(b) because it is a power series converges inside circle of convergence to analytic function.

(c) $f'(z) = -2\frac{z}{3!} + 4\frac{z^3}{5!} - 6\frac{z^5}{7!} + \cdots$

$\to f''(z) = \frac{-2}{3!} + \frac{4(3)z^2}{5!} - \frac{6(5)z^4}{7!} + \cdots$

$\to f^{(3)}(z) = \frac{24z}{5!} - \frac{6(5)(4)z^3}{7!} + \cdots$

$\Rightarrow f^{(3)}(0) = 0$

$f^{(4)} = 24/5! - \frac{6(5)(4)z^2}{7!} + \cdots$

$\to f^{(4)}(0) = 24/5! = 1/5$

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9) Let $P_n(z)$ be a sequence of polynomial converges uniformally to $g(z)$

we know $\int_{|z|=1} P_n(z)\,dz = 0 \quad \forall n$ as P_n analytic on loop

Now, by Theorem 8

$$\int P_n(z)\,dz \longrightarrow \int g(z)\,dz$$

but $\int P_n(z)\,dz = 0 \longrightarrow 0$

$$\therefore \int g(z)\,dz = 0$$

(سهولة نهائيا)

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10) let R_1 be the radius of convergence of $\sum_{k=0}^{\infty} a_k z^k$

and R_2 " " " " $\sum_{k=1}^{\infty} a_k(k) z^{k-1}$

هذا يعني ان نصف قطر التقارب للمتسلسلة ومشتقتها متساوية

Now, $R_1 = 1/L_1$, where $L_1 = \lim_{n\to\infty} \left|\frac{a_{k+1}}{a_k}\right|$

and $R_2 = 1/L_2$, " , $L_2 = \lim \left|\frac{a_{k+1}}{a_k}\frac{(k+1)}{k}\right| = \lim\left|\frac{a_{k+1}}{a_k}\right| = L_2$

$\Rightarrow R_2 = R_1$