(2)

(b) $\sum_{k=0}^{\infty} 2^k (z-1)^k$ "clear that $z_0 = 1$"

$$L = \lim_{k\to\infty} \left| \frac{2^{k+1}}{2^k} \right| = \lim_{k\to\infty} |2| = 2$$

$$\Rightarrow R = 1/L = 1/2$$

$\therefore$ the circle of convergence is $|z-1| = \frac{1}{2}$

(c) $\sum_{j=0}^{\infty} j!\, z^j$, $a_j = j!$

$$L = \lim_{j\to\infty} \left| \frac{a_{j+1}}{a_j} \right| = \lim_{j\to\infty} \left| \frac{(j+1)!}{j!} \right| = \lim_{j\to\infty} (j+1) = \infty$$

$$\Rightarrow R = 1/L = 1/\infty = 0$$

$\therefore$ the circle of convergence is $|z-0| = 0$

$$\Rightarrow |z| = 0 \Rightarrow z = 0$$

the series converge to only at $z=0$

(d) $\sum_{k=0}^{\infty} \frac{(-1)^k}{3^k} (z-i)^k$, $a_k = \frac{(-1)^k}{3^k}$

$$L = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k\to\infty} \left| \frac{(-1)^{k+1}}{3^{k+1}} \cdot \frac{3^k}{(-1)^k} \right| = 1/3$$

$$\Rightarrow R = 1/L = 3$$

$\therefore |z-i| < 3$

(e) $\sum_{k=1}^{\infty} \frac{(3-i)^k}{k^2} (z+2)^k$, $a_k = \frac{(3-i)^k}{k^2}$

$$L = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k\to\infty} \left| \frac{(3-i)^{k+1}}{(k+1)^2} \cdot \frac{k^2}{(3-i)^k} \right| = \lim_{k\to\infty} \left| \frac{(3-i)\, k^2}{(k+1)^2} \right|$$

$$= \lim_{k\to\infty} |3-i| = \sqrt{9+1} = \sqrt{10}$$

$$\Rightarrow R = 1/L = 1/\sqrt{10} \quad , \quad |z+2| = \sqrt{10}$$

(f) $\sum_{j=0}^{\infty} \frac{z^{2j}}{4^j}$, let $y = z^2$ then we have $\sum_{j=0}^{\infty} \frac{(y)^j}{4^j}$

$$L = \lim_{j\to\infty} \left| \frac{a_{j+1}}{a_j} \right| = \lim_{j\to\infty} \left| \frac{1}{4^{j+1}} \cdot 4^j \right| = 1/4 \Rightarrow R = 1/L = 4$$

$\therefore |y - 0| = 4 \Rightarrow |z^2| = 4 \Rightarrow |z| = 2$