5.5

(5)

[1] ⓐ Laurant series for $\frac{1}{z+z^2}$ in

ⓐ $0<|z|<1$, ((since $|z|<1 \Rightarrow \sum_{j=0}^{\infty} z^j = \frac{1}{1-z}$))

المطلوب متسلسلة قوى لـ $\underline{(z-0)}$

$$\therefore \frac{1}{z+z^2} = \frac{1}{z(z+1)} = \cancel{\frac{1}{z(z-(-1))}}$$

$$= \frac{1}{z}\frac{1}{1-(-z)} \quad ((\because |z|<1 \Rightarrow |-z|<1 \text{ as } |z|=|-z|))$$

$$= \frac{1}{z}\cdot\sum_{j=0}^{\infty}(-z)^j$$

$$= \sum_{j=0}^{\infty}(-1)^j z^{j-1}$$

ⓑ $|z|>1 \Rightarrow |1/z|<1$ (($\because \frac{1}{1-1/z} = \sum_{j=0}^{\infty}\left(\frac{1}{z}\right)^j$))

المطلوب متسلسلة قوى لـ z

أي مطلوب $\sum \square z^{\square}$

$$\therefore \frac{1}{z+z^2} = \frac{1}{z^2}\cdot\frac{1}{1+\frac{1}{z}} = \frac{1}{z^2}\cdot\frac{1}{1-(-\frac{1}{z})} \quad (\because |\tfrac{1}{z}| = |\tfrac{1}{-z}|<1)$$

$$= \frac{1}{z^2}\cdot\sum_{j=0}^{\infty}\left(-\frac{1}{z}\right)^j$$

$$= \sum_{j=0}^{\infty}\frac{(-1)^j}{z^{j+2}} = \sum_{j=0}^{\infty}(-1)^j z^{-j-2}$$

$$= \sum_{j=2}^{\infty}(-1)^j z^{-j}$$

ⓒ $0<|z+1|<1$ (($\Rightarrow \sum (z+1)^j = \frac{1}{1-(z+1)} = \frac{1}{\cancel{1}-z-\cancel{1}} = \frac{-1}{z}$))

↓

المطلوب متسلسلة قوى لـ $\underline{z+1}$

$$\therefore \frac{1}{z^2+z} = \frac{1}{(z+1)z} = \frac{1}{z+1}\cdot\left(\frac{1}{z}\right)$$

$$= \frac{1}{z+1}\cdot\left(\frac{-1}{1-(z+1)}\right)$$

$$= -\frac{1}{z+1}\cdot\sum_{j=0}^{\infty}(z+1)^j = \sum_{j=0}^{\infty}-(z+1)^{j-1} = -\sum_{j=-1}^{\infty}(z+1)^j$$