1. Find the Fourier series for the function:

$$f(x) = \begin{cases} -1 & -\pi < x < -\pi/2 \\ 0 & -\pi/2 < x < \pi/2 \\ 1 & \pi/2 < x < \pi \end{cases}$$

(Hint: Be careful with the discontinuity problems. Also in your solution make a plot of the function, it will help you a lot).

(5 marks)

2. In right circular cylindrical coordinates a particular vector function is given by $\mathbf{V}(\rho,\varphi) = \hat{\rho}_0 V_\rho(\rho,\varphi) + \hat{\varphi}_0 V_\varphi(\rho,\varphi)$. Show that $\vec{\nabla} \times \mathbf{V}$ has only a z-component.

(5 marks)

3. A calculation of the magneto-hydrodynamics pinch effect involves the evaluation of $\left(\mathbf{B} \cdot \vec{\nabla}\right)\mathbf{B}$. If the magnetic induction $\mathbf{B}$ is taken to be $\mathbf{B} = -\hat{\varphi}_0 B_\varphi(\rho)$, show that $\left(\mathbf{B} \cdot \vec{\nabla}\right)\mathbf{B} = -\hat{\rho}_0 B_\varphi^2 / \rho$.

(5 marks)

4. Working in spherical coordinates prove that:

$$\vec{\nabla} \cdot \hat{\mathbf{r}} f(r) = \frac{2f(r)}{r} + \frac{df}{dr}$$

(5 marks)