eq. (16.44),

$$P = \frac{q}{90} [H(\theta_2 - \theta_1)] = \frac{40}{90} [6(26.57 - 9.46)] = 45.63 \text{ kN/m}$$



# EXAMPLE 16.10

## EXAMPLE 16.10

Refer to Example 16.9. Determine the location of the resultant  $\bar{z}$ .

### SOLUTION

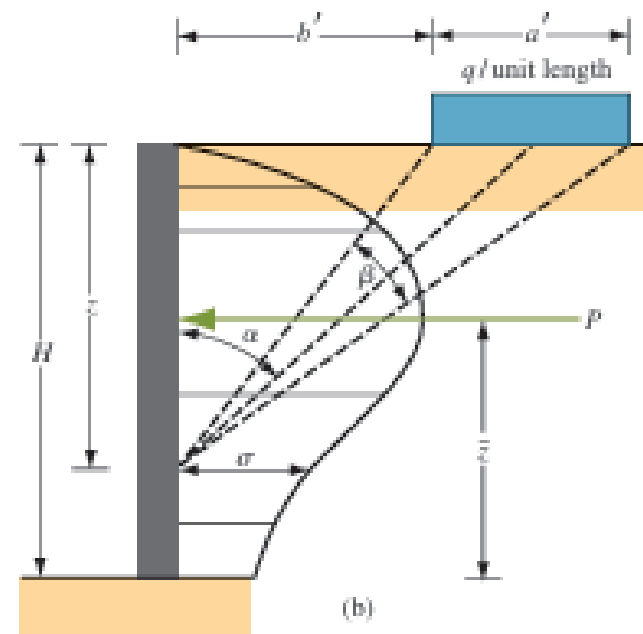
From Eqs. (16.48) and (16.49),

$$R = (a' + b')^2(90 - \theta_2) = (2 + 1)^2(90 - 26.57) = 570.87$$

$$Q = b'^2(90 - \theta_1) = (1)^2(90 - 9.46) = 80.54$$

From Eq. (16.47),

$$\begin{aligned}\bar{z} &= H - \left[ \frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)} \right] \\ &= 6 - \left[ \frac{(6)^2(26.57 - 9.46) + (570.87 - 80.54) - (57.3)(2)(6)}{(2)(6)(26.57 - 9.46)} \right] = 3.96 \text{ m}\end{aligned}$$

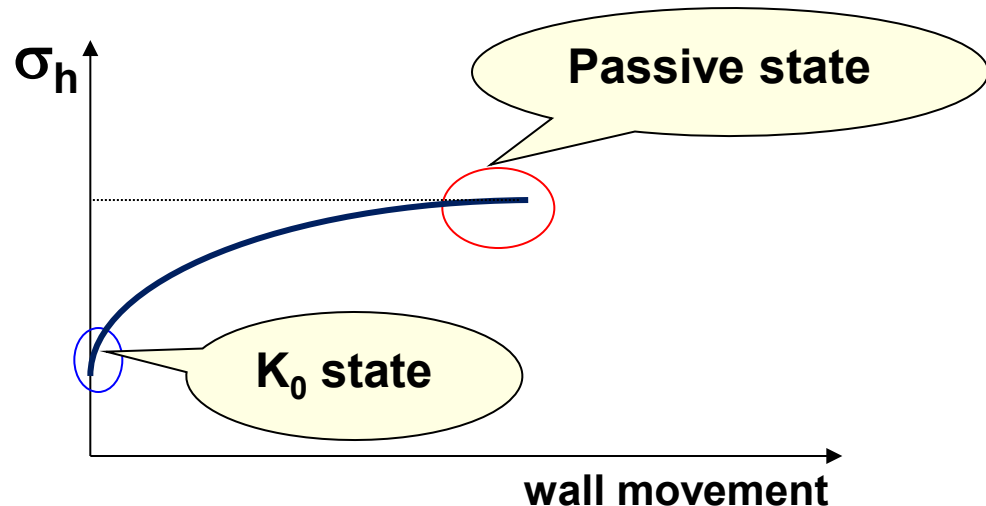
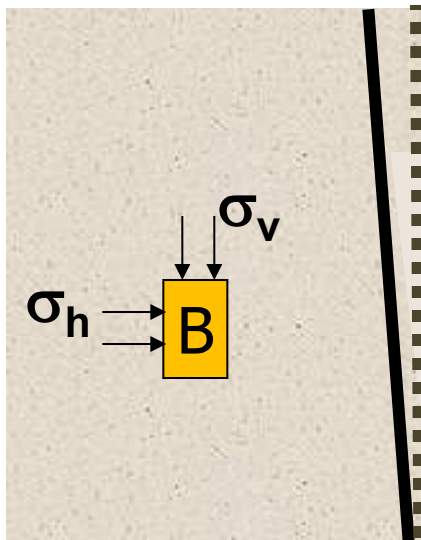


# Passive Earth Pressure

## Passive earth pressure

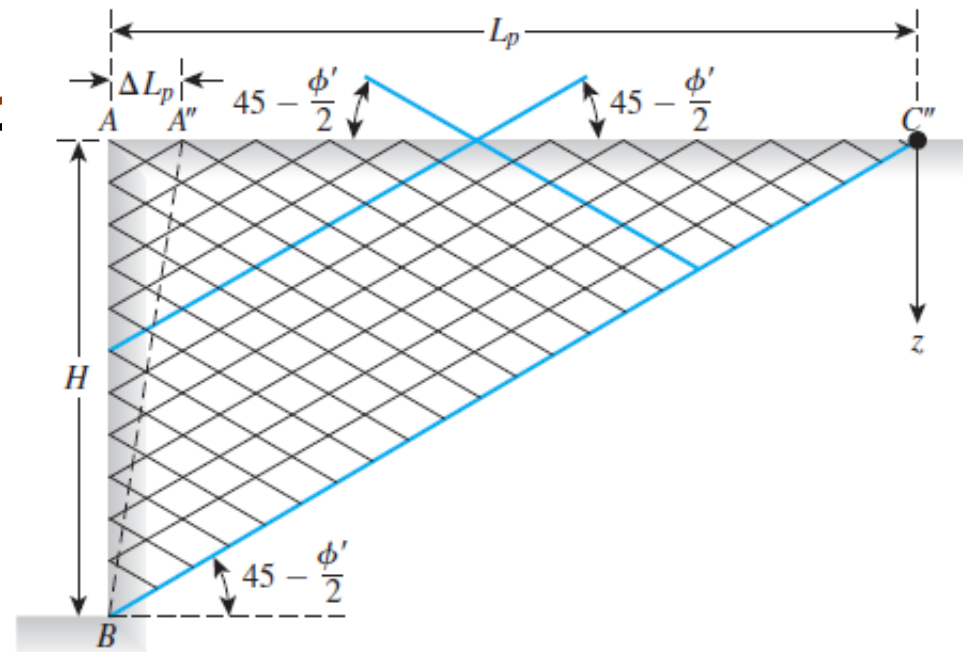
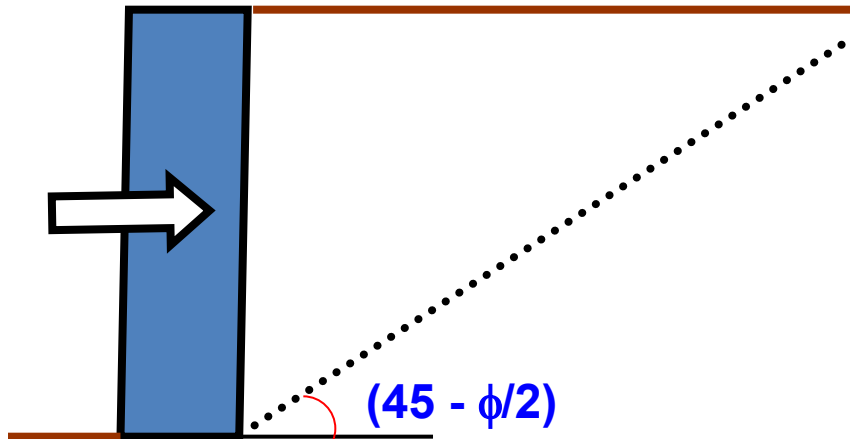
- Initially, soil is in  $K_0$  state.
- As the wall moves towards (pushed into) the soil mass,
- $\sigma_v$  remains the same, and
- $\sigma_h$  increases till failure occurs.  $\sigma_h \longrightarrow \sigma_p$

