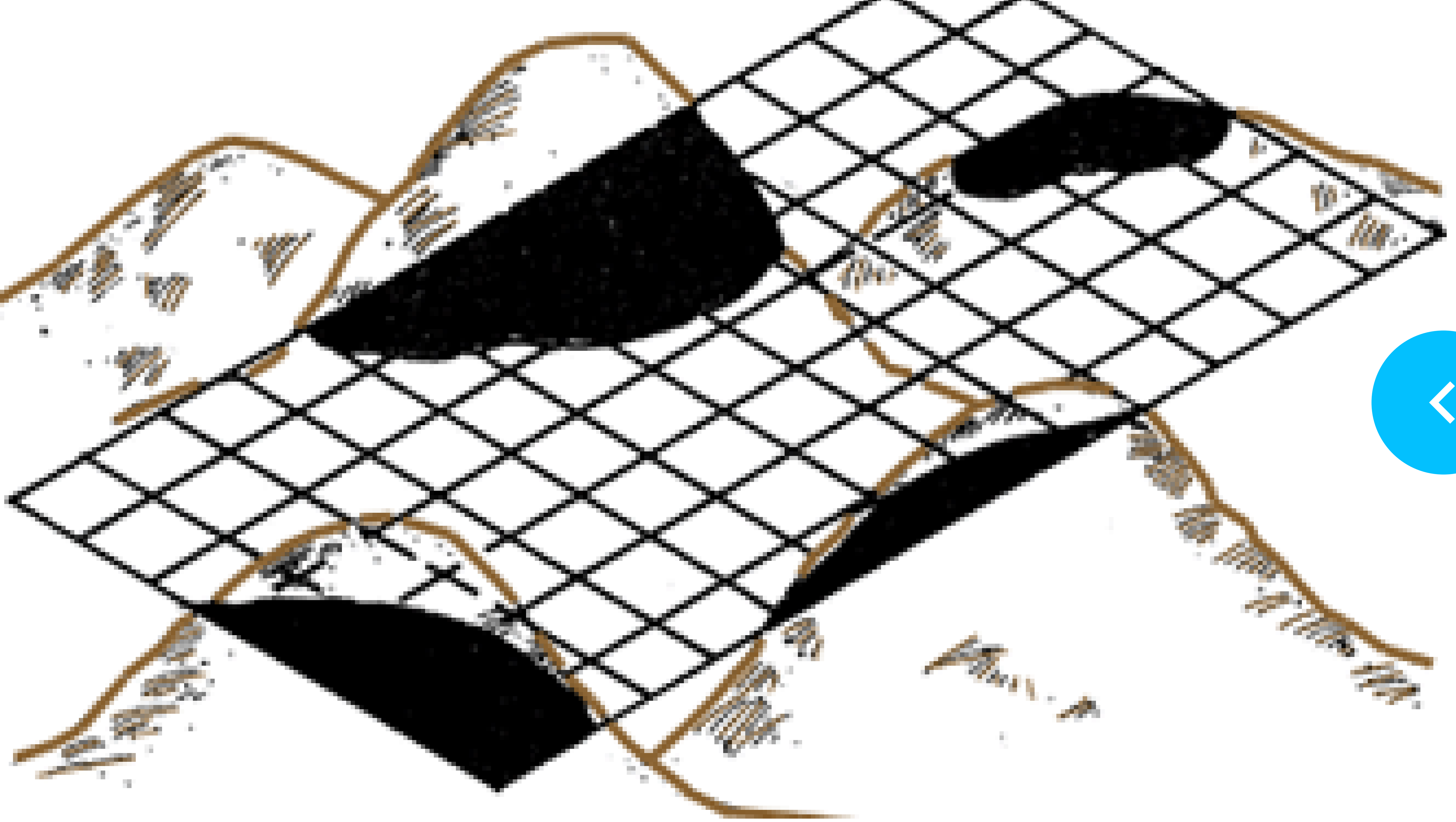
MEASUREMENT OF AREAS

Ibrahim Almohanna, 2019

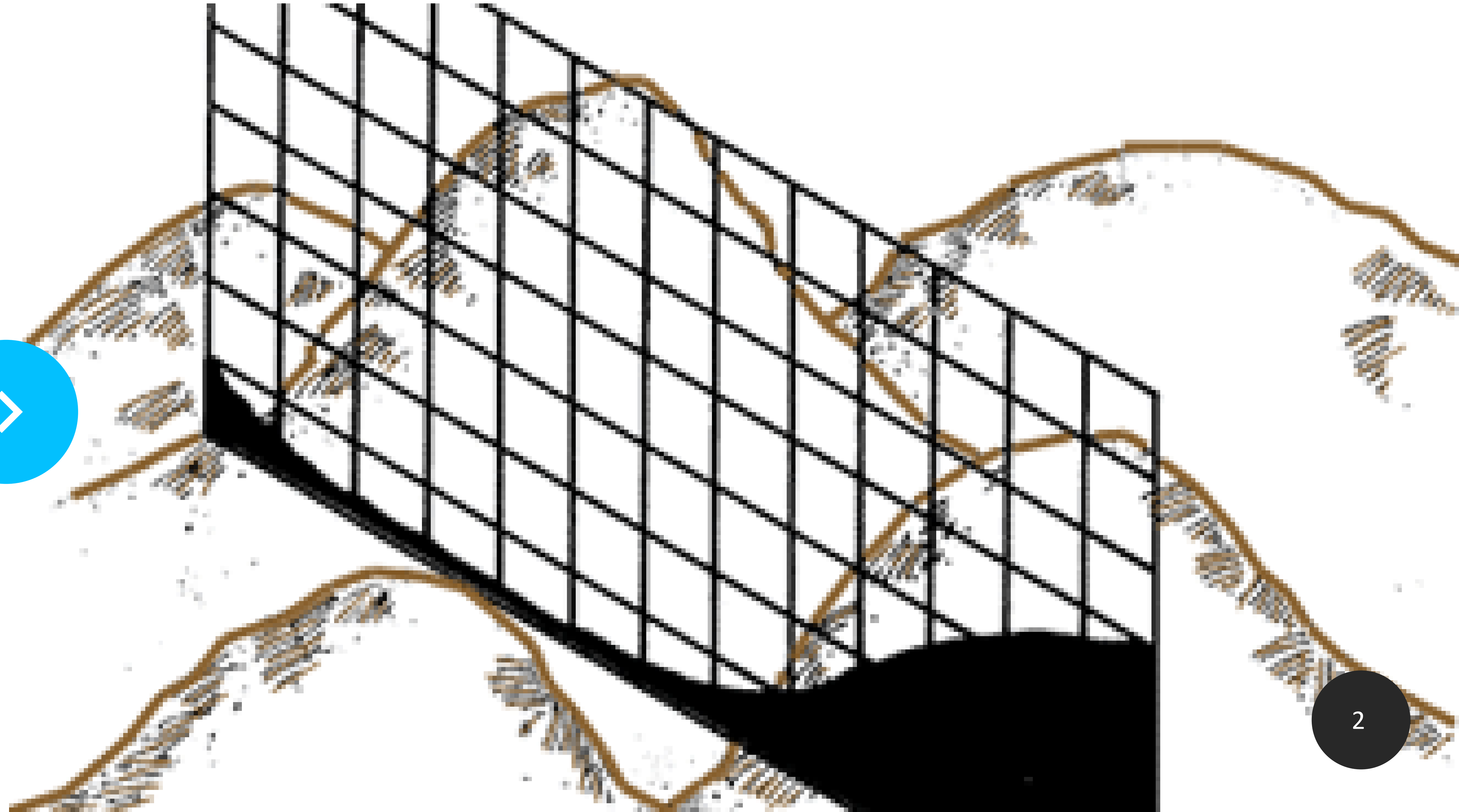


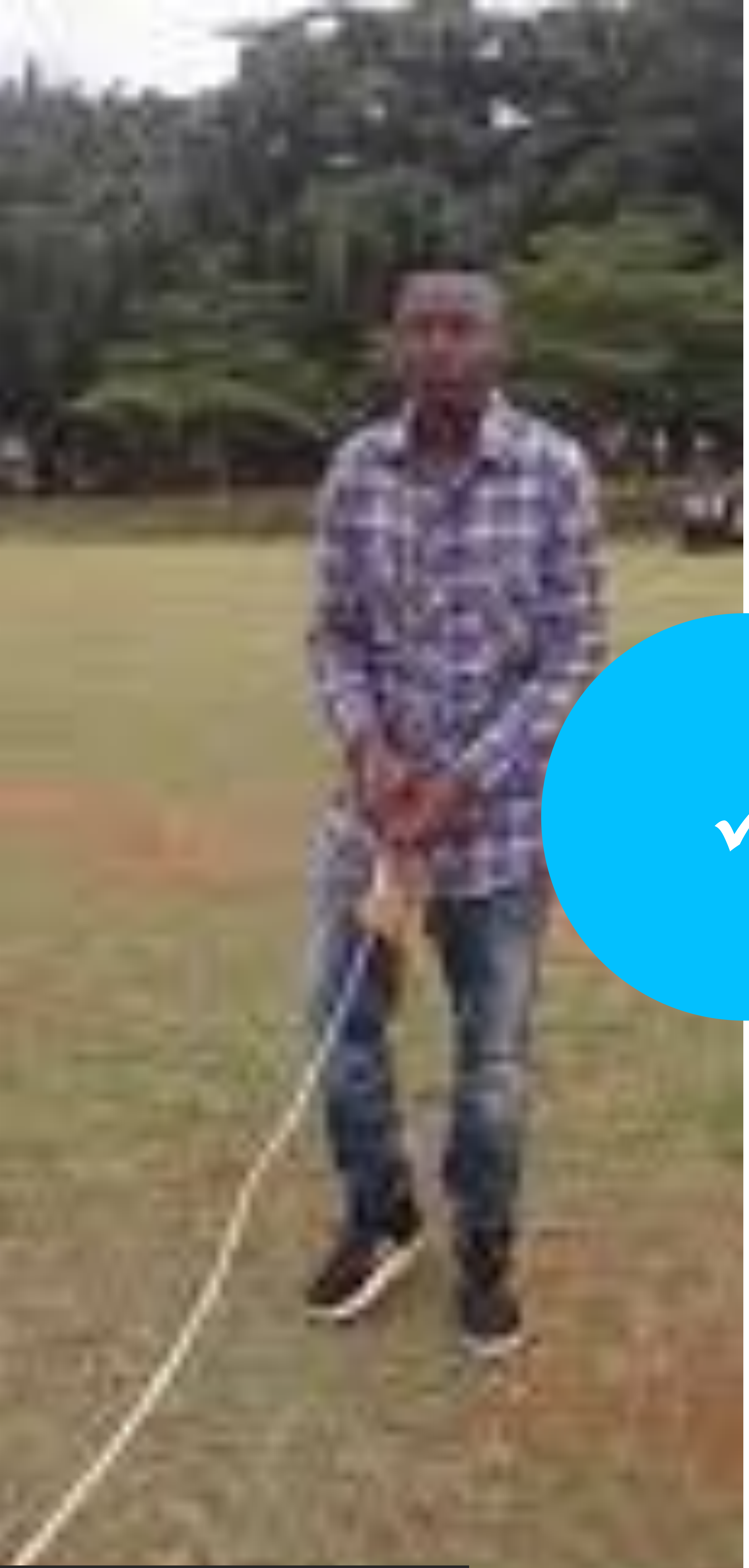
Horizontal Area

in land surveying, you should regard land areas as horizontal surfaces, not as the actual area of the ground surface. You always measure horizontal distances.

