(3) $\oint_{|z|=5} \frac{\sin z}{z^2-4}\,dz$

$f(z) = \frac{\sin z}{z^2-4}$ has isolated singularities at $2, -2$ in $|z|=5$

So, $\int \frac{\sin z}{z^2-4}\,dz = 2\pi i\,(\text{Res}(2) + \text{Res}(-2))$

$\because f(z) = \frac{\sin z}{z-2}/z+2$ ∧ $f(z) = \frac{\frac{\sin z}{z+2}}{z-2}$

$\Rightarrow f$ has simple pole at $z = \pm 2$

$\therefore$ Res$(2) = \lim_{z\to 2}(z-2)f(z) = \frac{\sin(2)}{4}$

$\text{Res}(-2) = \lim_{z\to -2} \frac{\sin z}{z-2} = \frac{\sin -2}{-4} = \frac{\sin 2}{4}$

hence, $\int f = 2\pi i\left(\frac{\sin(2)}{4} + \frac{\sin(2)}{4}\right)$

$= 2\pi i\left[\frac{\sin(2)}{2}\right]$

$= \sin(2)\pi i$

(5) $\int_{|z|=3} \frac{e^{iz}}{z^2(z-2)(z+5i)}\,dz$ → $5i \notin |z| = 5$

$= 2\pi i\left[\text{Res}(0) + \text{Res}(2)\right]$

(0: ~~simple~~ pole of order 2; 2: simple pole)

$\text{Res}(0) = \frac{-12+5i}{100}$ → $= \lim_{z\to 0} \frac{1}{1!}\frac{d}{dz}\left[\frac{e^{iz}}{(z-2)(z+5i)}\right]$

$\text{Res}(2) = \frac{e^{2i}}{4(2+5i)}$