Passive state



# Orientation of Failure Planes

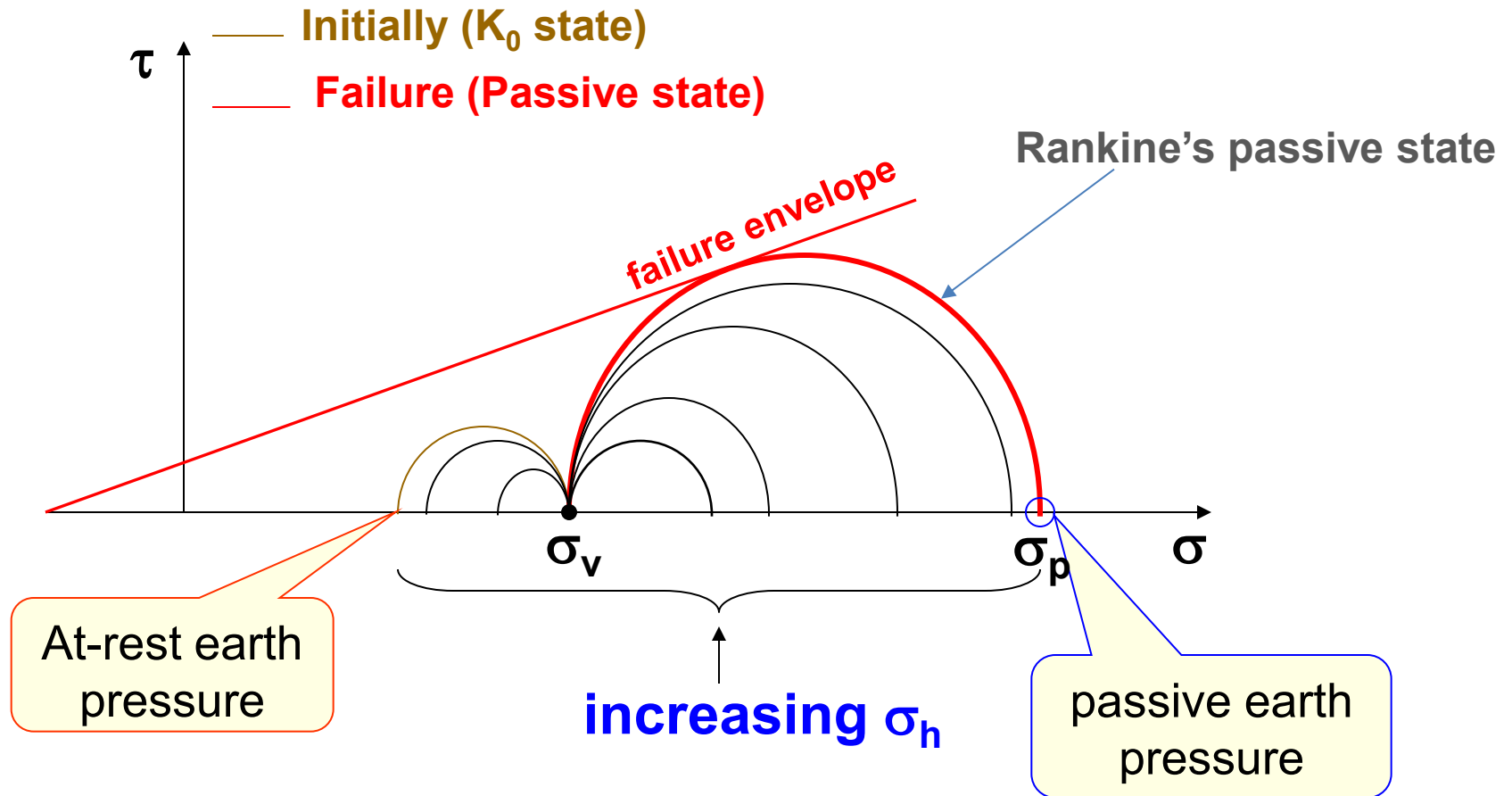
- From Mohr Circle the failure planes in the soil make  $\pm (45 - \phi/2)$ -degree angles with the direction of the major principal plane—that is, the horizontal.
- These are called potential *slip planes*.



- Because the slip planes make angles of  $(45 - \phi/2)$  degrees with the major principal plane, the soil mass in the state of plastic equilibrium is bounded by the plane  $BC'$ . The soil inside the zone  $ABC''$  undergoes the same unit deformation in the horizontal direction everywhere, which is equal to  $\Delta L_p/L_p$ .

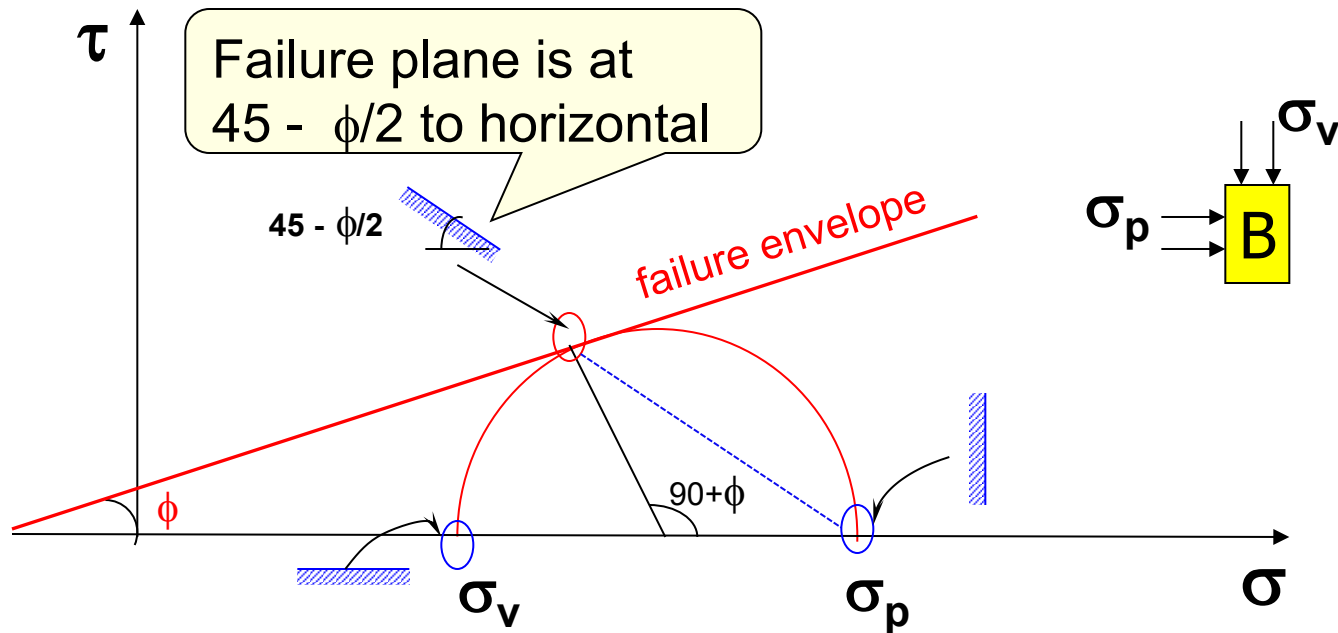
# Passive Earth Pressure

- As the wall moves towards the soil,

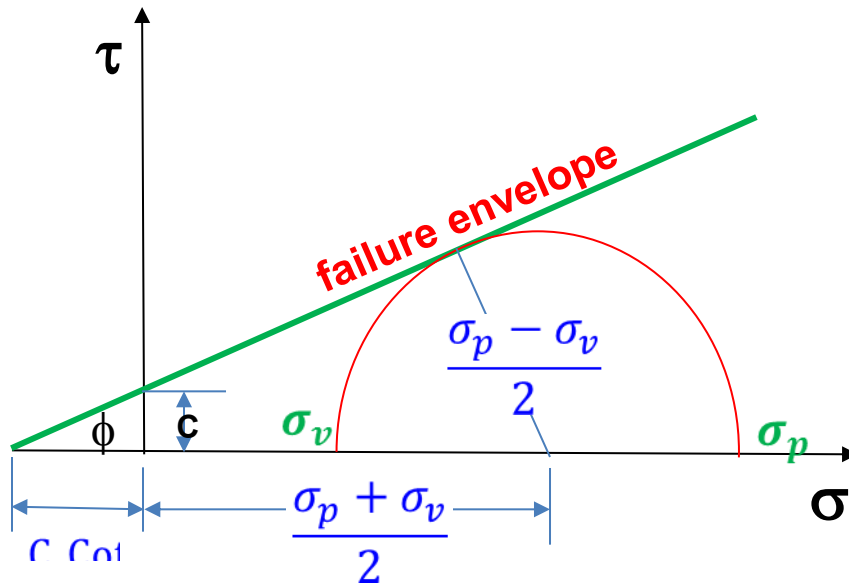


# Passive Earth Pressure

## Failure plane



# Passive Earth Pressure



$$\sin \phi = \frac{\frac{\sigma_p - \sigma_v}{2}}{c \cot \phi + \frac{\sigma_p + \sigma_v}{2}}$$