Cross-Section Area

You may often need to know the areas of cross-section profiles to calculate the amount of earthwork you need to do. This will be dealt with later in the levelling applications.

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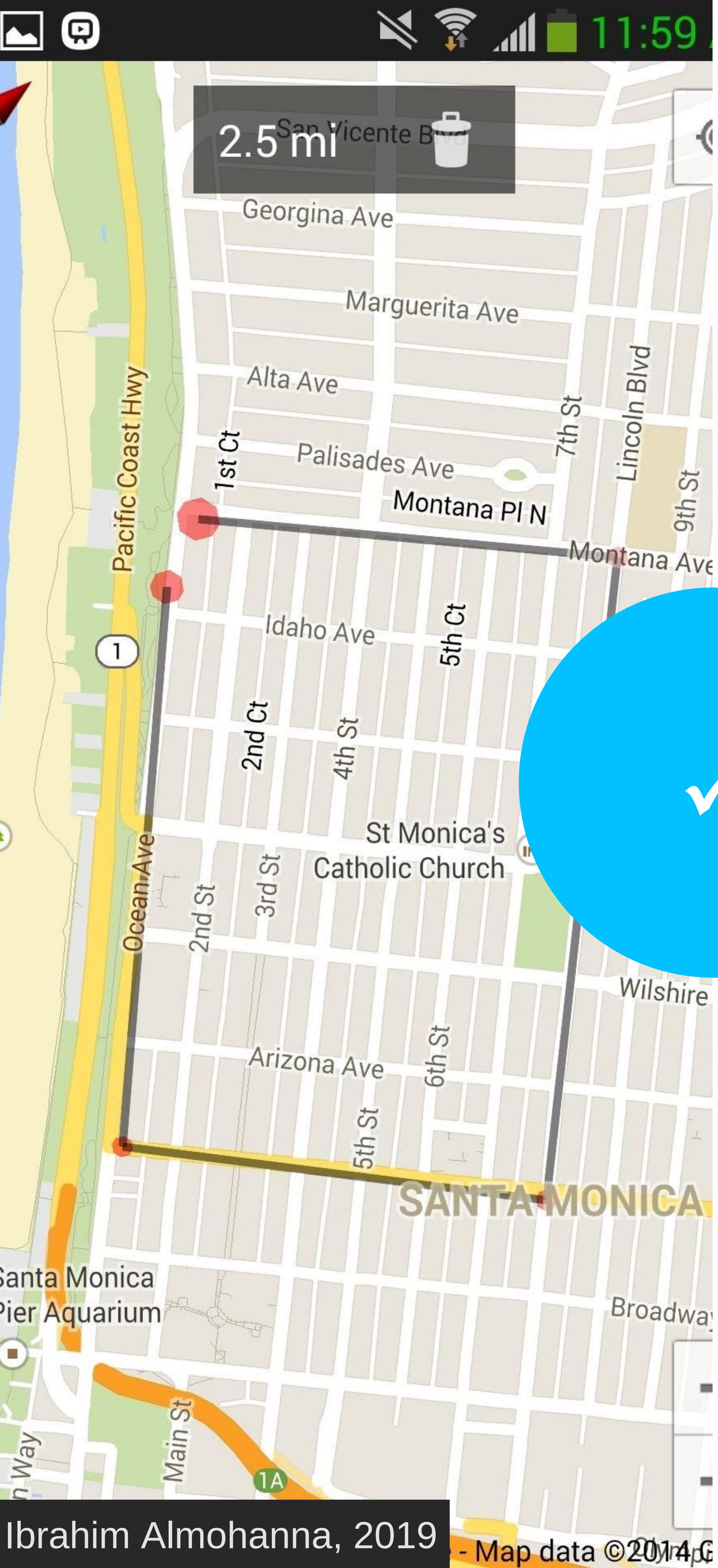
Field and Map Measurements

- Directly from field measurements, you will find all the measurements of distances and angles you need by surveying, and you will calculate the areas from them.



Field and Map Measurements

- Indirectly, from a plan or map. the boundary of the parcel is already plotted on a map of a given scale. You will need to extract the dimensions from the map, compute the map area and reduce to ground using the map scale or reduce the dimensions to ground scale and compute the ground area.

Methods of Measuring Areas

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1. **Graphical**: square grid or strip method
2. **Numerical**: average height, trapezoidal and Simpson's
3. **Instrumental**: mechanical or digital planimeter

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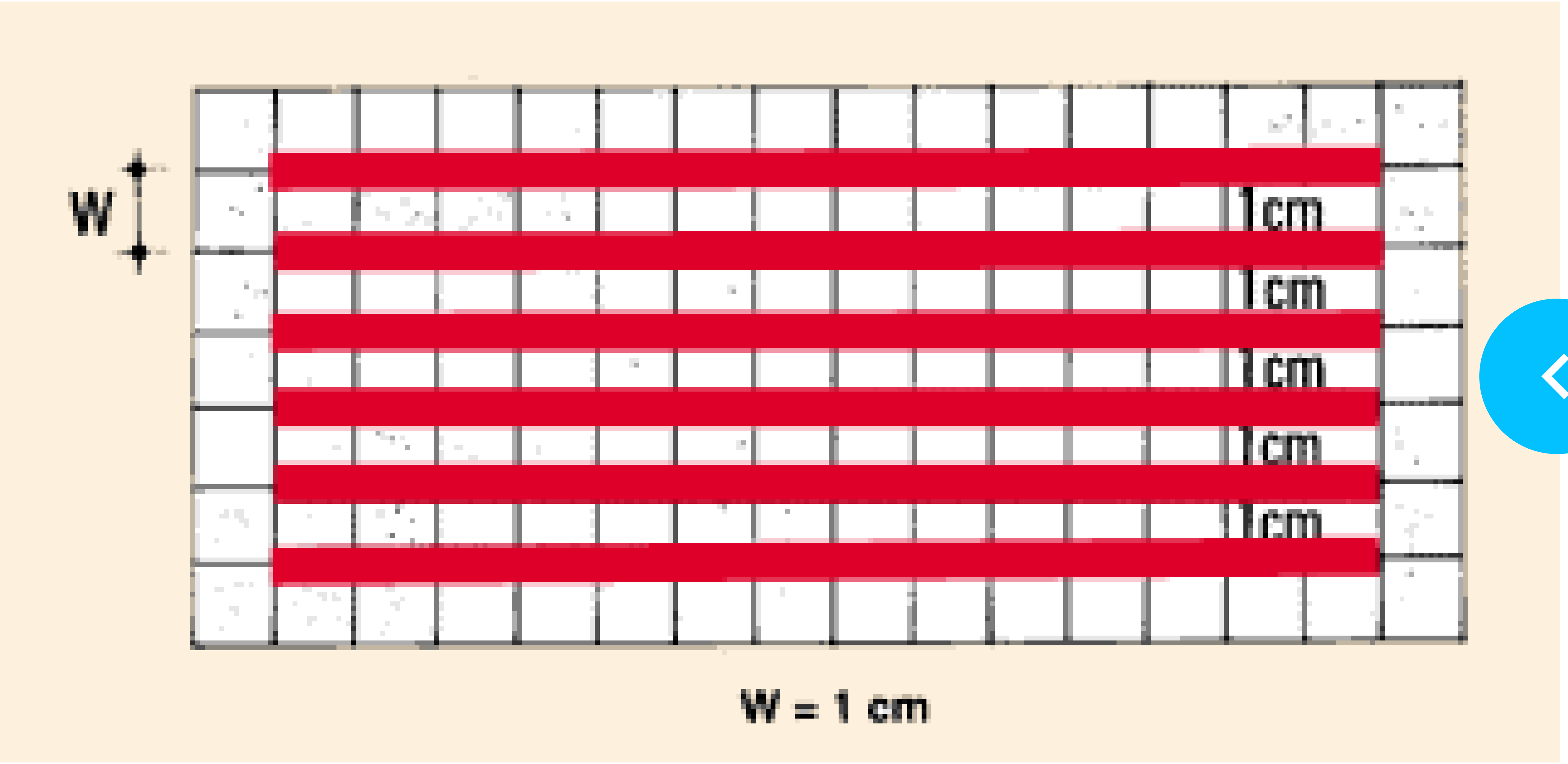
Methods of Measuring Areas

• Strips	• Graphic method giving rough estimate
• Square-grid	• Graphic method giving good estimates
• Subdivision into regular geometric figures	• Geometric method giving very good estimates
• Trapezoidal rule, Simpson's rule	• Geometric method giving good to very good estimates Suitable for curved boundary

Strips Method for Measuring Areas

- 1. Get a piece of transparent paper, such as tracing paper or light-weight square-ruled millimetric paper. Its size will depend on the size of the mapped area you need to measure.



