①

Simillar Solutions of Laminar Boundary Layer.

In boundary Layer it is known that velocity Profile

$$\frac{u}{U_\infty} = f(Re) \qquad U_\infty \rightarrow U_0$$

If $\frac{dU_\infty}{dx} = 0$ (no pressure gradient) or constant outer velocity U_0. It generates simillar velocity profiles at different values of x. if $U = f(x)$ then partial differential equation can be changed into ordinary differential equation for one independant variable. Such solution are called simillar solution. These solutions require ~~[illegible]~~

$$\frac{u}{U_\infty} = \phi\left(\frac{y}{\varphi(x)}\right) = f(\eta)$$ for which boundary solution can be done in terms of η. In general

$$\eta = C_1 y \sqrt{\frac{U}{U_\infty}} \qquad C_1 = \text{constant}$$

which results in $U_\infty = Cx^m$ or $U_\infty = C e^{\alpha x}$

m is constant α is positive

consider $U_\infty = U_0 \left(\frac{x}{L}\right)^m$ — ① U_0 some reference velocity, L = Reference length.

$$\eta = \frac{y}{2}\sqrt{\frac{U_\infty}{\nu x}} = \frac{y}{2}\sqrt{\frac{U_0}{\nu}}\ \frac{x^{\frac{m}{2}-1}}{L^{m/2}} \quad \text{—②}$$

now $\psi = \sqrt{U_\infty \nu x}\, F(\eta) = \frac{\sqrt{U_0 \nu}\ x^{\frac{m-1}{2}}}{L^{m/2}} F(\eta)$ —③

$$u = \frac{d\psi}{dy} = \frac{U_\infty F'}{2} \qquad \frac{u}{U_1} = \frac{F'}{2}$$

This satisfies similarity condition, Evaluating $u, v, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial y^2}$ and substituting in boundary layer Equation

$$F''' + (m+1)FF'' + 2mF^2 + 8m = 0 \quad \text{—④}$$

This is ordinary differential equation in η.

Boundary conditions are $\eta = 0$ $F = F' = 0$ $\eta = \infty$ $F' = 2$

In such situations the variable is changed to

$$\eta_1 = \eta\sqrt{2(m+1)}$$

$$f(\eta_1) = \sqrt{\frac{m+1}{2}}\, f(\eta)$$

Equation (4) becomes $F''' + FF'' + \beta(1+F'^2) = 0$ (5)

when $\beta = \frac{2m}{m+1}$ and boundary conditions are

$\eta = 0$ $F = F' = 0$ $\eta_1 = \infty$ $F' = 0$

Different solutions of these have been arrived at one of these are Falkan and Skan ~~etc~~ of flow over a wedge

They assumed η function as $\eta = cyx^a$.

free stream flow above boundary layer $U(x) = kx^m$

$m = 2a+1$ (m lies between 0 – 0.009) for wedge

They found $C = \sqrt{\frac{k(1+m)}{2\nu}}$ & $\eta = y\sqrt{\frac{m+1}{2}\left(\frac{U(x)}{\nu x}\right)}$ $\boxed{Eq\ 4.70}$

Got equation $f''' + ff'' + \beta(1+f'^2) = 0$

$\beta = \frac{2m}{1+m}$ $f(\eta=0) = f'(\eta=0) = 0$ $f'(\infty) = 1$

$\beta = 0$ is flat plate equation

For heat transfer use equation 4.79

$$\frac{Nu_x}{\sqrt{Re_x}} = \begin{cases} 0.22\, P_r^{0.27} & \beta = -0.1988 \\ 0.332\, P_r^{0.333} & \beta = 0 \quad \text{Flat plate} \\ 0.57\, Pr^{0.4} & \beta = 1 \quad m = 1\ a = 0 \quad \eta = cy \end{cases}$$

Stagnation point

Note: m lies between 0 – 0.009

Some cases

① Flat Plate Heat transfer with constant wall Temperature

Define $\theta(\eta) = \frac{T - T_e}{T_w - T_e}$ 4.56 $\qquad T_e \qquad\qquad T_w$