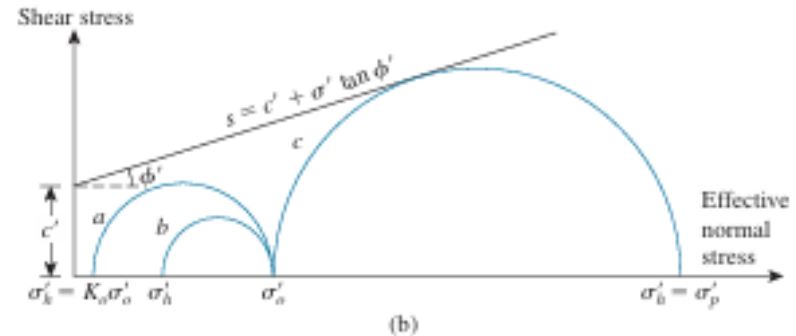
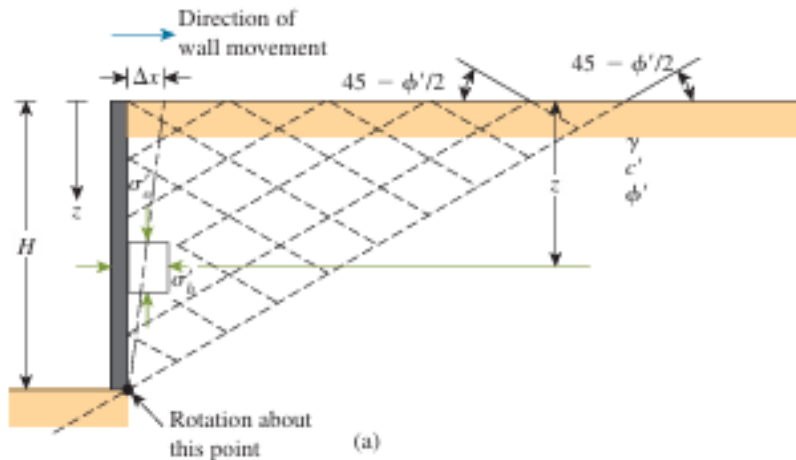
$$\sigma'_p = \sigma'_o \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

$$K_p = \text{Rankine passive earth pressure coefficient} \\ = \tan^2 \left( 45 + \frac{\phi'}{2} \right)$$

$$\sigma'_p = \sigma'_o K_p + 2c' \sqrt{K_p}$$

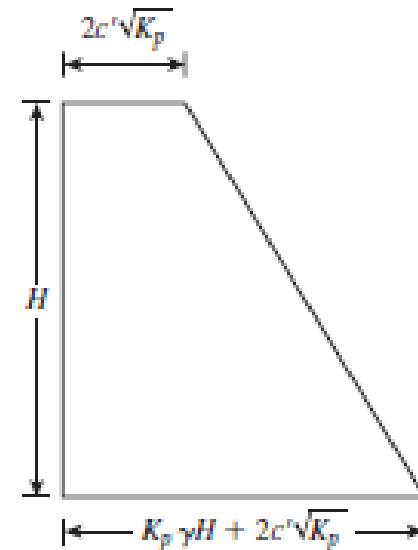
$$P_p = \frac{1}{2} \gamma H^2 K_p + 2c' H \sqrt{K_p}$$

# Passive Earth Pressure



$$\sigma_p' = \sigma_o' K_p + 2c'\sqrt{K_p}$$

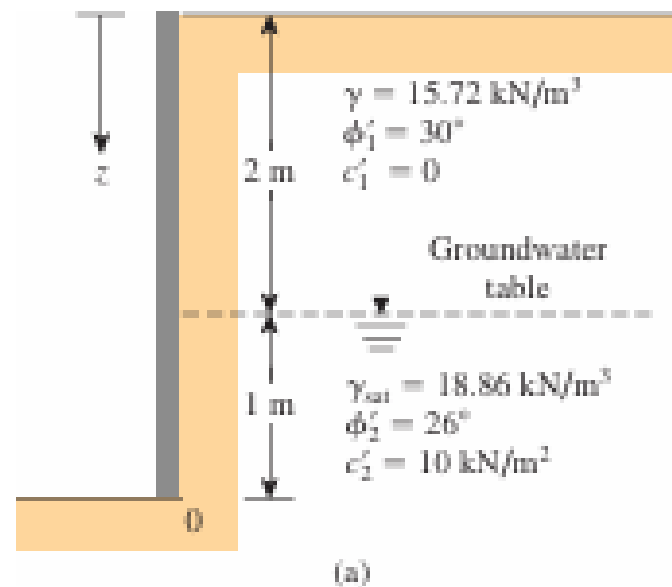
$$P_p = \frac{1}{2} \gamma H^2 K_p + 2c'H\sqrt{K_p}$$



# EXAMPLE 16.13

## EXAMPLE 16.13

A 3 m high wall is shown in Figure 16.22a. Determine the Rankine passive force per unit length of the wall.



# EXAMPLE 16.13

## SOLUTION

For the top layer,

$$K_{p(1)} = \tan^2\left(45 + \frac{\phi'_1}{2}\right) = \tan^2(45 + 15) = 3$$

From the bottom soil layer,

$$K_{p(2)} = \tan^2\left(45 + \frac{\phi'_2}{2}\right) = \tan^2(45 + 13) = 2.56$$

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$

where

$\sigma'_o$  = effective vertical stress

at  $z = 0$ ,  $\sigma'_o = 0$ ,  $c'_1 = 0$ ,  $\sigma'_p = 0$

at  $z = 2$  m,  $\sigma'_o = (15.72)(2) = 31.44$  kN/m<sup>2</sup>,  $c'_1 = 0$

So, for the top soil layer,

$$\sigma'_p = 31.44 K_{p(1)} + 2(0)\sqrt{K_{p(1)}} = 31.44(3) = 94.32 \text{ kN/m}^2$$

At this depth (that is,  $z = 2$  m), for the bottom soil layer,

$$\begin{aligned}\sigma'_p &= \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 31.44(2.56) + 2(10)\sqrt{2.56} \\ &= 80.49 + 32 = 112.49 \text{ kN/m}^2\end{aligned}$$

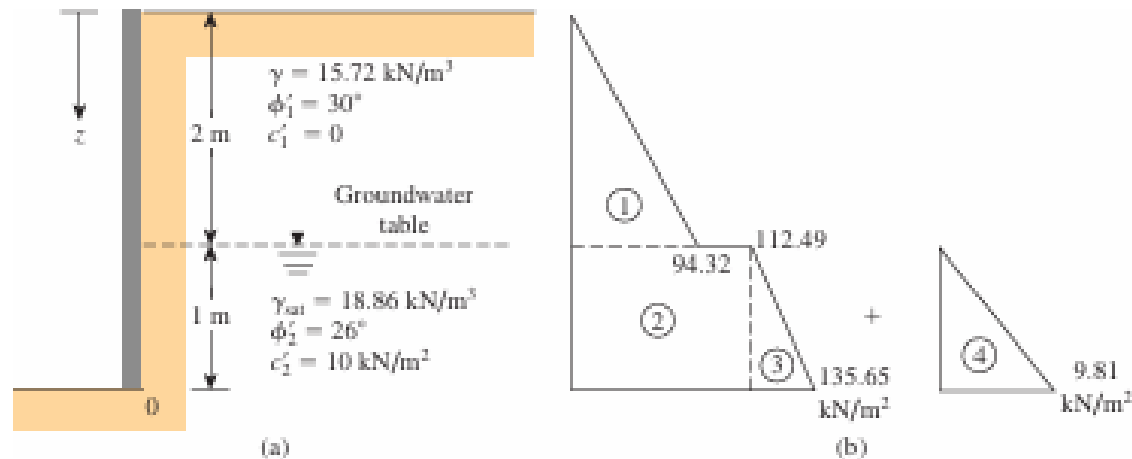
Again, at  $z = 3$  m,

$$\begin{aligned}\sigma'_o &= (15.72)(2) + (\gamma_{\text{sat}} - \gamma_w)(1) \\ &= 31.44 + (18.86 - 9.81)(1) = 40.49 \text{ kN/m}^2\end{aligned}$$

Hence,

$$\begin{aligned}\sigma'_p &= \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 40.49(2.56) + (2)(10)(1.6) \\ &= 135.65 \text{ kN/m}^2\end{aligned}$$

# EXAMPLE 16.13



| Area no.                         | Area                                           |          |
|----------------------------------|------------------------------------------------|----------|
| 1                                | $\left(\frac{1}{2}\right)(2)(94.32)$           | = 94.32  |
| 2                                | $(112.49)(1)$                                  | = 112.49 |
| 3                                | $\left(\frac{1}{2}\right)(1)(135.65 - 112.49)$ | = 11.58  |
| 4                                | $\left(\frac{1}{2}\right)(9.81)(1)$            | = 4.905  |
| $P_r \approx 223.3 \text{ kN/m}$ |                                                |          |

# Rankine Passive Earth Pressure

## Vertical Wall and Inclined Backfill

Granular backfill  $c' = 0$

$$\sigma'_p = \gamma z K_p$$

$$P_p = \frac{1}{2} \gamma H^2 K_p$$

$$K_p = \cos \alpha \frac{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi'}}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi'}}$$