2. On this paper, draw a series of strips, by drawing a series of parallel lines at a regular, fixed interval. Choose this strip width W to represent a certain number of metres.

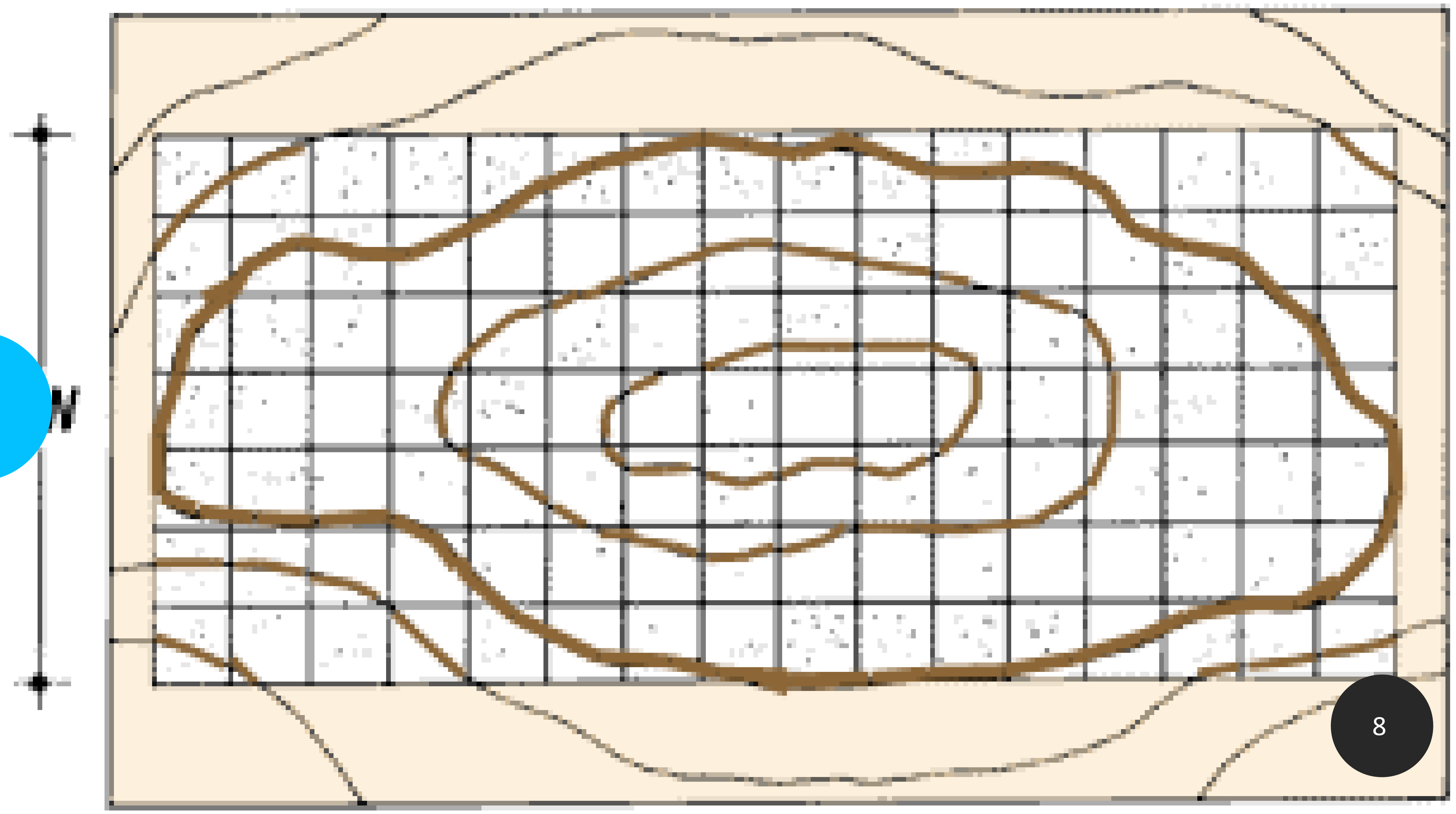
Example

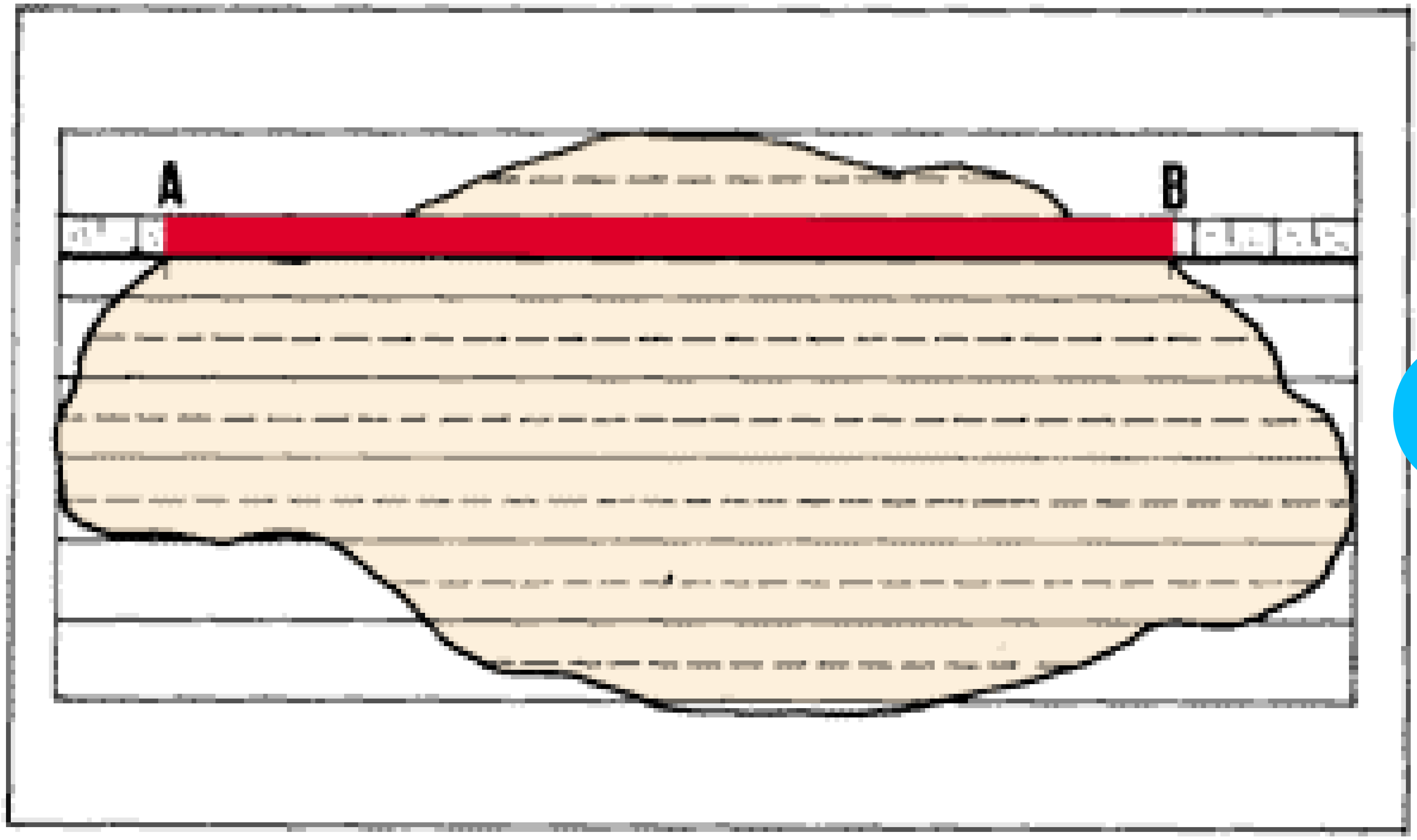
Scale 1: 2 000; strip width W = 1 cm = 20 m.

Scale 1: 50 000; strip width W = 1 cm = 500 m.

Note: the smaller the strip width, the more accurate your estimate of the land area will be.

3. Place the sheet of transparent paper over the plan or map of the area you need to measure, and attach it securely with drawing pins or transparent tape.





4. For each strip, measure the distance AB in centimetres along a central line between the boundaries of the area shown on the map.