B.L equation becomes

$$\theta'' + P_r f(\eta)\theta' = 0 \quad \theta(0) = 1 \quad \theta(\infty) = 0 \quad 4.57$$

Solution $\theta = \dfrac{\int_\eta^\infty e^{\left(-Pr \int_0^\eta f ds\right)} d\eta}{\int_0^\infty e^{\left[-Pr \cdot \int_0^\eta f ds\right]} d\eta}$

If both Boundary Layer and thermal boundary layers is thin $\quad \delta_T = \delta \, P_r^{0.4} \quad P_r = \frac{\nu}{\alpha} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}$

Heat Transfer $q_w = -k(T_w - T_e)\,\theta'(0)\sqrt{\frac{U}{2\nu x}}$

$$Nu_x = -\frac{\theta'(0)}{\sqrt{2}} Re_x^{1/2} \quad Re_x = \frac{Ux}{\nu}$$

$$Nu_x = 0.332\, Re_x^{0.5}\, P_r^{1/3}$$

② Falkner-Skan Wedge Flows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad v = -\frac{\partial}{\partial x}\int_0^y u\, dy$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \nu\frac{\partial^2 u}{\partial y^2} \qquad \frac{\partial P}{\rho} = -U\frac{dU}{\partial x} \qquad \frac{P}{\rho} + \frac{1}{2}\rho U^2 = 0$$

$$u\frac{du}{dx} - \frac{\partial u}{\partial y}\left(\frac{\partial}{\partial x}\int_0^y u\,dy\right) = U\frac{dU}{dx} + \nu\frac{d^2 u}{\partial y^2}$$

Let $u(x,y) = U(x)\, f'(\eta) \qquad \eta = f(x,y)$

If $\eta = c\,y\,x^a \qquad U(x) = K x^m \quad m = 2a+1 \quad C = \frac{k(1+m)}{2\nu}$

m is Falkner Skan Power law parameter

$$\eta = y\sqrt{\frac{m+1}{2}\frac{U(x)}{\nu x}}$$

$$f''' + ff'' + \beta(1 - f'^2) = 0 \quad \beta = \frac{2m}{1+m} \quad (4)$$

$$f(0) = f'(0) = 0 \quad f'(\infty) = 1$$

This is suitable for wedge β is measure of $\frac{dP}{dx}$

$+\beta \rightarrow \frac{dP}{dx}$ (-ve) $\quad -\beta \rightarrow \frac{dP}{dx}$ +ve $\quad \beta = 0$ Flat Plate

Heat Transfer $Nu_x = \sqrt{\frac{m+1}{2}}\, G(Pr, \beta)\, Re_x^{1/2}$

(3) ~~Flat Plate with Wall Suction~~

The value of m lies between 0 and -0.009

$$0 \not< m \not> -0.009$$

For wedge for semicone angle $\frac{\pi\beta}{2}$

for $m > 0 \quad U_1(x) = U_0 x^m \quad \beta = \frac{2m}{m+1}$

Special case $m = 1 \quad \beta = 1 \quad \theta$ wedge angle $= \frac{\pi}{2}$

is flow over flat plate near stagnation point

$$\eta = \frac{y}{2}\sqrt{\frac{U_0}{\nu}} \cdot \frac{x^{m+1}}{L^{m/2}}$$

(3) Wall Suction or Blowing

Examples: mass transfer, Drying, Cooling, boundary layer control

In this case $V_w(x)$ is assumed.

$\therefore$ at $\eta = 0 \quad U_w = -f(0)\sqrt{\frac{\nu U}{2x}} \quad f(0) \sim \sqrt{x}$

$U_w \propto \sqrt{x}$

$\therefore$ In Blasius Solution $f''' + ff'' = 0$

$$f'(0) = 0 \quad f'(\infty) = 1 \quad f(0) \neq 0$$

Momentum Integral Equation

Boundary layer equation

continuity $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ — (1)

Momentum $\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{d x}+\frac{1}{\rho}\frac{d\tau}{\partial y}$ — (2)

