(d) $|z+1|>1 \Rightarrow \left|\frac{1}{z+1}\right|<1 \Rightarrow \sum\left(\frac{1}{z+1}\right)^j = \frac{1}{1-\frac{1}{z+1}}$ 6

$$\therefore \frac{1}{z(z+1)} = \frac{1}{z+1}\left[\frac{1}{z+1-1}\right]$$

$$= \frac{1}{(z+1)^2}\left[\frac{1}{1-1/z+1}\right]$$

$$= \frac{1}{(z+1)^2}\cdot\sum_{j=0}^{\infty}\left(1/z+1\right)^j$$

$$= \sum_{j=0}^{\infty}(z+1)^{-j-2}$$

$$= \sum_{j=0}^{\infty}(z+j)^{-j}$$

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3) $$\frac{z}{(z+1)(z-2)} = \frac{(1/3)}{z+1} + \frac{(2/3)}{z-2}$$

(a) $|z|<1 \Rightarrow \frac{1}{1-z} = \sum z^j$

نبحث عن متسلسلة $\sum a_j z^j$

$$\therefore \frac{z}{(z+1)(z-2} = \frac{1/3}{z+1} + \frac{(2/3)}{z-2}$$

$$= \frac{1}{3}\left[\frac{1}{1-(-z)}\right] + \frac{2}{3}\left[\frac{1}{-2(1-z/2)}\right] \quad (\because |z|<1 \Rightarrow |z/2|<\tfrac{1}{2}<1$$

$$= \frac{1}{3}\cdot\sum_{j=0}^{\infty}(-z)^j + \frac{1}{3}\sum_{j=0}(z/2)^j$$

$$= \frac{1}{3}\sum_{j=0}^{\infty}(-1)^j z^j - \frac{1}{3}\sum z^j/2^j$$

$$= 1/3\cdot\sum_{j=0}^{\infty}\left((-1)^j - 2^{-j}\right)z^j$$

(b) $1<|z|<2$

$(1-(-\frac{1}{z}))$

$$\therefore \frac{z}{(z+1)(z-2)} = \frac{(1/3)}{(z+1)} + \frac{(2/3)}{(z-2)} = \frac{1}{3}\left[\frac{1}{-z(\quad)}\right] + \frac{(2/3)}{-2(1+z/2)}$$

$$= \frac{1}{3z}\cdot\sum_{j=0}^{\infty}\left(-\frac{1}{z}\right)^j + \frac{1}{3}\sum_{j=0}^{\infty}(+z/2)^j$$

as $|\frac{1}{z}|<1$ as $|2/z|<1$