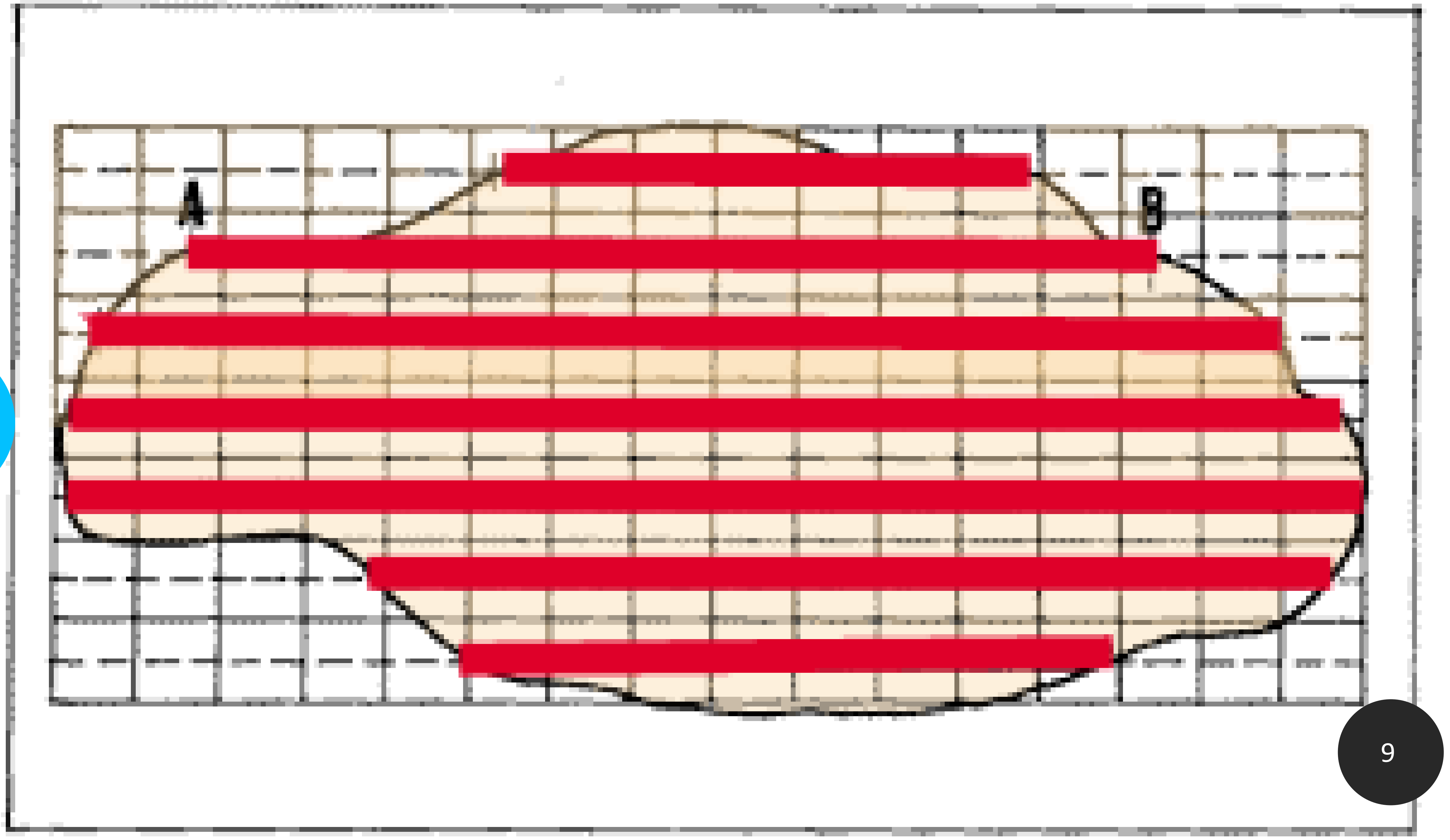
5. Calculate the sum of these distances in centimetres. Then, according to the scale you are using, multiply to find the equivalent distance in the field, in metres.

Example

Scale is 1 :2000 and 1 cm = 20 m.

Sum of distances = 16 cm.

Equivalent ground distance: 16 x 20 m = 320 m.



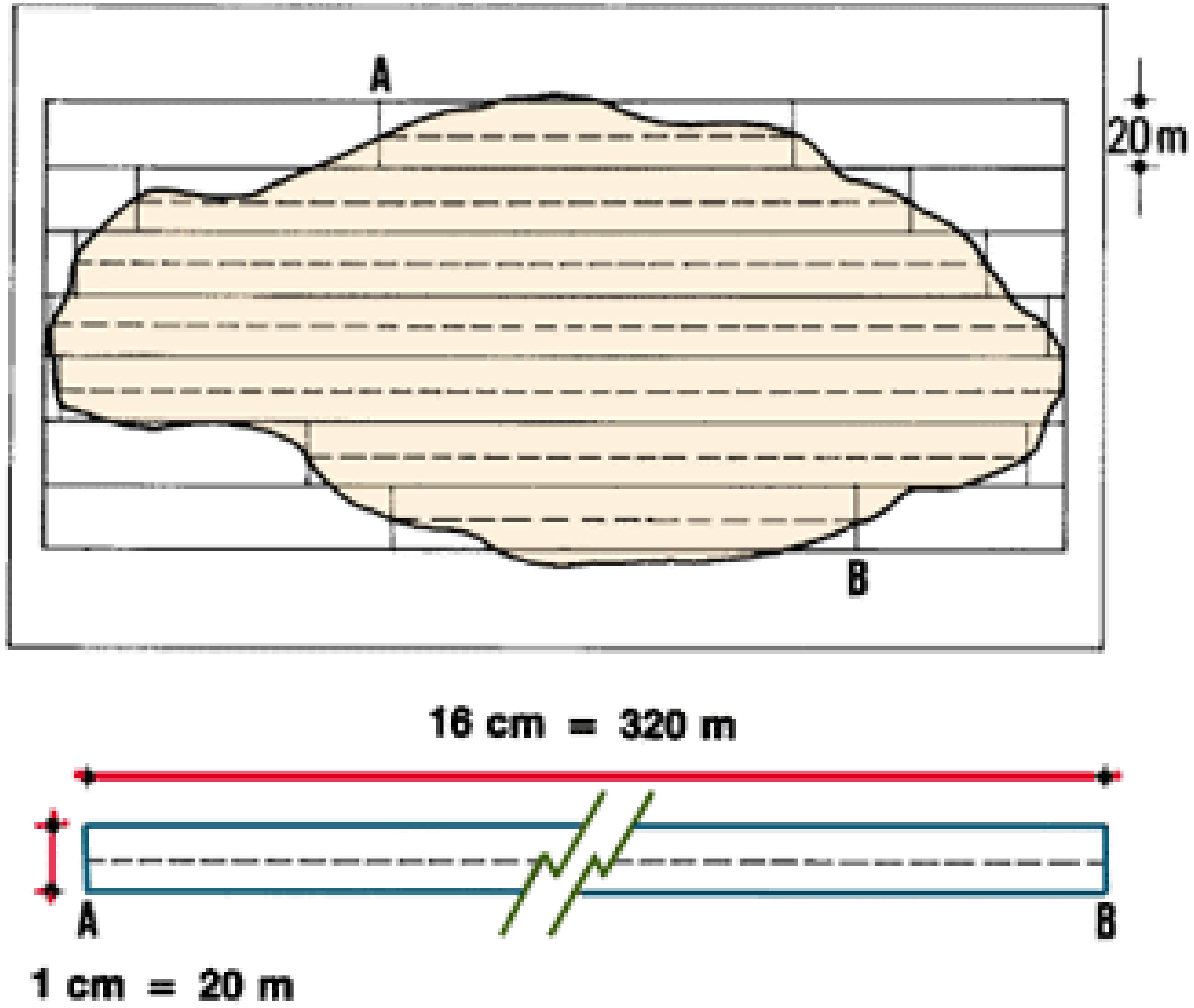
6. Multiply this sum of real distances (in metres) by the equivalent width of the strip W (in metres) to obtain a rough estimate of the total area in square metres.

Example

Sum of equivalent distances is	320 m.
Strip width (1 cm) is equivalent to	20 m.
Land area: 320 m x 20 m =	6 400 m2 or 0.64 ha

Note: 10000 m2 = 1 hectare (ha)

7. Repeat this procedure at least once to check on your calculations.



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Square-grid Method for Measuring Areas