~~Multiply (1) by (u-U) and subtract from (2)~~

~~$-\frac{1}{\rho}\frac{\partial\tau}{\partial y}=$~~

if we ~~de~~ consider steady incompressible flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dP}{\partial x}+\frac{1}{\rho}\frac{\partial\tau}{\partial y}$$

$$P+\frac{1}{2}\rho U^2=\text{constant}$$

$$\frac{\partial P}{\partial x}=-\rho U\frac{dU}{dx}\qquad \frac{1}{\rho}\frac{dP}{\partial x}=-U\frac{dU}{\partial x}$$

$$u\frac{du}{\partial x}+v\frac{du}{\partial y}=-U\frac{dU}{\partial x}+\frac{1}{\rho}\frac{\partial\tau}{\partial y} \quad (3)$$

Multiply (1) by $u-U$ & Subtract from (3)

$$-\frac{1}{\rho}\frac{\partial\tau}{\partial y}=\frac{\partial}{\partial x}(uU-u^2)+(U-u)\frac{\partial U}{\partial x}+\partial(vU-uv)$$

integrate from $y=0$ $y=\infty\simeq\delta$

$$\frac{\tau_w}{\rho}=\frac{\partial}{\partial x}\int_0^\infty u(U-u)\,\partial y+\frac{\partial U}{\partial x}\int_0^\infty(U-u)dy-Uv_w$$

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho U^2}$$

if Suction or Blowing is absent $v_w=0$

we get

$$\boxed{\frac{C_f}{2} = \frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx} + (H+2)\frac{\theta}{U}\frac{dU}{dx}}$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy \qquad H = \frac{\delta^*}{\theta} \quad \delta^* = \int_0^\infty \left(1-\frac{u}{U}\right)dy$$

This is ~~the~~ Momentum Integral Equation or Karman Integral Equation for boundary layer.

H = Shape factor H>1

H varies from 2.: for Stagnation flow to 3.5 at separation point

For Laminar flow.

For flat plate $\frac{dU}{dx} = 0$

$$\therefore \frac{C_f}{2} = \frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx}$$

Example Assume linear profile

$$\frac{u}{U} = \frac{y}{\delta} \qquad u=0 \quad y=0$$
$$u=U \quad y=\delta$$

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu U}{\delta}$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \delta/2$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \delta/6$$

$$\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx} = \frac{d\,\delta/6}{dx} = \frac{1}{6}\frac{d\delta}{dx} = \frac{\mu U/\delta}{\rho U^2}$$

$$\frac{1}{6}\frac{d\delta}{dx} = \frac{\mu}{\rho\delta U} = \frac{\nu}{\delta U} \quad ⑦$$

$$\delta\, d\delta = \frac{6\nu}{U} dx \quad \text{Integrate}$$

$$\frac{\delta^2}{2} = \frac{6\nu}{U} x \qquad \delta^2 = \frac{12\nu x}{U}$$

$$\frac{\delta^2}{x^2} = 12\frac{\nu}{Ux} \qquad \frac{\delta}{x} = \frac{\sqrt{12}}{\sqrt{Re_x}} = \frac{3.4164}{Re_x}$$

$$\frac{\delta}{x} = \frac{3.4164}{Re_x}$$

$$\delta^* = \frac{\delta}{2} \qquad \frac{\delta^*}{x} = \frac{\delta}{x2} = \frac{1}{2}\frac{\delta}{x} = \frac{1.7082}{\sqrt{Re_x}}$$

$$\frac{\delta^*}{x} = \frac{1.7082}{\sqrt{Re_x}}$$

$$\theta^* = \frac{\delta}{6} \qquad \frac{\theta}{x} = \frac{1}{6}\frac{\delta}{x} = \frac{1}{6}\,\frac{3.4164}{\sqrt{Re_x}}$$

$$\frac{\theta}{x} = \frac{0.5694}{Re_x}$$

	Integral	Blasius
$\frac{\delta}{x}$	$3.4164/\sqrt{Re_x}$	$5/\sqrt{Re_x}$
$\frac{\delta^*}{x}$	$1.7082/\sqrt{Re_x}$	$1.7208/\sqrt{Re_x}$
$\frac{\theta}{x}$	$0.5694/\sqrt{Re_x}$	$0.664/\sqrt{Re_x}$

$$\sqrt{Re_x} = \sqrt{\frac{Ux}{\nu}}$$

These methods are based on momentum Integral method. These methods are

Pohlaussen Method
Thawaits Method
Young's Method

Pohlaussen Method

This method extends momentum Integral Method.