**Table 16.9**

# Rankine Passive Earth Pressure

TABLE 16.9 Passive Earth Pressure Coefficient  $K_p$  [from Eq. (16.70)]

| $\downarrow \alpha$ (deg) | $\phi'$ (deg) $\rightarrow$ |       |       |       |       |       |       |
|---------------------------|-----------------------------|-------|-------|-------|-------|-------|-------|
|                           | 28                          | 30    | 32    | 34    | 36    | 38    | 40    |
| 0                         | 2.770                       | 3.000 | 3.255 | 3.537 | 3.852 | 4.204 | 4.599 |
| 5                         | 2.715                       | 2.943 | 3.196 | 3.476 | 3.788 | 4.136 | 4.527 |
| 10                        | 2.551                       | 2.775 | 3.022 | 3.295 | 3.598 | 3.937 | 4.316 |
| 15                        | 2.284                       | 2.502 | 2.740 | 3.003 | 3.293 | 3.615 | 3.977 |
| 20                        | 1.918                       | 2.132 | 2.362 | 2.612 | 2.886 | 3.189 | 3.526 |
| 25                        | 1.434                       | 1.664 | 1.894 | 2.135 | 2.394 | 2.676 | 2.987 |

# Rankine Passive Earth Pressure

## Vertical Wall and Inclined Backfill

$c' - \phi'$  Soil

$$\sigma'_p = \gamma z K'_p = \gamma z K'_p \cos \alpha$$

$$K'_p = \frac{1}{\cos^2 \phi'} \left\{ \frac{2 \cos^2 \alpha + 2 \left( \frac{c'}{\gamma z} \right) \cos \phi' \sin \phi'}{\sqrt{4 \cos^2 \alpha (\cos^2 \alpha - \cos^2 \phi') + 4 \left( \frac{c'}{\gamma z} \right)^2 \cos^2 \phi' + 8 \left( \frac{c'}{\gamma z} \right) \cos^2 \alpha \sin \phi' \cos \phi'}} \right\} - 1$$

Table 16.10

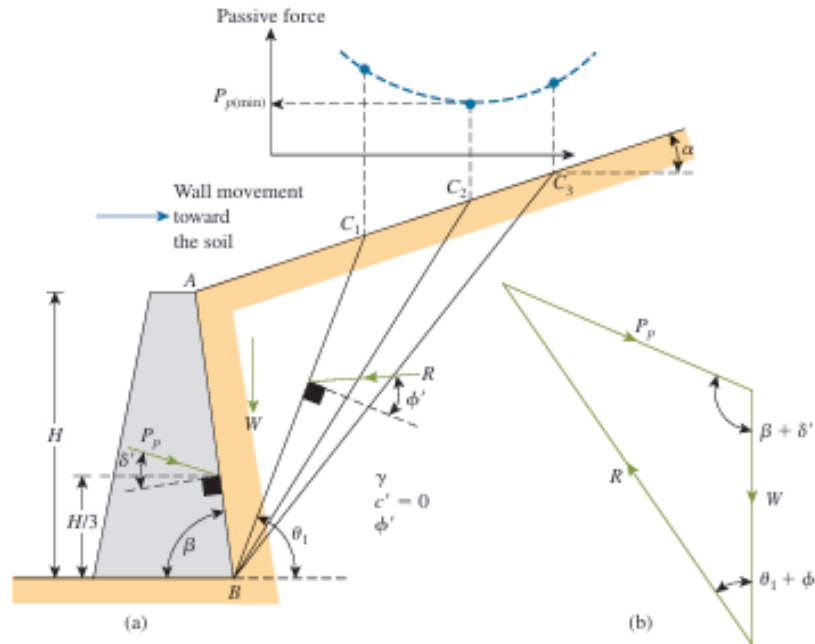
# Rankine Passive Earth Pressure

TABLE 16.10 Values of  $K_p'$

| $\alpha$ (deg) | $\phi'$ (deg) | $c'/\gamma z$ |        |        |        |        |        |        |
|----------------|---------------|---------------|--------|--------|--------|--------|--------|--------|
|                |               | 0             | 0.025  | 0.05   | 0.075  | 0.1    | 0.2    | 0.5    |
| 0              | 15            | 1.6984        | 1.7636 | 1.8287 | 1.8939 | 1.9590 | 2.2197 | 3.0016 |
| 0              | 20            | 2.0396        | 2.1110 | 2.1824 | 2.2538 | 2.3252 | 2.6109 | 3.4678 |
| 0              | 25            | 2.4639        | 2.5424 | 2.6209 | 2.6994 | 2.7778 | 3.0918 | 4.0336 |
| 0              | 30            | 3.0000        | 3.0866 | 3.1732 | 3.2598 | 3.3464 | 3.6928 | 4.7321 |
| 0              | 35            | 3.6902        | 3.7862 | 3.8823 | 3.9783 | 4.0744 | 4.4586 | 5.6112 |
| 0              | 40            | 4.5989        | 4.7061 | 4.8134 | 4.9206 | 5.0278 | 5.4567 | 6.7434 |
| 5              | 15            | 1.6477        | 1.7156 | 1.7830 | 1.8501 | 1.9169 | 2.1823 | 2.9709 |
| 5              | 20            | 1.9940        | 2.0669 | 2.1396 | 2.2122 | 2.2846 | 2.5734 | 3.4353 |
| 5              | 25            | 2.4195        | 2.4989 | 2.5782 | 2.6575 | 2.7367 | 3.0529 | 3.9986 |
| 5              | 30            | 2.9543        | 3.0416 | 3.1288 | 3.2159 | 3.3030 | 3.6511 | 4.6935 |
| 5              | 35            | 3.6412        | 3.7378 | 3.8342 | 3.9307 | 4.0271 | 4.4126 | 5.5678 |
| 5              | 40            | 4.5445        | 4.6521 | 4.7597 | 4.8672 | 4.9747 | 5.4046 | 6.6935 |
| 10             | 15            | 1.4841        | 1.5641 | 1.6408 | 1.7153 | 1.7882 | 2.0700 | 2.8799 |
| 10             | 20            | 1.8539        | 1.9323 | 2.0097 | 2.0863 | 2.1622 | 2.4615 | 3.3392 |
| 10             | 25            | 2.2854        | 2.3680 | 2.4502 | 2.5320 | 2.6135 | 2.9370 | 3.8950 |

(continued)

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1. The weight of the wedge,  $W$
2. The resultant,  $R$ , of the normal and shear forces on the plane  $BC_1$
3. The passive force,  $P_p$

$$P_p = \frac{1}{2} \gamma H^2 K_p$$

$$K_p = \text{Coulomb's passive pressure coefficient}$$

$$= \frac{\sin^2(\beta - \phi')}{\sin^2 \beta \sin(\beta + \delta') \left[ 1 - \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' + \alpha)}{\sin(\beta + \delta') \sin(\beta + \alpha)}} \right]^2}$$

# Coulomb's Passive Earth Pressure

$$K_p = \text{Coulomb's passive pressure coefficient}$$

$$= \frac{\sin^2(\beta - \phi')}{\sin^2 \beta \sin(\beta + \delta') \left[ 1 - \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' + \alpha)}{\sin(\beta + \delta') \sin(\beta + \alpha)}} \right]^2}$$

TABLE 16.11 Values of  $K_p$  [from Eq. (16.74)] for  $\beta = 90^\circ$  and  $\alpha = 0^\circ$

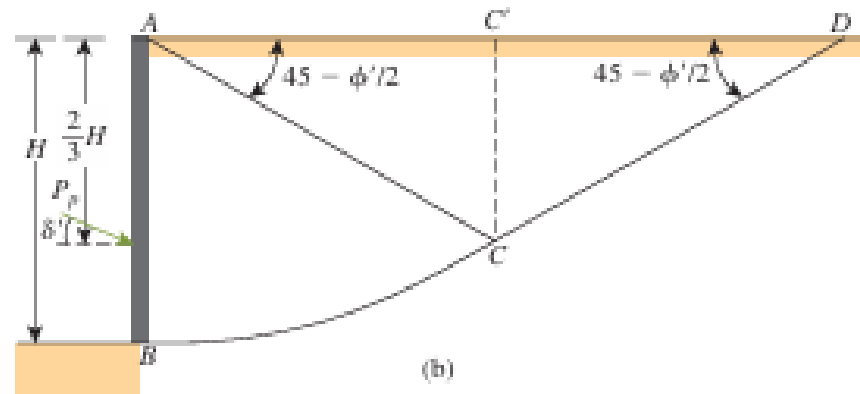
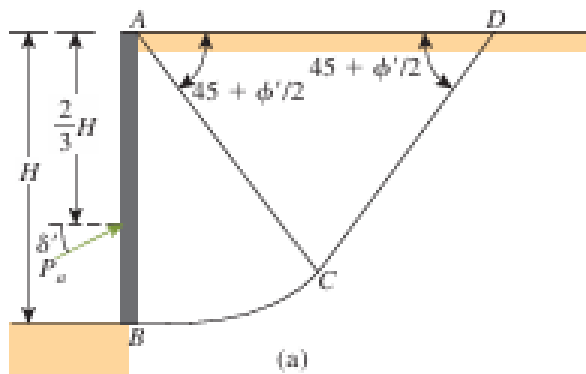
| $\phi'$ (deg) | $\delta'$ (deg) |       |       |       |        |
|---------------|-----------------|-------|-------|-------|--------|
|               | 0               | 5     | 10    | 15    | 20     |
| 15            | 1.698           | 1.900 | 2.130 | 2.405 | 2.735  |
| 20            | 2.040           | 2.313 | 2.636 | 3.030 | 3.525  |
| 25            | 2.464           | 2.830 | 3.286 | 3.855 | 4.597  |
| 30            | 3.000           | 3.506 | 4.143 | 4.977 | 6.105  |
| 35            | 3.690           | 4.390 | 5.310 | 6.854 | 8.324  |
| 40            | 4.600           | 5.590 | 6.946 | 8.870 | 11.772 |

# Comments on the Failure Surface Assumption for Coulomb's Pressure Calculations

The fundamental assumption in Coulomb's pressure calculation methods for active and passive pressure is the acceptance of *plane failure surface*.

