

2nd Mid Exam

Answer the following questions:

(5 Marks)

1 Show that Laguerre's ODE, Eq.

$$xy''(x) + (1 - x)y'(x) + ny(x) = 0.$$

may be put into self-adjoint form by multiplying by e^{-x} and that

$$w(x) = e^{-x} \text{ is the weighting function.}$$

(5 Marks)

2 To a good approximation, the interaction of two nucleons may be described by a mesonic potential attractive V for A negative.

Develop a series solution of the resultant Schrödinger wave equation

$$\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + (E - V)\psi = 0 \quad , \quad V = \frac{Ae^{-ax}}{x}$$

through the first three nonvanishing coefficients.

$$\psi = a_0 \left\{ x + \frac{1}{2}A'x^2 + \frac{1}{6} \left[\frac{1}{2}A'^2 - E' - aA' \right] x^3 + \dots \right\},$$

where the prime indicates multiplication by $2m/\hbar^2$.

(5 Marks)

3 The rate of a particular chemical reaction $A + B \rightarrow C$ is proportional to the concentrations of the reactants A and B :

$$\frac{dC(t)}{dt} = \alpha [A(0) - C(t)][B(0) - C(t)].$$

Find $C(t)$ for $A(0) \neq B(0)$. The initial condition is that $C(0) = 0$.

(5 Marks)

4 The motion of a body falling in a resisting medium may be described by

$$m \frac{dv}{dt} = mg - bv$$

when the retarding force is proportional to the velocity, v . Find the velocity. Evaluate the constant of integration by demanding that $v(0) = 0$.

مع اطيب التمنيات بالنجاح

Mathematical supplement

1-
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$