M.Integral Equation is

$$\frac{C_f}{2} = \frac{\tau_o}{\frac{1}{2}\rho U^2} = \frac{d\theta}{dx} + \frac{\theta}{U}(H+2)\frac{dU}{dx}$$

This method assumes finite thickness of boundary layer. Boundary conditions $u(0)=0$ $u(\delta)=U$ $\frac{\partial u}{\partial y}\Big|_{y=\delta}=0$

Besides this there is a condition that external velocity gradient caused by pressure gradient to boundary layer flow i.e. $\nu\frac{\partial^2 u}{\partial y^2} = -U\frac{dU}{dx}$ $\left[\frac{dP}{dx} = U\frac{dU}{dx}\right]$

Pohlaussen defined a parameter called Form Parameter

$$\lambda = \frac{\delta^2}{\nu} U'$$

and a velocity profile as a 4 degree polynomial

$$\frac{u}{U} = a\eta + b\eta^2 + c\eta^3 + d\eta^4 \qquad \eta = y/\delta$$

This along with boundary conditions gives

$$a = 2+\frac{\lambda}{6} \quad b = -\frac{\lambda}{2} \quad c = -2+\frac{\lambda}{2} \quad d = 1-\frac{\lambda}{2}$$

Substitute this in velocity profile gives

$$\frac{u}{U} = F(\eta) + \lambda(G(\eta))$$

$$F(\eta) = 2\eta - 2\eta^3 + \eta^4$$

$$G(\eta) = \frac{\eta}{6}(1-\eta^2)^3$$

The Problem reduces to find boundary layer thickness δ and using it in momentum Integral Equation to find other thickness

$$\frac{\tau}{\frac{1}{2}\rho U^2} = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{1}{2}\rho U^2} = \frac{\nu}{U}\left[F'(0) + \lambda G'(0)\right]$$

$$\boxed{\frac{\tau}{\frac{1}{2}\rho U^2} = \frac{\nu}{\delta U}\left(2 + \frac{\lambda}{6}\right)} \quad \text{—(1)}$$

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) d\eta = \frac{3}{10} - \frac{\lambda}{120} \quad \boxed{\frac{\delta^*}{\delta} = \frac{3}{10} - \frac{\lambda}{120}} \quad \text{—(2)}$$

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta = \frac{37}{315} - \frac{\lambda}{945} - \frac{\lambda^2}{7094}$$

$$\boxed{\frac{\theta}{\delta} = \frac{37}{315} - \frac{\lambda}{945} - \frac{\lambda^2}{7094}} \quad \text{—(3)}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{\nu}{U\delta}\left(2 + \frac{\lambda}{8}\right) \quad \text{—(4)}$$

This method was modified by Bohlan by introducing Z Parameter

$$Z = \frac{\delta^2}{\nu} = \frac{\lambda}{\frac{dU}{dx}}$$

He Proved that $\lambda = 7.0252$ Only and $\lambda = 12$ gives separation

Different velocity Profiles are used.

① $f(\eta) = \eta$ Boundary Condition $f(1)=1$

② $f(\eta) = \frac{3\eta}{2} - \frac{\eta^3}{2}$ Boundary condition $f(1)=1$ $f''(1)=0$

③ $f(\eta) = 2\eta - 2\eta^3 + \eta^4$ Boundary condition $f(1)=1$ $f'(1)=0$ $f''(1)=0$

④ $f(\eta) = A_0 + A_1 \sin BY$

⑤ $f(\eta) = \sin \frac{\pi\eta}{2}$

Values of different parameter using these velocity Profile

Profile	$\delta^*\sqrt{\frac{U}{\nu x}}$	$\theta\sqrt{\frac{U}{\nu x}}$	$\delta\sqrt{\frac{U}{\nu x}}$	$H = \frac{\delta^*}{\theta}$	$C_f\sqrt{\frac{xU}{\nu x}}$
1	1.732	0.577	3.40	3	1.555
2	1.740	0.646	4.64	3.7	1.292
3	1.75	0.686	5.836	2.55	1.372
4	1.741	0.656	4.705	2.66	1.310
Exact	1.721	0.664	5	2.54	1.328