However, for walls with friction, this assumption does not hold in practice. The nature of *actual* failure surface in the soil mass for active and passive pressure is shown in Figure below, respectively (for a vertical wall with a horizontal backfill).

Note that the failure surface  $BC$  is curved and that the failure surface  $CD$  is a plane.



# Comments on the Failure Surface Assumption for Coulomb's Pressure Calculations

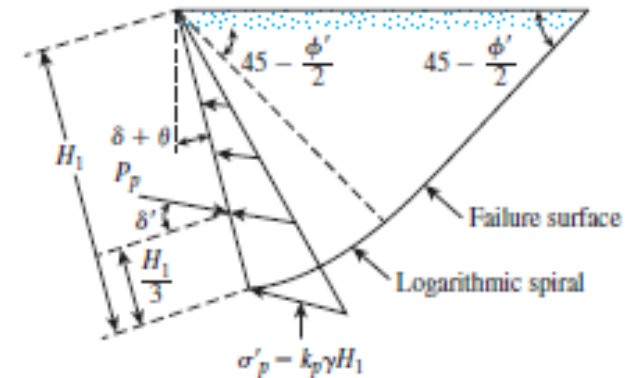
Although the actual failure surface in soil for the case of active pressure is somewhat different from that assumed in the calculation of the Coulomb pressure, the results are not greatly different.

However, in the case of passive pressure, as the value of  $\delta'$  increases, Coulomb's method of calculation gives increasingly erroneous values of  $P_p$ .

This factor of error could lead to an unsafe condition because the values of  $P_p$  would become higher than the soil resistance.

Several studies have been conducted to determine the passive force  $P_p$ , assuming that the curved portion  $BC$  is an arc of a circle, an ellipse, or a logarithmic spiral (e.g., Caquot and Kerisel, 1948; Terzaghi and Peck, 1967; Shields and Tolunay, 1973; Zhu and Qian, 2000).

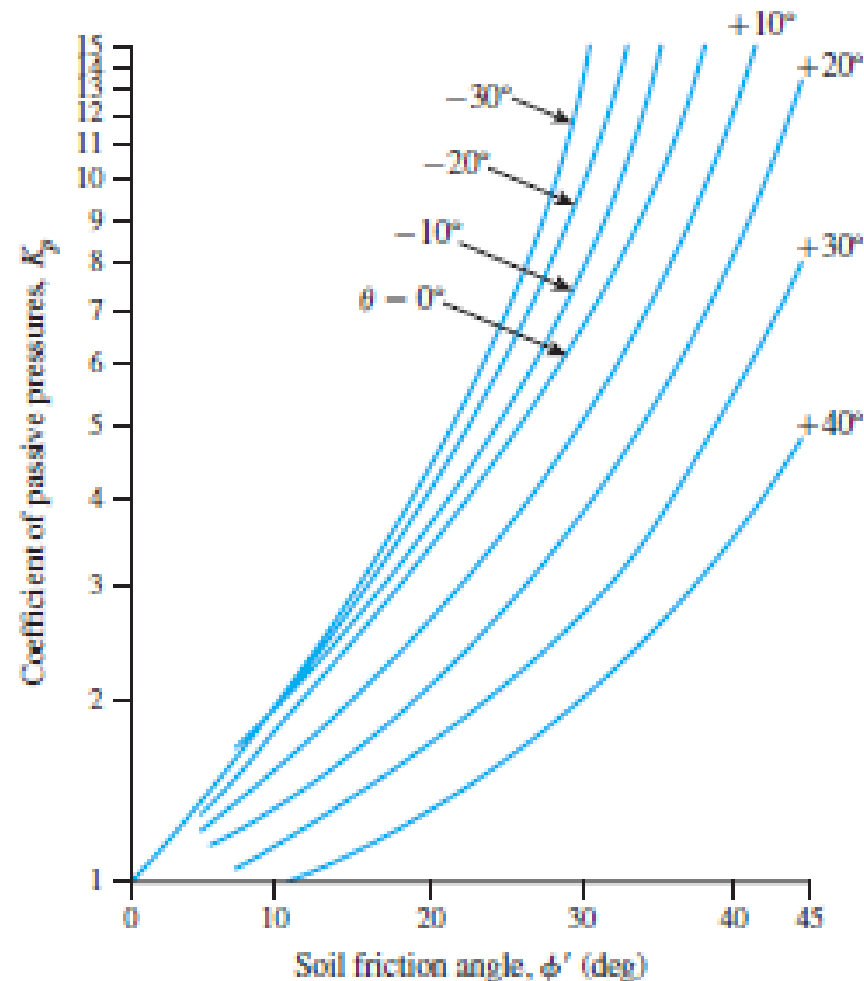
## Inclined wall with horizontal backfill



for  $\delta'/\phi' = 1$ .

# Caquot and Kerisel Solution for Passive Earth Pressure (Granular Backfill)

for  $\delta'/\phi' = 1$ .



# Caquot and Kerisel Solution for Passive Earth Pressure (Granular Backfill)

If  $\delta' / \phi' \neq 1$ ,

1. Assume  $\delta'$  and  $\phi'$ .
2. Calculate  $\delta' / \phi'$ .
3. Using the ratio of  $\delta' / \phi'$  (step 2), determine the reduction factor,  $R'$ , from Table 16.12.
4. Determine  $K_p$  from Figure 16.25 for  $\delta' / \phi' = 1$ .
5. Calculate  $K_p$  for the required  $\delta' / \phi'$  as

$$K_p = (R')[K_{p(\delta' / \phi' = 1)}] \quad (16.76)$$

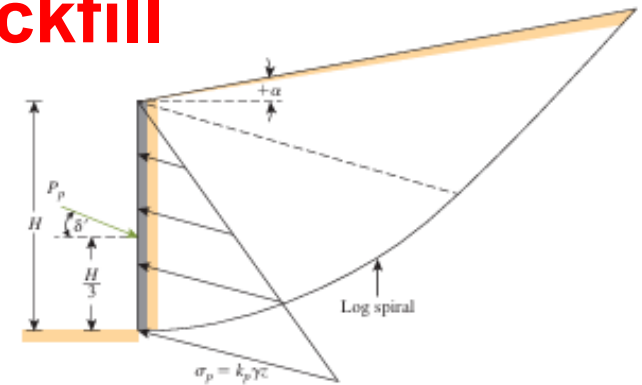
TABLE 16.12 Caquot and Kerisel's Reduction Factor,  $R'$ , for Passive Pressure Calculation

| $\phi'$ | $\delta' / \phi'$ |       |       |       |       |       |       |       |
|---------|-------------------|-------|-------|-------|-------|-------|-------|-------|
|         | 0.7               | 0.6   | 0.5   | 0.4   | 0.3   | 0.2   | 0.1   | 0.0   |
| 10      | 0.978             | 0.962 | 0.946 | 0.929 | 0.912 | 0.898 | 0.881 | 0.864 |
| 15      | 0.961             | 0.934 | 0.907 | 0.881 | 0.854 | 0.830 | 0.803 | 0.775 |
| 20      | 0.939             | 0.901 | 0.862 | 0.824 | 0.787 | 0.752 | 0.716 | 0.678 |
| 25      | 0.912             | 0.860 | 0.808 | 0.759 | 0.711 | 0.666 | 0.620 | 0.574 |
| 30      | 0.878             | 0.811 | 0.746 | 0.686 | 0.627 | 0.574 | 0.520 | 0.467 |
| 35      | 0.836             | 0.752 | 0.674 | 0.603 | 0.536 | 0.475 | 0.417 | 0.362 |
| 40      | 0.783             | 0.682 | 0.592 | 0.512 | 0.439 | 0.375 | 0.316 | 0.262 |
| 45      | 0.718             | 0.600 | 0.500 | 0.414 | 0.339 | 0.276 | 0.221 | 0.174 |

# Caquot and Kerisel Solution for Passive Earth Pressure (Granular Backfill)

## Vertical wall with Inclined backfill

$$P_p = \frac{1}{2} \gamma H^2 K_p$$



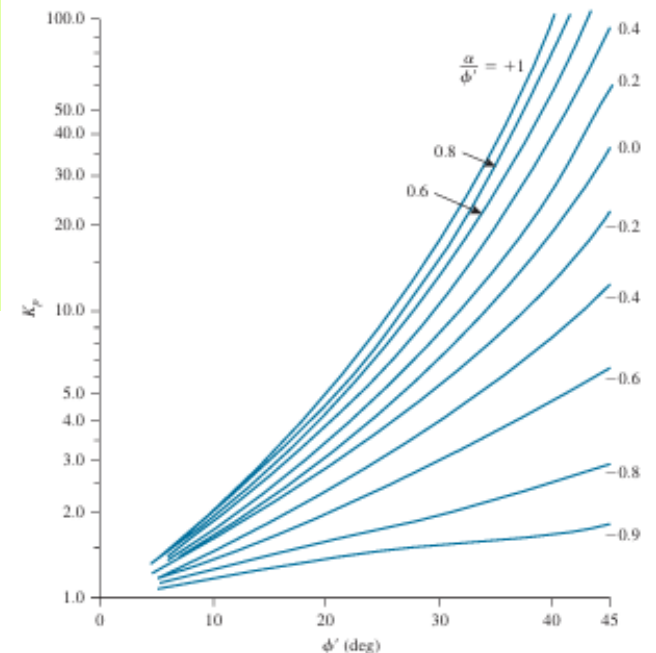
**Step 1.** Determine  $\alpha/\phi'$  (note the sign of  $\alpha$ ).