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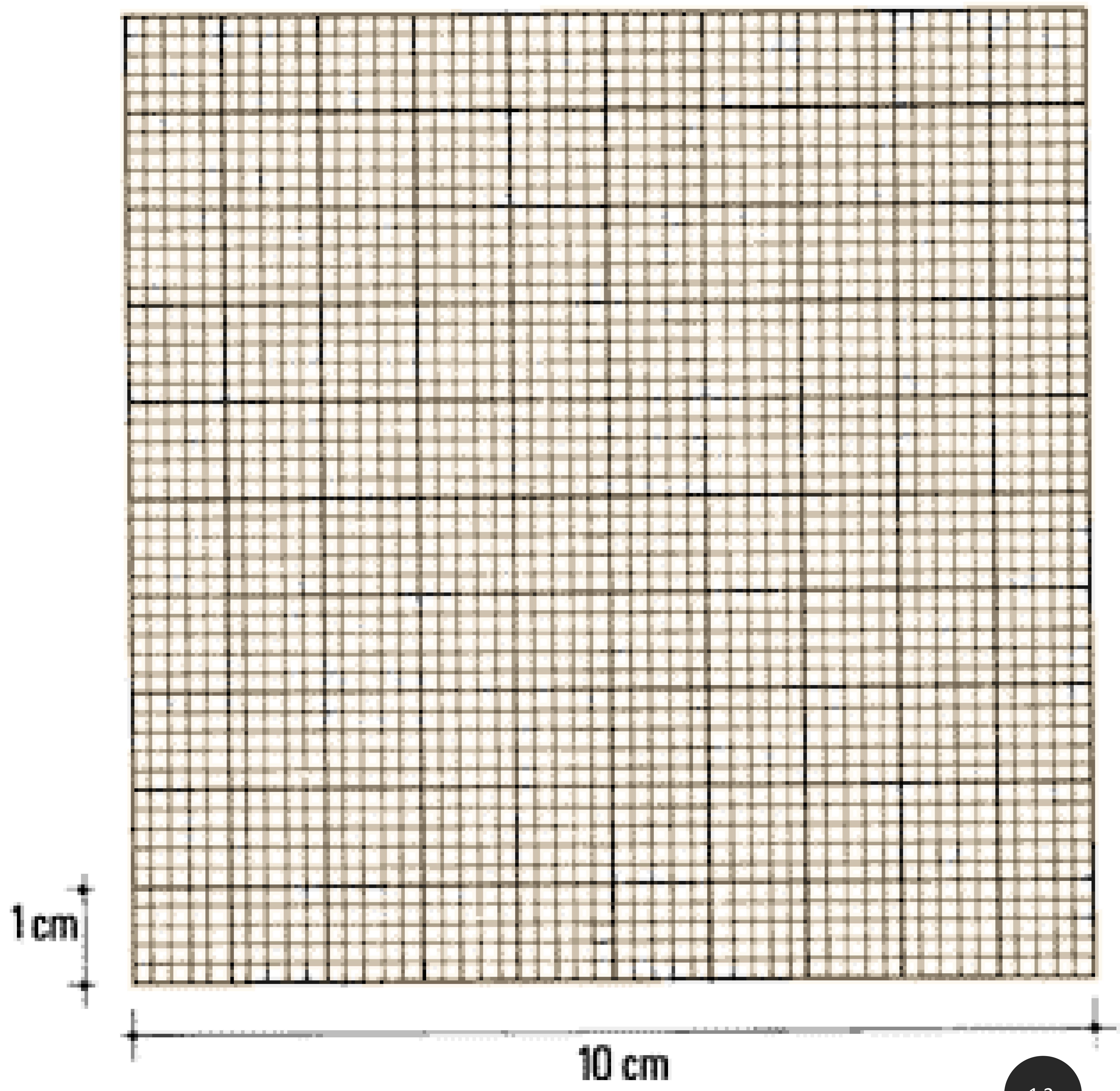
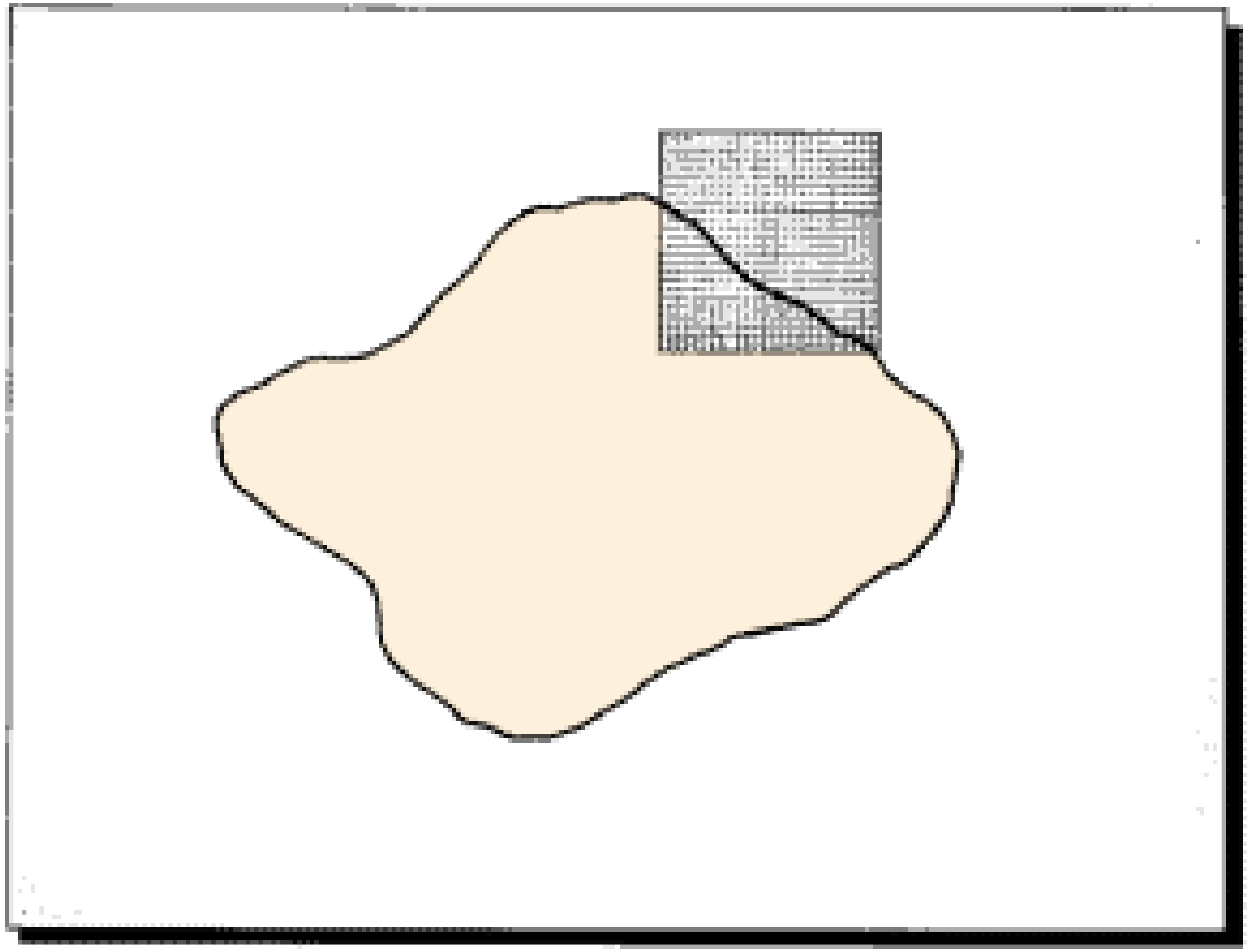
1. Get a piece of transparent square-ruled paper, or draw a square grid on transparent tracing paper yourself. To do this, trace a grid made of 2 mm x 2 mm squares inside a 10 cm x 10 cm square, using the example given on the page.

Note: if you use smaller unit squares on the grid, your estimate of the land area will be more accurate; but the minimum size you should use is 1 mm x 1 mm = 1 mm2.



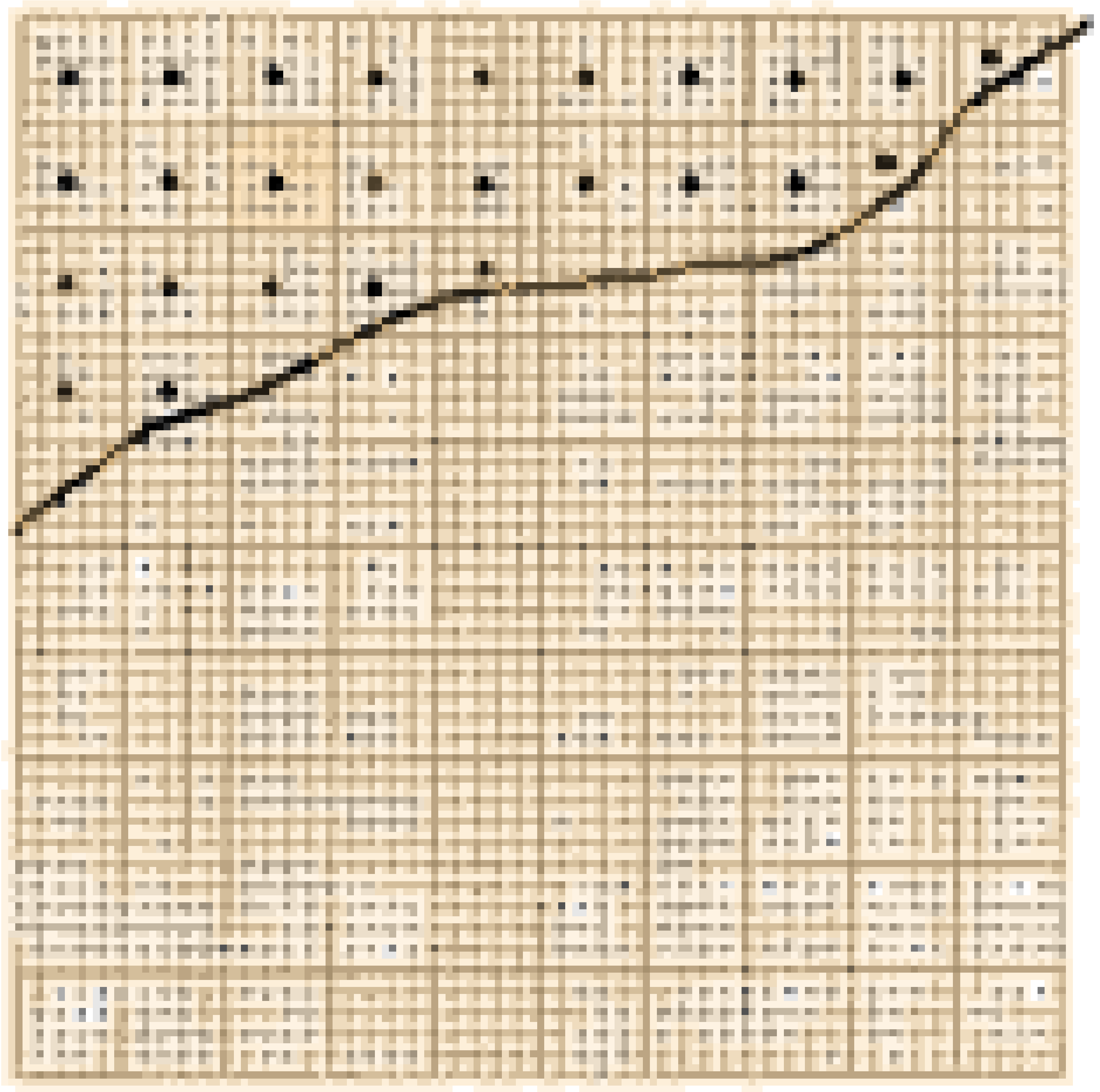
2. Place this transparent grid over the drawing of the area you need to measure, and attach it to the drawing securely with thumbtacks or tape. If your grid is smaller than this area, start at one edge of the drawing. Clearly mark the outline of the grid, then move to the next section and proceed in this way over the entire area.



3. Count the number of full squares included in the area you need to measure. To avoid mistakes, mark each square you count with your pencil, making a small dot.

Note: towards the centre of the area, you may be able to count larger squares made, for example, of 10 x 10= 100 small squares. This will make your work easier.

4. Look at the squares around the edge of the drawing. If more than one-half of any square is within the drawing, count and mark it as a full square. Ignore the rest.