$\delta\sqrt{\frac{U}{\nu x}}$ = Growth of boundary layer thickness

$\delta^*\sqrt{\frac{U}{\nu x}}$ = Growth of boundary Layer displacement thickness

$\theta\sqrt{\frac{U}{\nu x}}$ = Growth of boundary Layer momentum thickness

$C_f\sqrt{\frac{U}{\nu x}}$ = Growth of drag

Pohlhausen Method is suited for flow with favourable pressure gradient $\frac{dP}{dx} = -ve$.

4.6.6 Thwaites Method

This is further improvement of original Pohlhausen method.

Twaits Proposed a parameter

$$\lambda_1 = \frac{\theta^2}{\nu}\frac{dU}{dx} = \left(\frac{\theta}{\delta}\right)^2 \lambda \qquad \lambda = \frac{\delta^2}{\nu}\frac{dU}{dx}$$

If momentum integral Equation is multiplied by $\frac{U\theta}{\nu}$

$$\frac{\tau_w\,\theta}{\mu U} = \frac{U\theta}{\nu}\frac{d\theta}{dx} + \frac{\theta^2}{\nu}\frac{dU}{dx}(H+2) \quad \text{(A)}$$

$$\frac{\tau_w\,\theta}{\mu U} = S(\lambda_1) \qquad H = \frac{\delta'}{\theta} = H(\lambda_1)$$

$S(\lambda)$ = Shear function

$H(\lambda)$ = Shape function

$$\theta\, d\theta = d\left(\frac{\theta^2}{2}\right)$$

Equation (A) becomes

$$U\frac{d}{dx}\left(\frac{\lambda_1}{\frac{dU}{dx}}\right) \simeq 2\left[S(\lambda) - \lambda(2+H)\right] = F(\lambda_1)$$

He proposed $F(\lambda_1) = 0.45 - 6\lambda_1$

which gave solution

$$\frac{\theta^2}{\cancel{\delta^2}} = \frac{0.45\,\nu}{U^6}\int_0^x U^5\,dx$$

Knowing θ $\quad \lambda = \frac{\theta^2}{\nu}\frac{dU}{dx}$

$$\tau_w = \frac{\mu U}{\theta} S(\lambda) \qquad \delta^* = \theta\, H(\lambda)$$

one curve fit is $\quad S(\lambda) = (\lambda + 0.09)^{0.62}$

Young's Method

12 BR

This method uses relationship between τ_w and Parameter λ

According to Polhausen's Method

$$\frac{\tau_w}{\rho U_\infty^2} = \frac{\nu}{U_\infty \delta}\left(2 + \frac{\lambda}{6}\right) \quad (1)$$

writing $\frac{\delta}{\theta} = f$ $\qquad \lambda = \frac{\delta^2}{\nu} U_\infty' = \frac{\delta^2}{\nu}\frac{dU_\infty}{dx}$

Momentum equation becomes

$$\frac{d\theta}{dx} + \frac{\theta}{U_\infty}(H+2)\frac{dU_\infty}{dx} = \frac{\tau_w}{\rho U_\infty^2} = \frac{\nu\theta}{6 U_\infty}\frac{dU_\infty}{dx} + \frac{2\nu}{f\theta U_\infty} \quad (2)$$

$$\cancel{\theta\frac{d\theta}{dx} = \frac{\theta^2}{U_\infty}\frac{dU_\infty}{dx}}$$

$$\theta\frac{d\theta}{dx} + \frac{\theta^2}{U_\infty}\frac{dU_\infty}{dx}\left(H+2-\frac{f}{6}\right) = \frac{2\nu}{fU_\infty}$$

$$\frac{d\theta^2}{dx} + \frac{\theta^2}{U_\infty} g \frac{dU_\infty}{dx} = \frac{4\nu}{fU_\infty} \quad (3)$$

