(4)

$\text{Res}(f, z_0) = a_{-1}$ from Laurent

$f(z) = \sum_{j=-\infty}^{\infty} a_j (z - z_0)^j$

$\Rightarrow f'(z) = \sum_{j=-\infty}^{\infty} j a_j (z - z_0)^{j-1}$

at $j = 0 \rightarrow \text{Res}(f', z_0) = 0 \cdot a_0 = 0$

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(5) No

if f has a simple pole at z_0

$\Rightarrow f(z) = \frac{g(z)}{z - z_0}$, $g(z_0) \neq 0$, g is analy at z_0

$\Rightarrow \text{Res}(f, z_0) = \lim_{z \to z_0} (z - z_0) f(z)$

$= \lim_{z \to z_0} g(z)$

$= g(z_0) \neq 0$

$f(z) = \frac{1}{z^2}$ has a pole of order 2 at 0

and $\text{Res}(f, 0) = a_{-1} = 0$

مفكوك لوران