If f is analytic inside and on Γ $\Rightarrow \int \frac{f(z)}{z-z_0} = 2\pi i f(z_0)$

(2) هي حالة خاصة من نظرية الباقي وذلك عندما

$f(z_0) \neq 0$ (1)

فإن الدالة

$$g(z) = \frac{f(z)}{z-z_0}$$

has a simple pole at z_0

$$\Rightarrow \operatorname{Res}(z_0, g) = \lim_{z \to z_0} (z-z_0)\, g(z)$$

$$= \lim_{z \to z_0} f(z)$$

$$= f(z_0)$$

$$\therefore \int \frac{f}{z-z_0} = 2\pi i f(z_0)$$

$f(z_0) = 0$ (2)

Taylor expansion for f about z_0 is

$$f(z) = \sum_{j=0}^{\infty} \frac{f^{(j)}(z_0)(z-z_0)^j}{j!}$$

$$\Rightarrow g(z) = \frac{f(z)}{z-z_0} = \sum_{j=0} \frac{f^{(j)}(z_0)(z-z_0)^{j-1}}{j!} \qquad j-1=-1$$

$$\Rightarrow a_{-1} = f^{(0)}(z_0)$$

$$= f(z_0)$$

اذا من نظرية الباقي نجد

So, $\int f = 2\pi i f(z_0)$