**Step 2.** Knowing  $\phi'$  and  $\alpha/\phi'$ , use Figure 16.26a to determine  $K_p$  for  $\delta'/\phi' = 1$ .

**Step 3.** Calculate  $\delta'/\phi'$ .

**Step 4.** Go to Table 16.12 to determine the reduction factor,  $R'$ .

**Step 5.**  $K_p = (R') [K_{p(\delta'/\phi' = 1)}]$ . (16.78)



# EXAMPLE 16.14

## EXAMPLE 16.14

Consider a 3 m high ( $H$ ) retaining wall with a vertical back ( $\theta = 0^\circ$ ) and a horizontal granular backfill. Given:  $\gamma = 15.7 \text{ kN/m}^3$ ,  $\delta' = 15^\circ$ , and  $\phi' = 30^\circ$ . Estimate the passive force,  $P_p$ , by using

- Coulomb's theory
- Caquot and Kerisel's theory

### SOLUTION

#### Part a

From Eq. (16.73),

$$P_p = \frac{1}{2} K_p \gamma H^2$$

From Table 16.11, for  $\phi' = 30^\circ$  and  $\delta' = 15^\circ$ , the value of  $K_p$  is 4.977. Thus,

$$P_p = \left( \frac{1}{2} \right) (4.977) (15.7) (3)^2 = 351.6 \text{ kN/m}$$

#### Part b

From Eq. (16.75), with  $\theta = 0$ ,  $H_1 = H$ ,

$$P_p = \frac{1}{2} \gamma H^2 K_p$$

From Figure 16.25a, for  $\phi' = 30^\circ$  and  $\delta'/\phi' = 1$ , the value of  $K_{p(\delta'/\phi'=1)}$  is about 5.9. Also, from Table 16.12, with  $\phi' = 30^\circ$  and  $\delta'/\phi' = 0.5$ , the value of  $R'$  is 0.746.

Hence,

$$P_p = \frac{1}{2} \gamma H^2 K_p = \frac{1}{2} (15.7) (3)^2 (0.746 \times 5.9) = 311 \text{ kN/m}$$

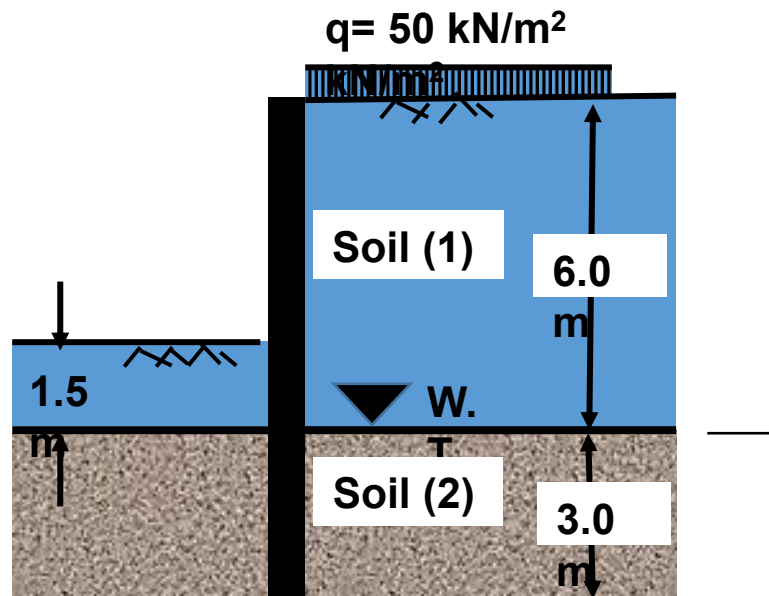
# EXAM

The soil conditions adjacent to a sheet pile wall are given in the Figure below. A surcharge pressure of  $50 \text{ kN/m}^2$  being carried on the surface behind the wall.

For soil 1, a sand above the water table,  $c' = 0 \text{ kN/m}^2$  and  $\phi' = 38^\circ$  and  $\gamma = 18 \text{ kN/m}^3$ .

For soil 2, a saturated clay,  $c' = 10 \text{ kN/m}^2$  and  $\phi' = 28^\circ$  and  $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$ .

- Calculate  $K_a$  and  $K_p$  for each of soils (1) and (2).
- Complete the given table for the Rankine active pressure at 6 and 9 m depth behind the wall shown in the Figure.
- Complete the given table for the Rankine passive pressure at 1.5 and 4.5 m depth in front of the wall shown in the Figure.



**Table . Active and passive earth pressures on sheet pile wall**

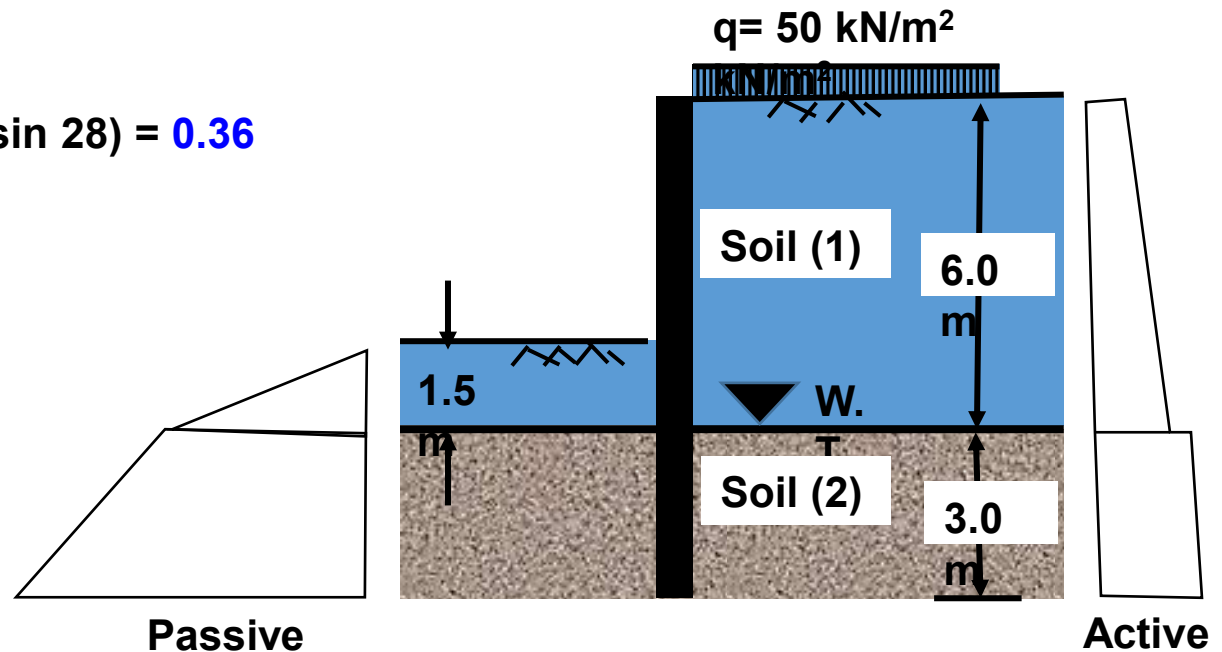
| Depth<br>(meter) | Soil |                                       |  |
|------------------|------|---------------------------------------|--|
|                  |      | Active Pressure (kN/m <sup>2</sup> )  |  |
| 0                | 1    |                                       |  |
| 6                | 1    |                                       |  |
| 6                | 2    |                                       |  |
| 9                | 2    |                                       |  |
|                  |      |                                       |  |
|                  |      | Passive Pressure (kN/m <sup>2</sup> ) |  |
| 0                | 1    |                                       |  |
| 1.5              | 1    |                                       |  |
| 1.5              | 2    |                                       |  |
| 4.5              | 2    |                                       |  |

# SOLUTION

## SOLUTION

Soil 1:  $K_a = (1 - \sin 38^\circ) / (1 + \sin 38^\circ) = 0.24$   
 $K_p = 1/K_a = 4.17$

Soil 2:  $K_a = (1 - \sin 28^\circ) / (1 + \sin 28^\circ) = 0.36$   
 $K_p = 1/K_a = 2.78$