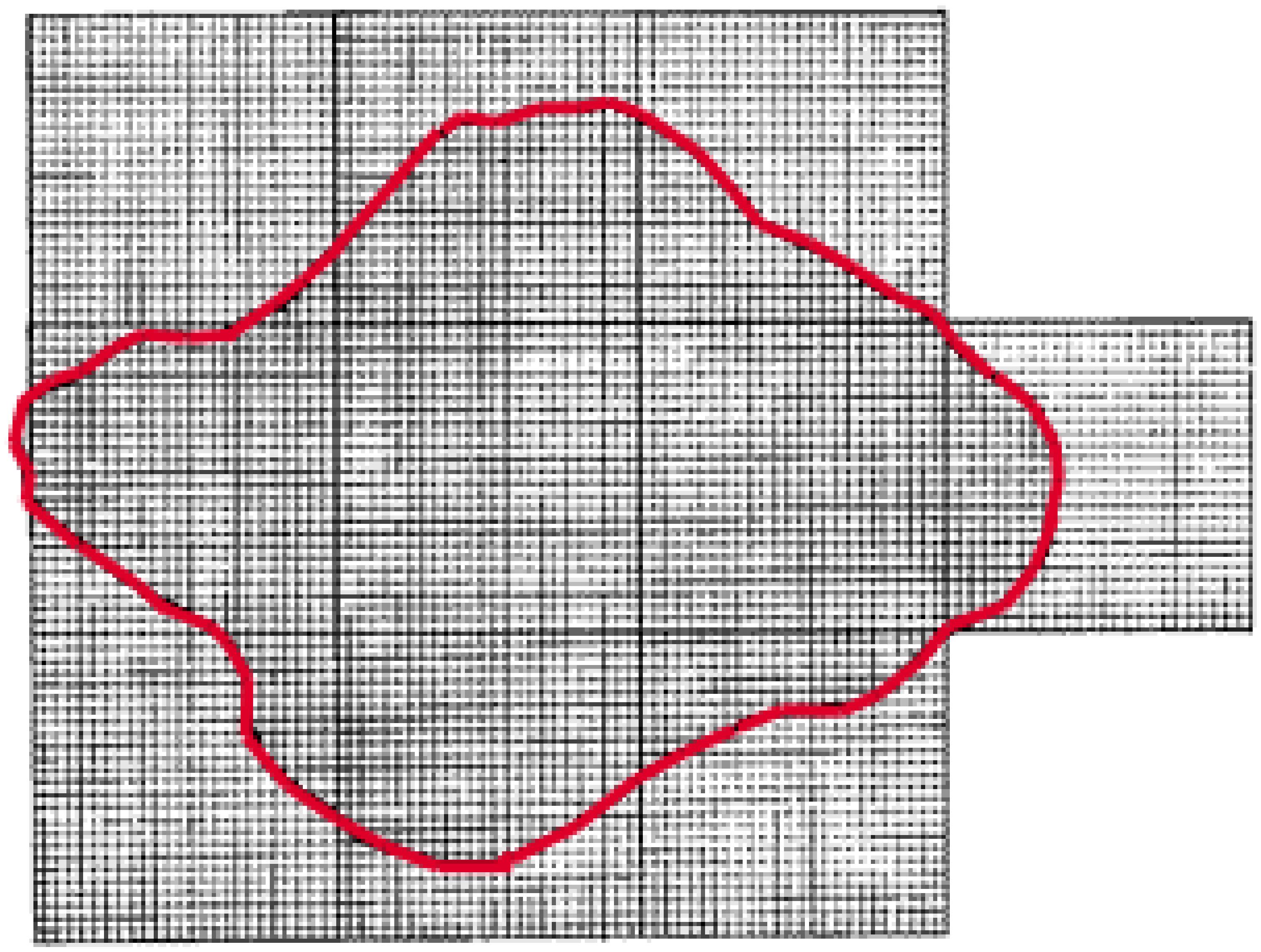
5. Add these two sums (steps 3 and 4), to obtain the total number T of full squares.

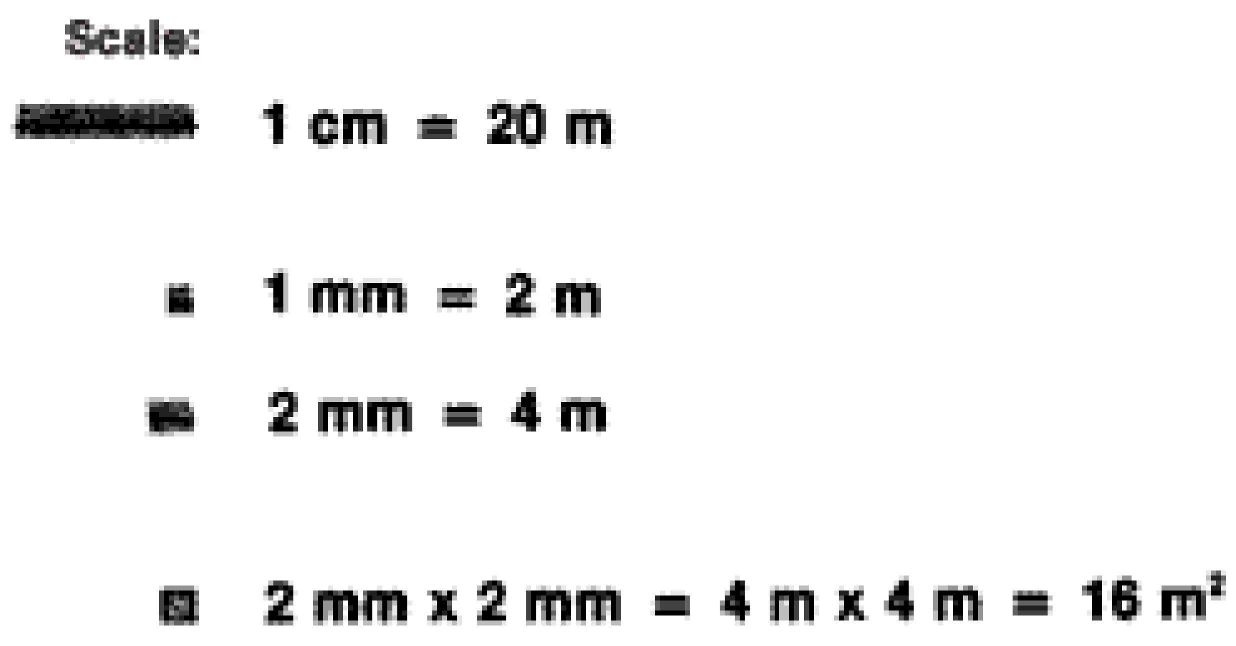
6. Add the sums again at least once to check them.

7. Using the distance scale of the drawing, calculate the equivalent unit area for your grid. This is the equivalent area of one of its small squares.

Example

- Scale 1:2000 or 1 cm = 20 m or 1 mm = 2 m
- Grid square size is 2 mm x 2 mm
- Equivalent unit area of grid = 4 m x 4 m = 16 m^2





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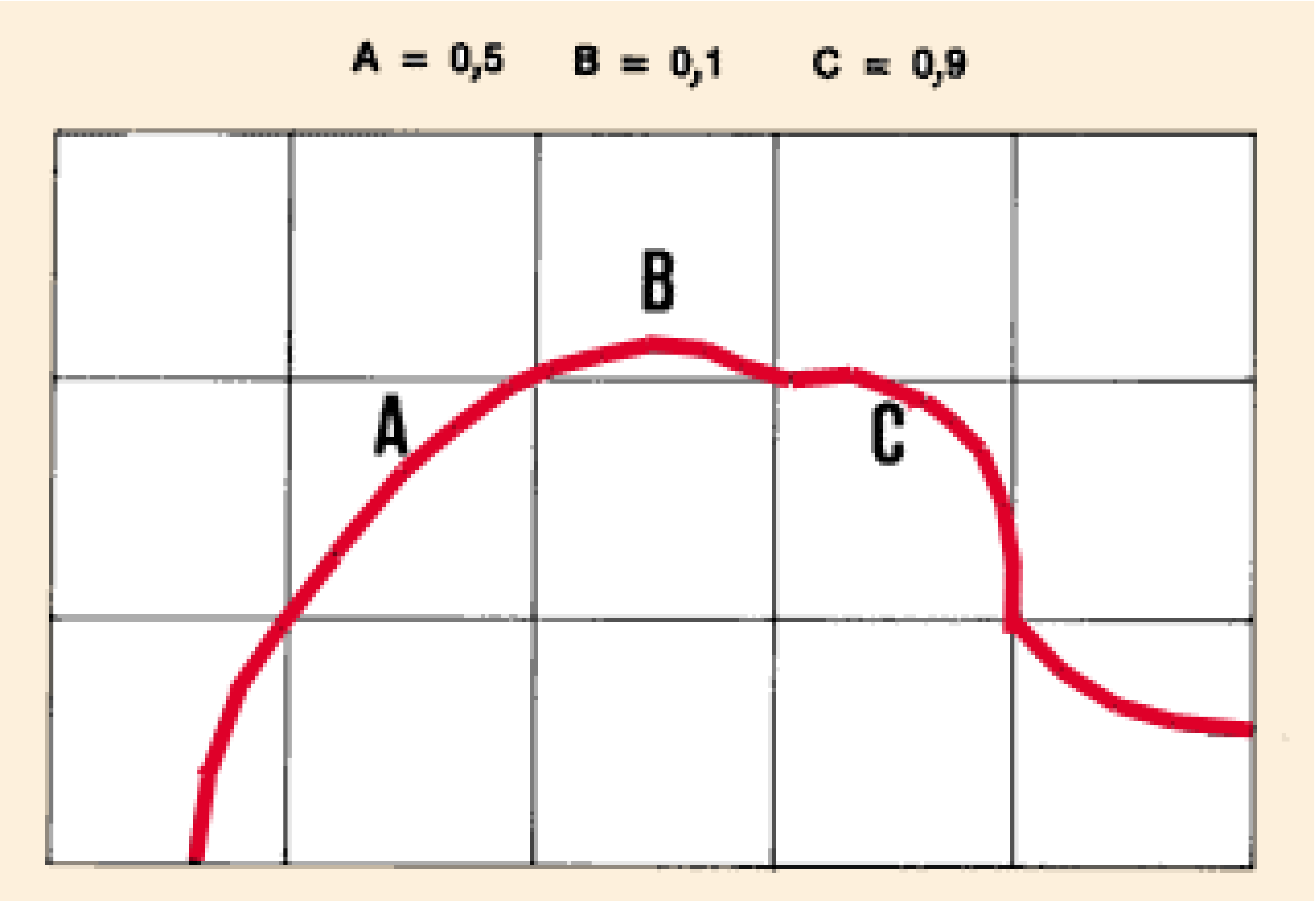
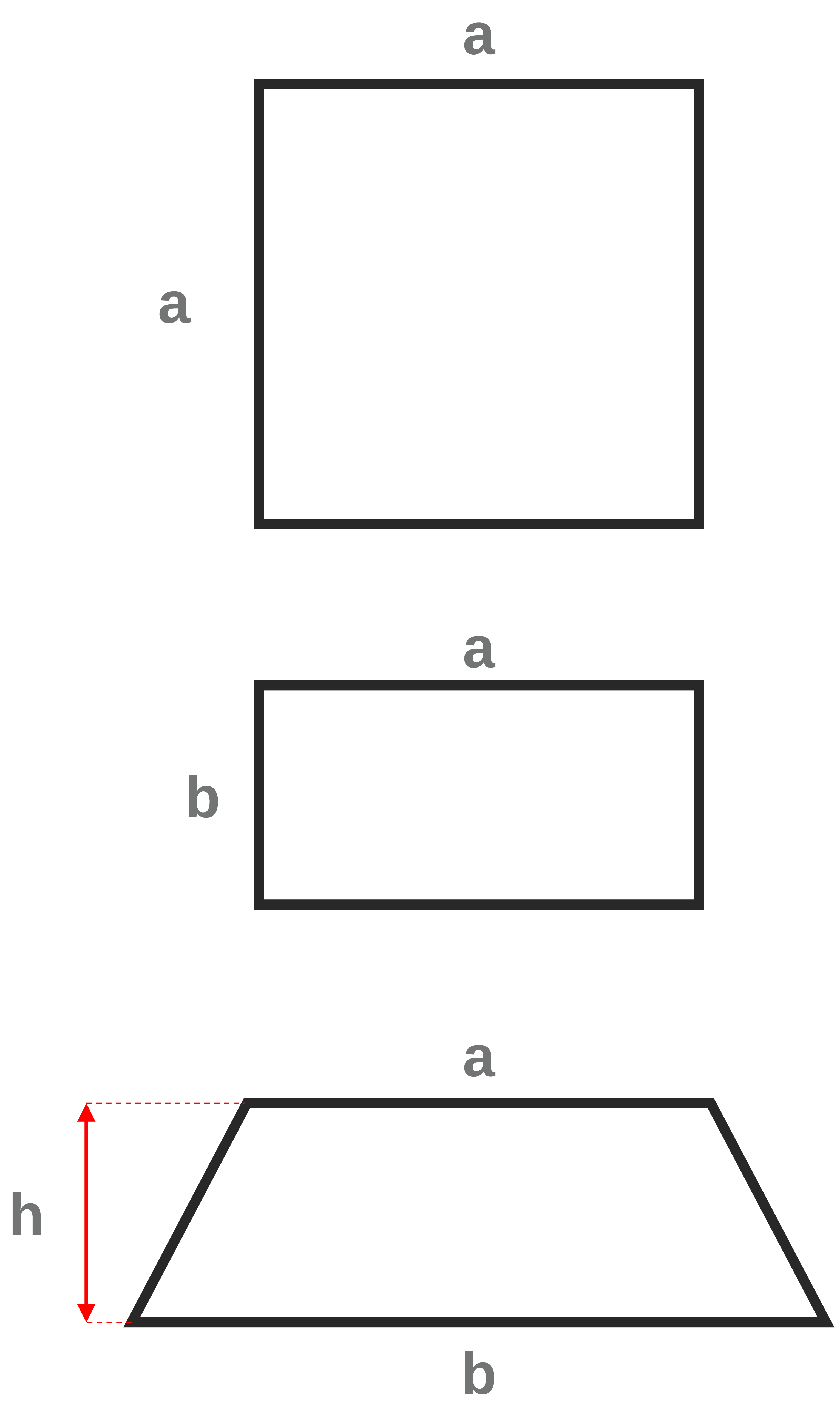
8. Multiply the equivalent unit area by the total number T of full squares to obtain a fairly good estimate of the measured area.

