1

(a) $\dfrac{e^{3z}}{z-2} = f$

the isolated singulraty is $\underline{\underline{2}}$

f has a simple pole ~~of order~~ at $\underline{\underline{2}}$

$\Rightarrow \operatorname{Res}(2) = \lim\limits_{z\to 2} (z-2)\cdot f$

$= \lim\limits_{z\to 2} e^{3z} = \underline{\underline{e^{6}}}$

(d) $f(z) = \left(\dfrac{z-1}{z+1}\right)^3 = \dfrac{(z-1)^3}{(z+1)^3}$

isolated singularity is $\underline{\underline{-1}}$

f has a pole of order $\underline{\underline{3}}$

$\therefore \operatorname{Res}(-1) = \lim\limits_{z\to -1} \dfrac{1}{2!} \dfrac{d^2}{dz^2}\left[(z+1)^3 f(z)\right]$

$= \lim\limits_{z\to -1} \dfrac{1}{2} \dfrac{d^2}{dz^2}\left[(z-1)^3\right]$

$= \frac{1}{2} \lim\limits_{z\to -1} 6(z-1)$

$= \cancel{6}\ \cancel{-7}\ -6$

(f) $f(z) = \sin\left(\dfrac{1}{3z}\right) \rightarrow$ singularity is $\underline{\underline{0}}$

$\sin\left(\dfrac{1}{3z}\right) = \sum\limits_{j=0}^{\infty} \dfrac{(-1)^j}{(2j+1)!}\left(\dfrac{1}{3z}\right)^{2j+1}$

$= \dfrac{1}{1}\,\dfrac{1}{3z} + \cdots$

$\Rightarrow \operatorname{Res}(0) = \underline{\underline{\dfrac{1}{3}}}$