① $\int_0^{2\pi} \frac{d\theta}{2+\sin\theta} = \frac{2\pi}{\sqrt{3}}$

$\boxed{d\theta = dz/iz}$

$\sin\theta = \frac{z-z^{-1}}{2i}$

$$\Rightarrow \int_0^{2\pi} \frac{d\theta}{2+\sin\theta} = \int_c \frac{dz/zi}{2+\left[\frac{z+z^{-1}}{2i}\right]} = \int_c \frac{dz/zi}{\frac{4i+\left[\frac{z^2-1}{z}\right]}{2i}} = \int_c \frac{dz/zi}{\frac{4iz+z^2-1}{\cancel{z}2i}}$$

$$= 2\int_c \frac{dz}{4iz+z^2-1}$$

$z^2 + 4iz - 1 = 0$

$$\Rightarrow z = \frac{-4i \pm \sqrt{-16+4}}{2} = \frac{-\overset{2}{\cancel{4}}i \pm 2\sqrt{3}i}{\cancel{2}} = (-2\pm\sqrt{3})i$$

$z_0 = (-2+\sqrt{3})i$ inside the circle $|z|=1$ "as $|(-2+\sqrt{3})i| = -2+\sqrt{3} < 1$"

$z_0' = (-2-\sqrt{3})i$ outside the circle $|z|=1$ "as $|(-2-\sqrt{3})i| = 2+\sqrt{3} > 1$"

Now, $$\int_0^{2\pi} \frac{d\theta}{2+\sin\theta} = 2\int_c \frac{dz}{4iz+z^2-1} = 2\left(2\pi i\left[\mathrm{Res}\left(\underbrace{(-2+\sqrt{3})i}_{\text{simple pole}}\right)\right]\right)$$

$$\mathrm{Res}((-2+\sqrt{3})i) = \lim_{z\to z_0} (z-\cancel{z_0})\cdot\frac{1}{(\cancel{z-z_0})(z-z_0')}$$

$$= \lim_{z\to z_0} \frac{1}{(z-z_0')} = \frac{1}{z_0-z_0'} = \frac{1}{2\sqrt{3}i}$$

So, $$\int_0^{2\pi} \frac{d\theta}{2+\sin\theta} = \overset{2}{\cancel{4}}\pi\cancel{i}\cdot\frac{1}{\cancel{2}\sqrt{3}\cancel{i}}$$

$$= \frac{2\pi}{\sqrt{3}}$$