# SOLUTION

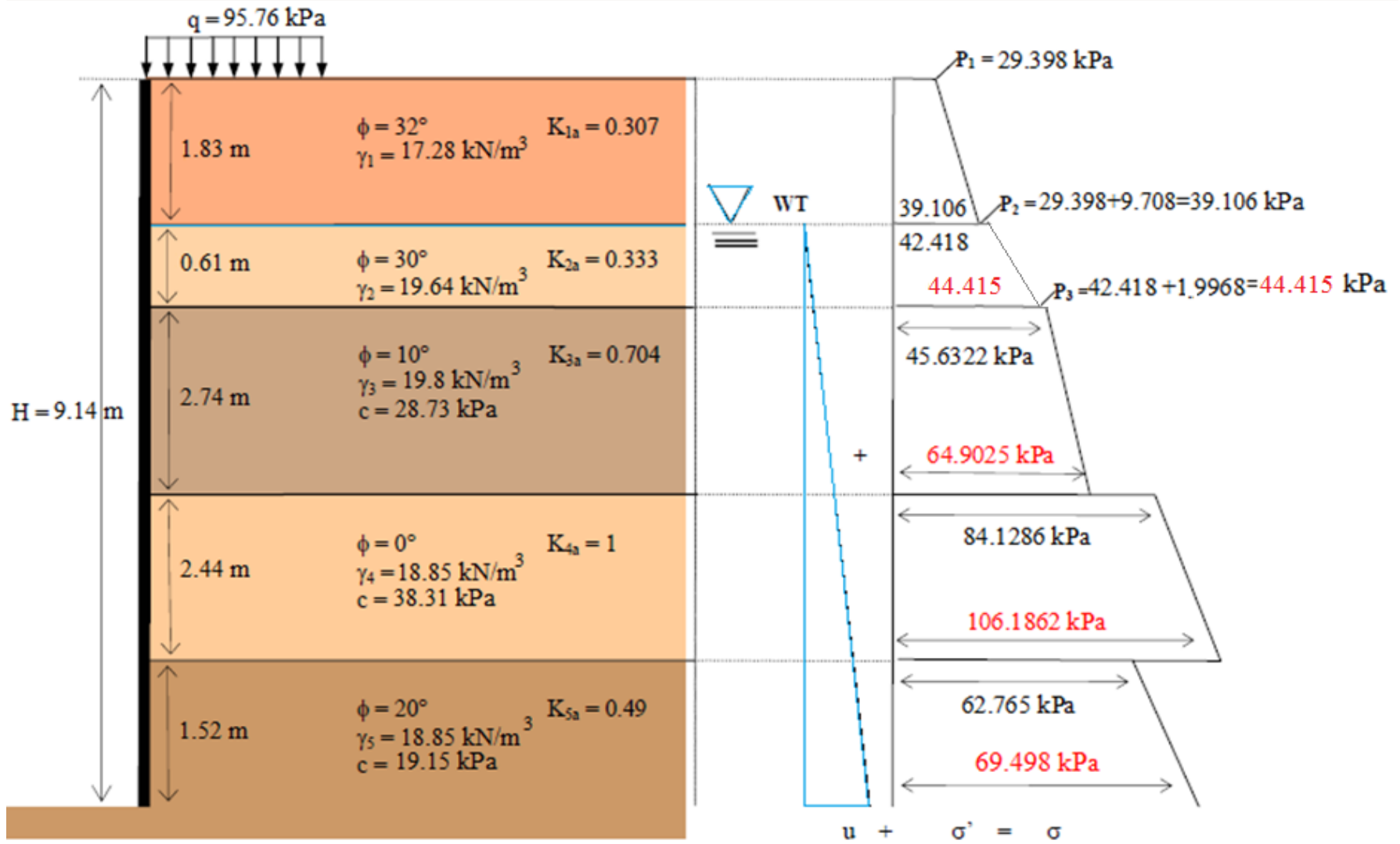
| Depth<br>(meter) | Soil |                                                                                                        |                 |
|------------------|------|--------------------------------------------------------------------------------------------------------|-----------------|
|                  |      | <b>Active Pressure (kN/m<sup>2</sup>)</b>                                                              |                 |
| 0                | 1    | $0.24 \times 50$                                                                                       | <b>= 12</b>     |
| 6                | 1    | $0.24 \times (50 + 18 \times 6)$                                                                       | <b>= 37.9</b>   |
| 6                | 2    | $0.36 \times (50 + 18 \times 6) - 2 \times \sqrt{0.36} \times 10$                                      | <b>= 44.9</b>   |
| 9                | 2    | $0.36 \times (50 + 18 \times 6 + 10.2 \times 3) - 2 \times \sqrt{0.36} \times 10$<br>$+ 9.81 \times 3$ | <b>= 85.33</b>  |
|                  |      |                                                                                                        |                 |
|                  |      | <b>Passive Pressure (kN/m<sup>2</sup>)</b>                                                             |                 |
| 0                | 1    |                                                                                                        | <b>= 0</b>      |
| 1.5              | 1    | $4.17 \times 18 \times 1.5$                                                                            | <b>= 112.6</b>  |
| 1.5              | 2    | $2.78 \times 18 \times 1.5 + 2 \times \sqrt{2.78} \times 10$                                           | <b>= 108.4</b>  |
| 4.5              | 2    | $2.78 \times (18 \times 1.5 + 10.2 \times 3) + 2 \times \sqrt{2.78} \times 10$<br>$+ 9.81 \times 3$    | <b>= 222.93</b> |
|                  |      |                                                                                                        |                 |

# RECOMMENDED PROCEDURE

1. Calculate the appropriate  $k$  for each soil
2. Calculate  $\sigma_v$  at a specified depth
3. Add  $q$  if any
4. Multiply the sum of  $(\sigma_v + q)$  by the appropriate  $k$  (for upper and lower soil) and subtract (or add for passive) cohesion part if exists.
5. Calculate water pressure
6. Divide each trapezoidal area into a rectangle and a triangle
7. Calculate areas and that give the lateral forces
8. Locate point of application for each force
9. Find the resultant force
10. Take moments about the base of the wall and find location of the resultant

# EXAMPLE

Plot the Rankine pressure diagram and find the resultant force **F** and its location under an active pressure condition.



# SOLUTION

$$\text{At } h=0, p_1 = q K_{1a} = (95.76)(0.307) = 29.398 \text{ kPa}$$

$$\text{At } h = -1.83 \Delta p_2 = \gamma_1 h K_{1a} = (17.28)(1.83)(0.307) = 9.708 \text{ kPa}$$

$$\text{At } h = -(1.83+dh) = [q + (\gamma_1) 1.83] K_{2a} = [95.76 + (17.28)(1.83)] (0.333) = 42.418 \text{ kPa}$$

$$\text{At } h = -2.44 \Delta p_3 = (\gamma_2 - \gamma_w) h K_{2a} = (19.64 - 9.81)(0.61)(0.333) = 1.9968 \text{ kPa} \rightarrow 42.418 + 1.9968 = 44.415 \text{ kPa}$$

$$\begin{aligned} \text{At } h = -(2.44+dh) &= [q + (\gamma_1) 1.83 + (\gamma_2 - \gamma_w) 0.61] K_{3a} - 2c \sqrt{K_{3a}} \text{ from } p = \gamma h K_a - 2c \sqrt{K_a} \\ &= [95.76 + (17.28)1.83 + (19.64 - 9.81)0.61] (0.704) - 2(28.73)(0.84) = 45.6322 \text{ kPa} \end{aligned}$$

$$\text{At } h = -5.18 \Delta p_4 = (\gamma_3 - \gamma_w) h K_{3a} = (19.8 - 9.81)(2.74)(0.704) = 19.27 \text{ kPa} \therefore 45.632 + 19.27 = 64.9025 \text{ kPa}$$

$$\text{At } h = -(5.18 + dh) = [95.76 + 31.62 + 5.996 + 27.3726] (1) - 2(38.31)(1) = 84.1286 \text{ kPa}$$

$$\text{At } h = -7.62 \Delta p_5 = (\gamma_4 - \gamma_w) h K_{4a} = (18.85 - 9.81)(2.44)(1) = 22.0576 \text{ kPa}$$