Example

- Total count of full squares T = 256
- Equivalent unit area = $16m^2$
- Total area = 256 x 16 m2 = 4096 m^2



Note: when you work with large-scale plans such as cross-sections, you can improve the accuracy of your area estimate by modifying step 5, above. To do this, look at all the squares around the edge of the drawing which are crossed by a drawing line. Then, estimate by sight the decimal part of the whole square that you need to include in the total count (the decimal part is a fraction of the square, expressed as a decimal, such as 0.5,0.1 and 0.9).



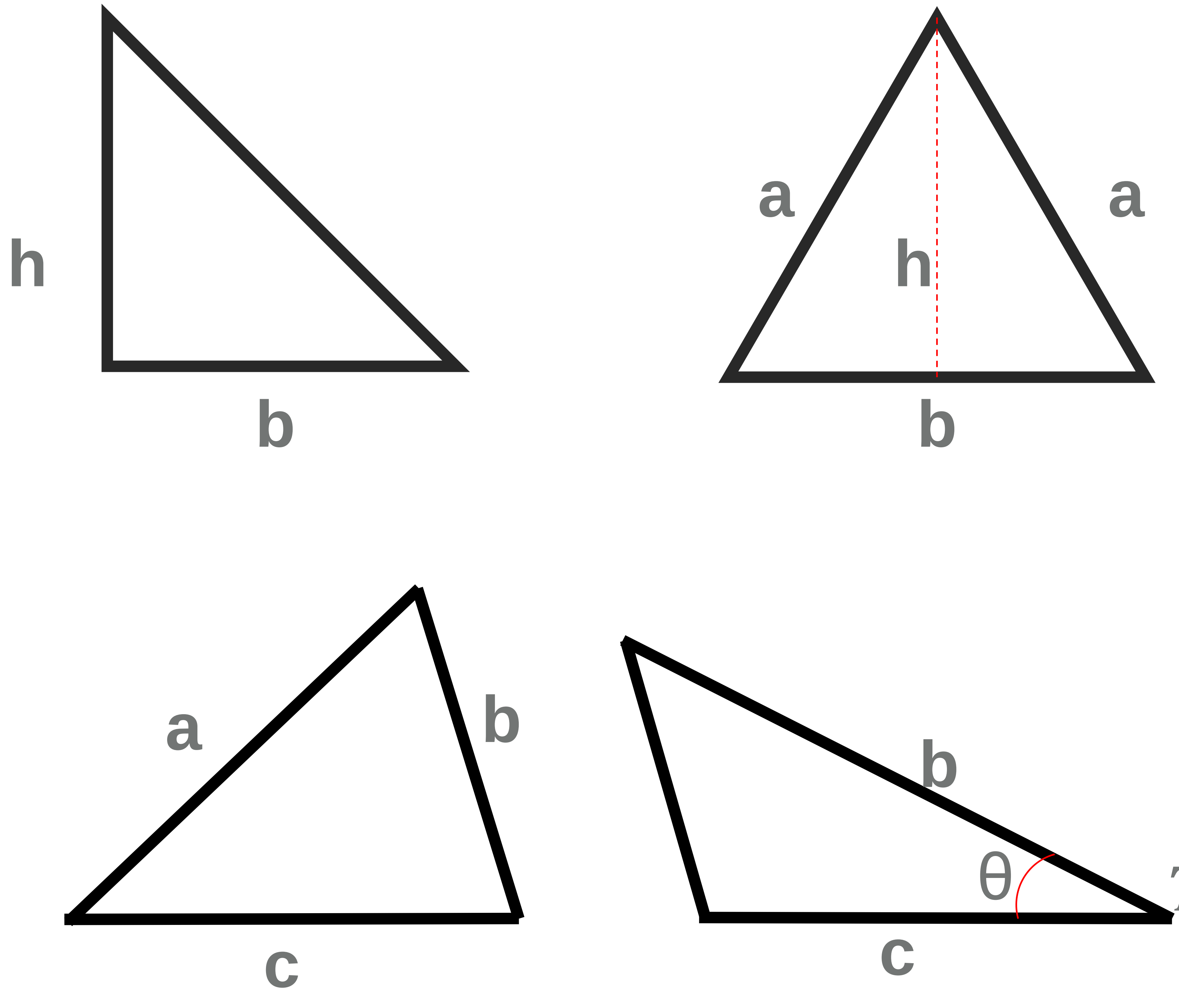
Area of Regular Geometric Figures Using Mathematical Models