$$g = 2\left[(H+2) - f/6\right]$$

According to Polhausen λ varies between

+7 near stagnation point $[H = 2.31]$ to

−12 near separation $\qquad (H = 2.74)$

Values of $\frac{\delta}{\theta}$ vary from 7 to 9.5

assume g to be constant

③ become Bto

$$\frac{d}{dx}\left(\theta^2 u_\infty^{g}\right) = \frac{4\nu}{f} u_\infty^{g-1}$$ (13)

$$\theta^2 u_\infty^{g} = \int_0^x \frac{4\nu}{f} u_\infty^{g-1} dx$$

$$\theta^2\Big|_{x_1} = \frac{4\nu}{|u_\infty^{g}|_{x_1}} \int_0^{x_1} \frac{u_\infty^{g-1}}{f} dx \quad (4)$$

Since g is power both at numerator and denominator therefor θ^2 is not very sensitive to g. There g = constant is not a bad assumption.

For flat plate at zero incidence

$$\theta^2 = \frac{4\nu x}{U_\infty f} \rightarrow \theta^2 \frac{U_\infty x}{\nu} = \frac{4x^2}{f}$$

$$\frac{\theta\sqrt{R_x}}{x} = \sqrt{\frac{4}{f}} = \frac{2}{\sqrt{f}}$$

Blasius solut $\frac{\theta\sqrt{R_x}}{x} = 0.664$

∴ To obtain correct value of θ

$$\frac{2}{\sqrt{f}} = 0.664 \quad f = 9.074$$

In flat plate case $H = 2.59$

$$\therefore g = 2\left((H+2) - f/_6\right) = 6.16$$

$$\therefore f = 9.074 \quad g = 6.16$$

(14) — BM

$$\theta^2_{x_1} = \frac{0.441\,\nu}{U_\infty^{6.16}} \int_0^{x_1} U_\infty^{5.16}\, dx$$

This equation is similar to Twaites. Two methods differ a little

Again $u = 0$ at $x = 0$. $U_\alpha = kx$

let $U_\alpha = kx$ and $x = 0$

$$\theta^2_{x_1} = \frac{0.441\,\nu}{(kx)^{6.16}} \int_0^{x} (kx)^{5.16}\, dx$$

$\theta^2 = 0.0716 \frac{\nu}{k}$ at $x = 0$

$$\lambda = \frac{\delta^2 U_\infty'}{\nu} = \frac{f^2 \theta^2 U_\infty'}{\nu}$$

$$C_f = \frac{\tau_w}{\rho U_\infty^2} = \frac{2\nu}{U_\infty f \theta}\left(2 + \frac{\lambda}{6}\right)$$ can be evaluated

Book Problems Hints

Prob 4.1 $\quad \frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

~~B. condition~~ Boundary conditions $\quad u=0 \; y=0 \rightarrow u(0)=0$

$U=U \; y=\delta \rightarrow u(\delta)=U$

$\frac{du}{dy}$ at $y=0 = 0 \left(\frac{d\mathcal{U}}{\partial y}\right)_{y=0} = 0$

Calculate $\delta \; \theta \; \delta^* \; H \; C_f \; C_D$

Compare with Blasius solution $\quad \frac{\partial u}{\partial y^2}(0) = 0$

$$\delta^* = \int_0^\delta \left(\cancel{1} - \frac{u}{U}\right) dy = \int_0^1 \left(1 - \frac{u}{U}\right)\delta\, d\eta \equiv \frac{3}{8}\delta \quad \text{(Prove)}$$

$$\theta = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)\delta\, d\eta = \frac{39}{280}\delta \qquad H = \frac{\delta^\circ}{\theta} = 2.69$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \qquad \tau_w = \mu \frac{du}{\partial y}\Big|_{y=0} \qquad C_f = \frac{3\mu}{\rho \delta U}$$

C_f for Plate is

$$C_f = 2\frac{d\theta}{dx} \qquad \frac{3\mu}{\rho\delta U} = 2\frac{d}{dx}\left(\frac{39\delta}{280}\right)$$

