(8)

(B) $f(z) = \sum_{j=0}^{\infty} a_j (z-z_0)^j + \sum_{j=1}^{\infty} a_{-j} (z-z_0)^{-j}$

$|f| \leq M$ in $r < |z-z_0| < R$

$\because a_j = \frac{1}{2\pi i} \oint_c \frac{f(z)}{(z-z_0)^{j+1}} dz$ and $f^{(j)}(z_0) = \frac{j!}{2\pi i} \oint_c \frac{f(z)}{(z-z_0)^{j+1}} dz$

$\therefore |f^{(n)}(z)| \leq \frac{n!\, M}{R^n}$

now:

\# $|z-z_0| < R$:

$|a_j| = \frac{|f^{(j)}(z_0)|}{j!} \leq \frac{\cancel{j!}\, M}{\cancel{j!}\, R^n} = \frac{M}{R^j}$

وهذا هو المطلوب

\# $|z-z_0| > r$

$|a_{-j}| = \frac{1}{2\pi} \left| \int \frac{f(z)}{(z-z_0)^{-j+1}} dz \right|$ Theorem 5 page. 121

$\leq \frac{1}{2\pi} \frac{M}{r^{-j+1}} \ell(c)$

Length of c
$c : |z-z_0| = r$
$\ell(c) = 2\pi r$

$= \frac{1}{\cancel{2\pi}} \frac{M}{\cancel{r^{-j+1}}} \cdot \cancel{2\pi r}$

$= M r^j$