الشراكة الطلابية
SPP

[e] $\int_{|z|=1} \frac{1}{z^2 \sin z} dz \to$ singularity $z=0$ and $z = k\pi, k \in \mathbb{Z}$

only if $k=0$ inside $|z|=1$

$\Rightarrow \int \frac{1}{z^2 \sin z} = 2\pi i [\underline{\text{Res}(0)}]$

Since $z^2 \sin z$ has zero of order 3 at $z=0$

$\Rightarrow \frac{1}{z^2 \sin z}$ has a pole of order 3 at $z=0$

$\Rightarrow \text{Res}(0) = \lim_{z \to 0} \frac{1}{2!} \cdot \frac{d^2}{dz^2}\left[(z-0)^3 f(z)\right]$

$= 1/6$

$\therefore \int = 2\pi i (1/6) = \frac{\pi i}{3}$

[g] $\int_{|z|=8} \frac{1}{z^2+z+1} dz$

$z^2+z+1=0 \to z_0 = \frac{-1+\sqrt{3}i}{2}, \quad z_0' = \frac{-1-\sqrt{3}i}{2}$

and insid $|z|=8$

$\text{Res}(z_0) = \frac{1}{\sqrt{3}i}, \quad \text{Res}(z_0') = -\frac{1}{\sqrt{3}i}$

$\Rightarrow \int dz = 0$