Integrate $0 - x$

$$\frac{\delta}{x} = \frac{?}{\sqrt{R_x}} \qquad R_x = \frac{Ux}{\nu} \quad \nu = \mu/\rho$$

Knowing $\frac{\delta}{x}$ find $\frac{\delta^\circ}{x}, \; \frac{\theta^\circ}{x} \quad \frac{C_f}{x}$

& compare with Blasius solution

4.3 $\quad \frac{u}{U} = \sin\left(\frac{\pi}{2} y/\delta\right)$

follow Question (1) and solve

4.10 Claussen Parameter $= \dfrac{\delta^*}{\tau_w}\dfrac{dP}{dx}$

For Falkner Scan flow (see Book)

$$\eta = y\left[(m+1)\frac{U}{2\nu x}\right]^{\frac{1}{2}} \qquad f(\eta) = \frac{u}{U} \qquad \eta = y/\delta$$

$$\delta^* = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy = \int_0^{1}\left(1-\frac{u}{U}\right)\delta\, d\eta$$

If $\eta^* \triangleq \int_0^1 (1-f'(\eta))\,d\eta \qquad \delta^* = \sqrt{\dfrac{2\nu x}{(m+1)U}}\,\eta^*$

$$\tau_w = \mu\left(\frac{du}{\partial y}\right)_{y=0} = \mu U f'(0)\,\frac{(m+1)U}{2\nu x}$$

outside ~~free~~ stream velocity

$$U = kx^m \qquad \frac{dP}{dx} = -\rho U\frac{dU}{dx}$$

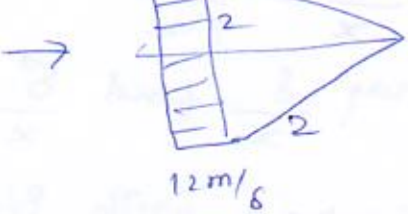
$$\frac{dP}{dx} = -\frac{\rho m U^2}{x}$$

$$\therefore \frac{\delta^*}{\tau_w}\frac{dP}{dx} \equiv \frac{m\,\eta^*}{(m+1)f'(0)}$$ which does not depend on x

and hence is constant

4.12 $T = 20^\circ C \quad P = 100\,kPa$

$\rho = ?$



$12\,m/s$

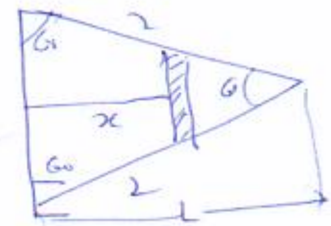
$$\tau_w = f(x) = \left(\mu\frac{du}{dy}\right)_{y=0}$$

τ_w as given in equation (4.52)

$$\tau_w = \frac{\mu U f''(0)}{\sqrt{\frac{2\nu x}{U}}}$$, $f''(0)$ from Table (4.1) $= 0.4690$

$$\tau_w = 0.332\sqrt{\frac{\rho U}{x}}\; U^{3/2}$$

Strip area $L\left(1-\frac{x}{L}\right)dx = (L-x)dx$



$L = 2\sin 60 = 1.732\ m$

$$F = \int_0^L \tau_w\, dA = 0.39\ N$$

Other Problems solve these

① Given velocity Profile for laminar boundary Layer over a flat Plate as

$$\frac{u}{U} = f(\eta) + \lambda G(\eta)$$

U = free stream flow $\eta = y/\delta$)

Using boundary conditions for boundary layer

Prove if $\eta(0)=0$ Then $G(0)=0$

$\eta(1)=1$ Then $G(1)=0$

$f'(0)=0$ Then $G'(0)=0$

$f''(0)=0$ Then $G''(0)=1$

② For The velocity Profile of question ①

if $f(\eta) = \sin\frac{\pi\eta}{2}$ Prove $G(\eta) = \frac{2}{\pi^2}\left[\sin\frac{\pi\eta}{2} + \sin^2\frac{\pi\eta}{2}\right]$

③ For The velocity Profile of question ①

if $f(\eta) = \sin\frac{\pi\eta}{2}$ Prove That Separation occurs at $\lambda = -\frac{\pi^2}{2}$.

(for these question read Approximate methods Theory)