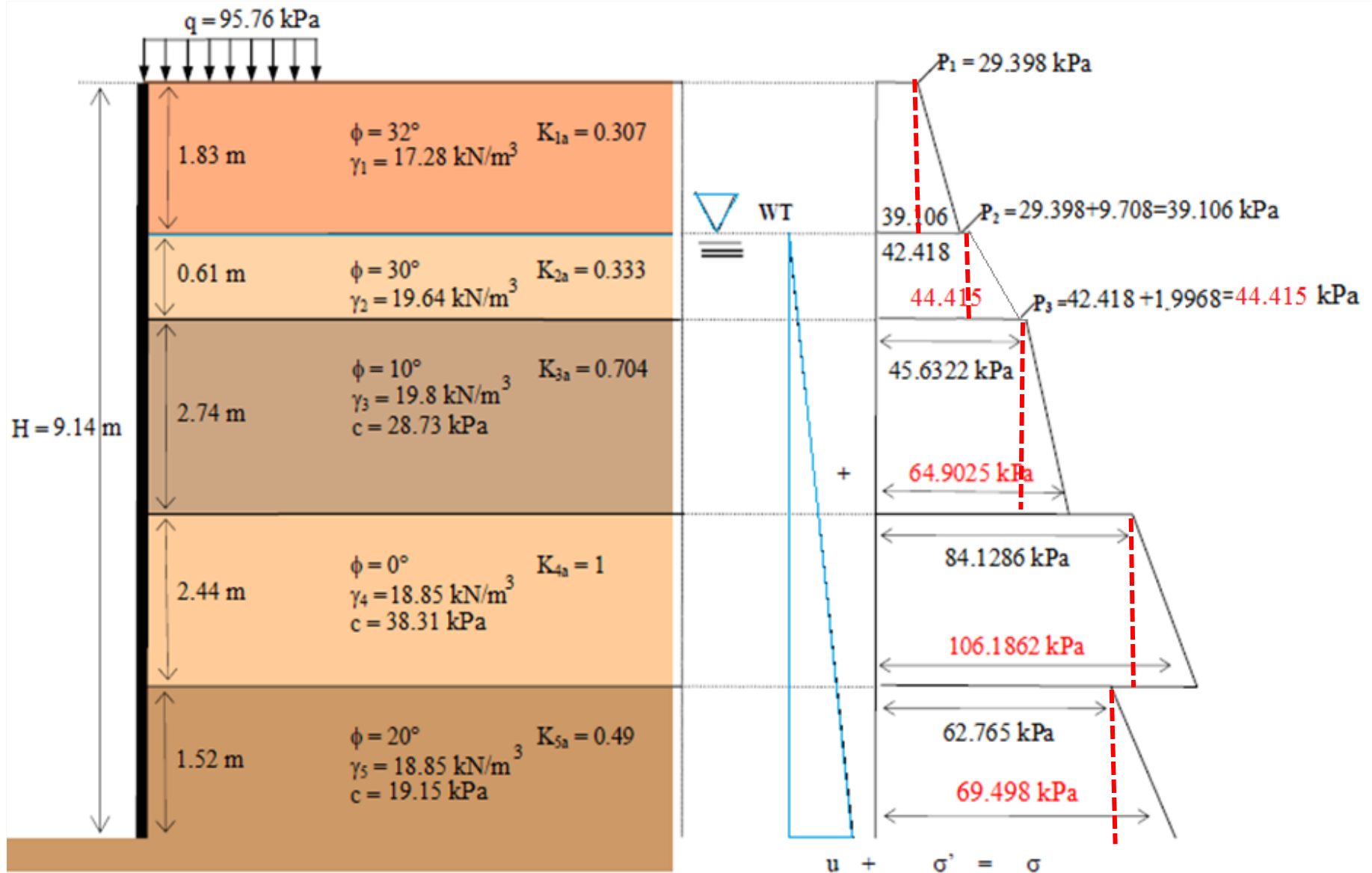
$$\therefore 84.1268 + 22.0576 = 106.1862 \text{ kPa}$$

$$\text{At } h = -(7.62 + dh) = [95.76 + 31.62 + 5.996 + 27.3726 + 22.0576] (0.49) - 2(19.15)(0.7) = 62.765 \text{ kPa}$$

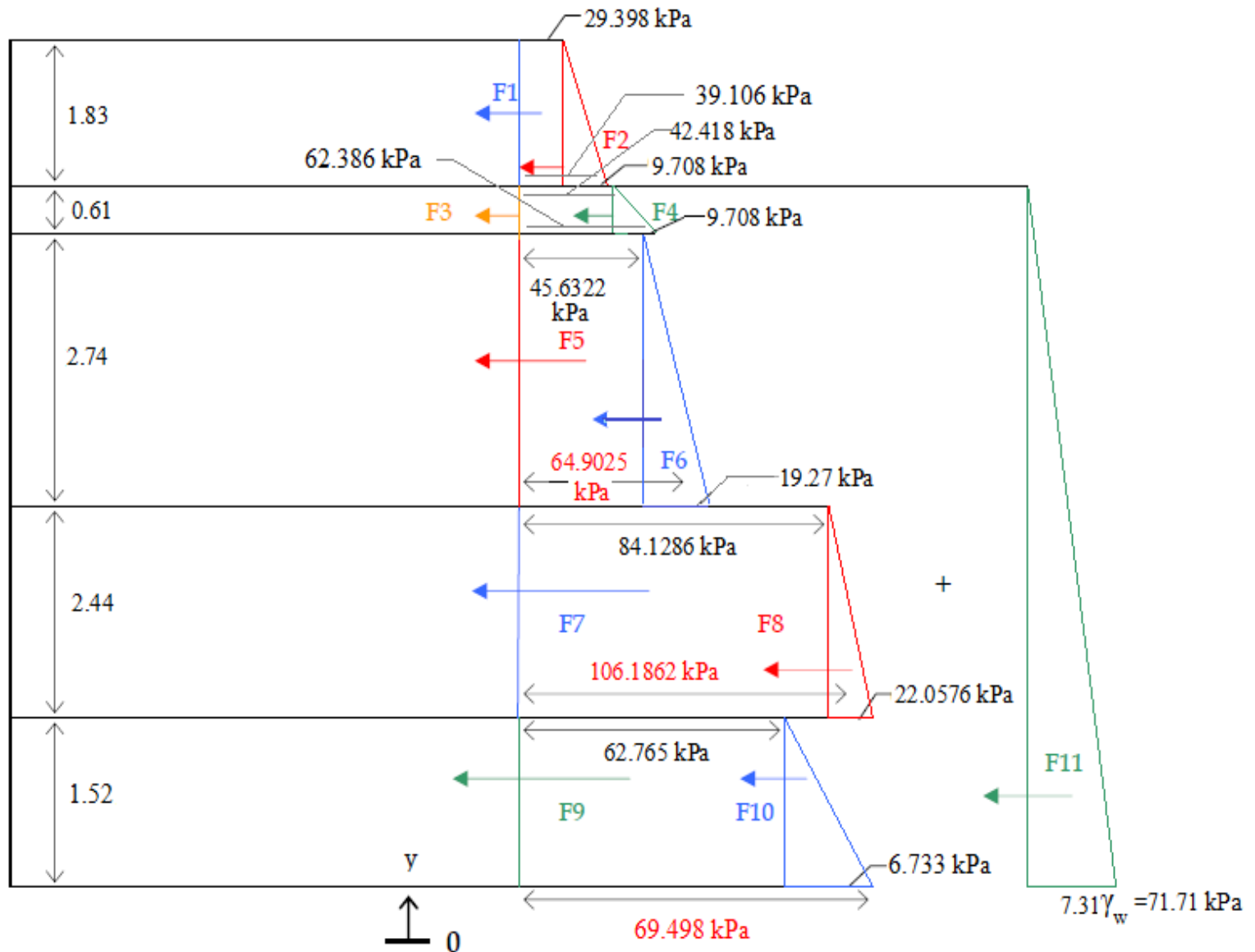
$$\text{At } h = -9.14 \Delta p_6 = (\gamma_5 - \gamma_w) h K_{5a} = (18.85 - 9.81)(1.52)(0.49) = 6.733 \text{ kPa}$$

$$\therefore 62.765 + 6.33 = 69.498 \text{ kPa}$$

# SOLUTION



# SOLUTION



# SOLUTION

$$F1 = (29.398 \text{ kPa})(1.83) = 53.798 \text{ kN}$$

$$F2 = 0.5(9.708 \text{ kPa})(1.83) = 8.8828 \text{ kN}$$

$$F3 = (42.418)(0.61) = 25.875 \text{ kN}$$

$$F4 = 0.5(19.968 \text{ kPa})(0.61) = 7.8919 \text{ kN}$$

$$F5 = (45.6322 \text{ kPa})(2.74) = 125.032 \text{ kN}$$

$$F6 = 0.5(19.27 \text{ kPa})(2.74) = 26.3999 \text{ kN}$$

$$F7 = (84.1286 \text{ kPa})(2.44) = 205.2738 \text{ kN}$$

$$F8 = 0.5(22.0576 \text{ kPa})(2.44) = 26.91 \text{ kN}$$

$$F9 = (62.765 \text{ kPa})(1.52) = 95.4 \text{ kN}$$

$$F10 = 0.5(6.733 \text{ kPa})(1.52) = 5.117 \text{ kN}$$

$$F11 = 0.5(71.71)(7.31) = 262.104 \text{ kN}$$

The resultant R is,  $R = \sum Fi = 842.68448 \text{ kN}$

The location of R is..... $\sum M_0 = 0$  (about 0)

$$842.68448(y) = (53.798)(8.225) + (8.8828)(7.92) + (25.875)(7.005) + (7.8919)(6.903) + (125.032)(5.33) + (26.3999)(4.873) + (205.273)(2.74) + (26.91)(2.33) + (95.4)(0.76) + (5.117)(0.506) + (262.104)(2.4366)$$

$$\therefore 57.1 y = 2882.53945 \rightarrow \rightarrow \rightarrow y = 3.4206 \text{ m above "0"}$$

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*The end*