$$Square\ Area = a^2$$

$$Rectangular\ Area = a \times b$$

$$Trapizoidal\ \ Area = (a + b) \times h \div 2$$

Area of Regular Geometric Figures Using Mathematical Models

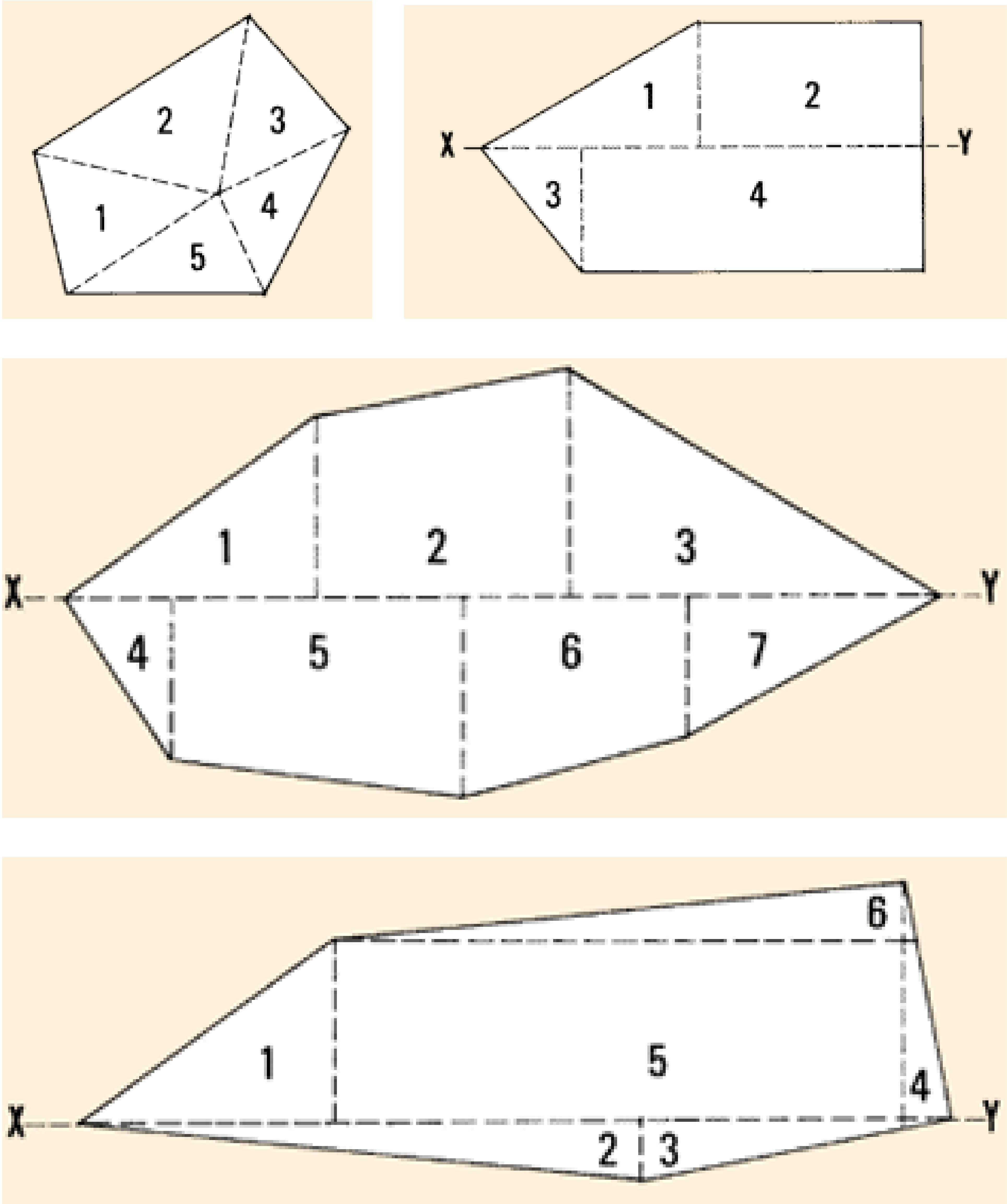


$$Right\ Angle\ Triangle\ Area = b \times h \div 2$$

$$Non - symetric\ Triangle\ Area = \sqrt{s \times (s - a) \times (s - b) \times (s - c)}$$

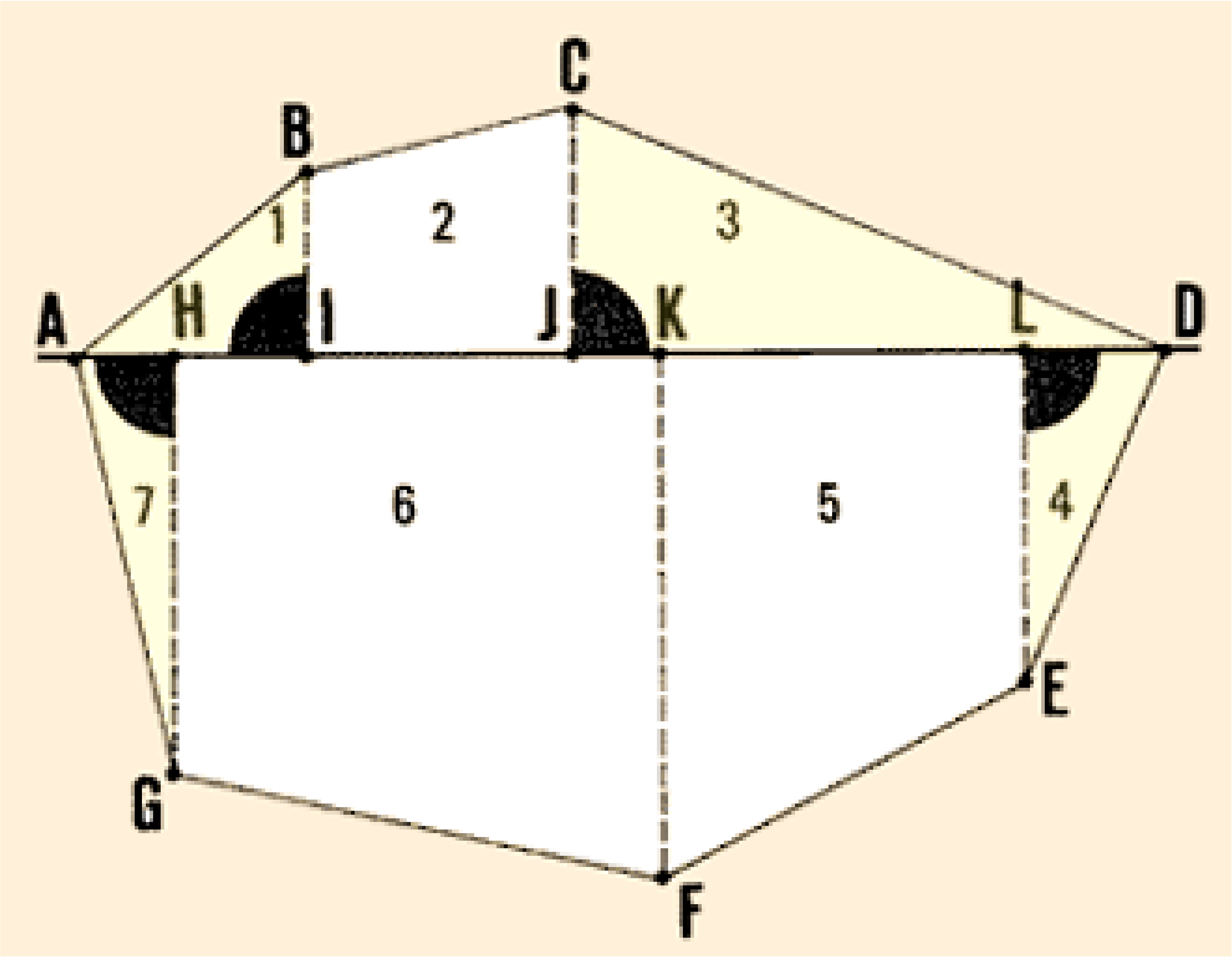
$$s = (a\ +\ b\ +\ c) \div 2$$

$$Triangle\ of\ two\ sides\ and\ angle\ Area = (b\ *\ c \sin \theta)\ \div\ 2$$



Subdivision of The Area Into Regular Geometrical Figures

When you need to measure areas directly in the field, divide the tract of land into regular geometrical figures, such as triangles, rectangles or trapeziums. Then take all the necessary measurements, and calculate the areas according to mathematical formulas. If a plan or map of the area is available, you can draw these geometrical figures on it, and find their dimensions by using the reduction scale.



Using a Base Line to Subdivide Land Areas

- When the shape of the land is polygonal, you should usually subdivide the total area you need to measure into a series of regular geometrical figures from a common base line AD.
- You will lay out offsets from the other summits of the polygon which are perpendicular to this base line to form:
- right triangles (1,3,4 and 7),
- trapeziums (2, 5 and 6).

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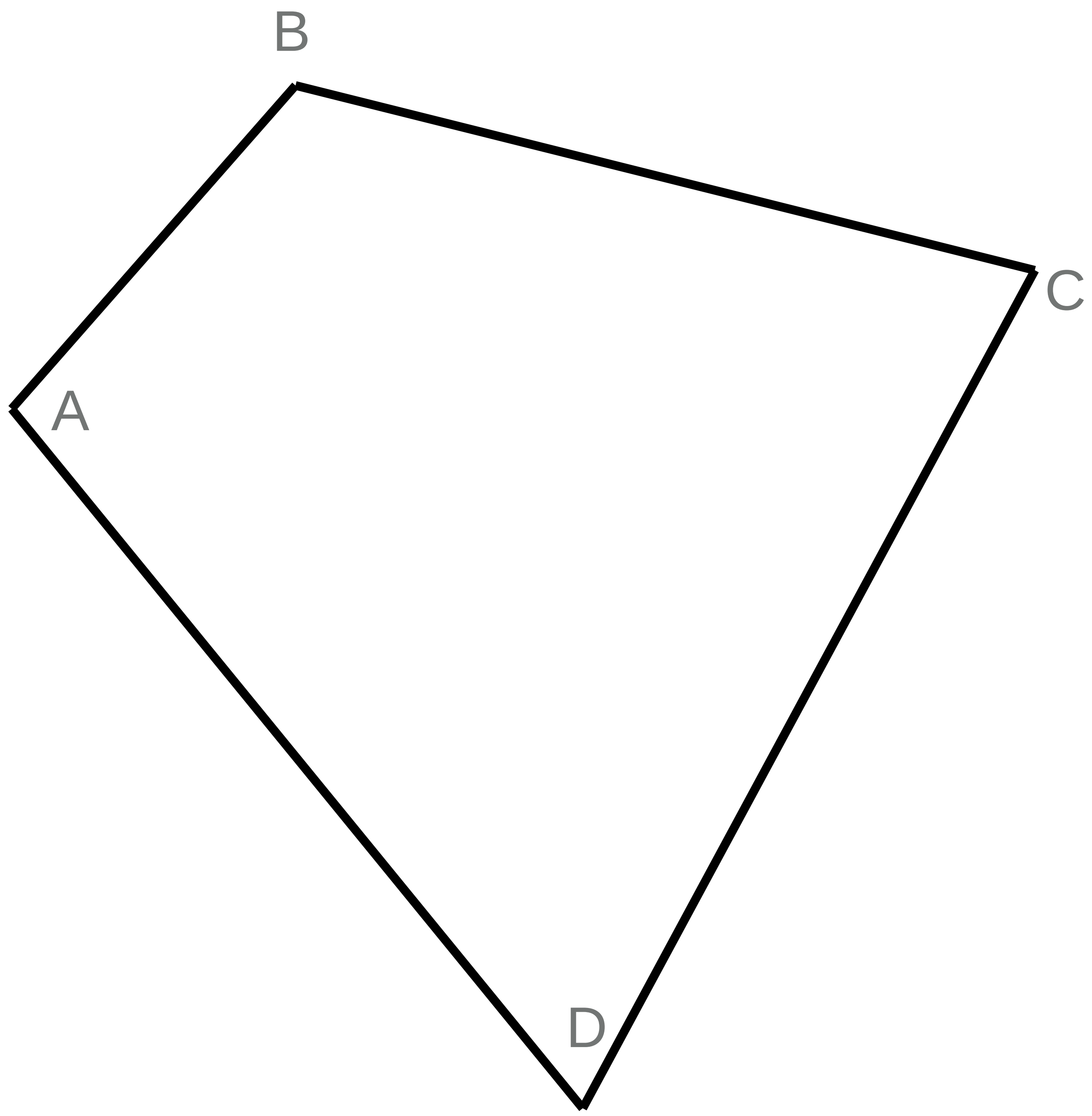
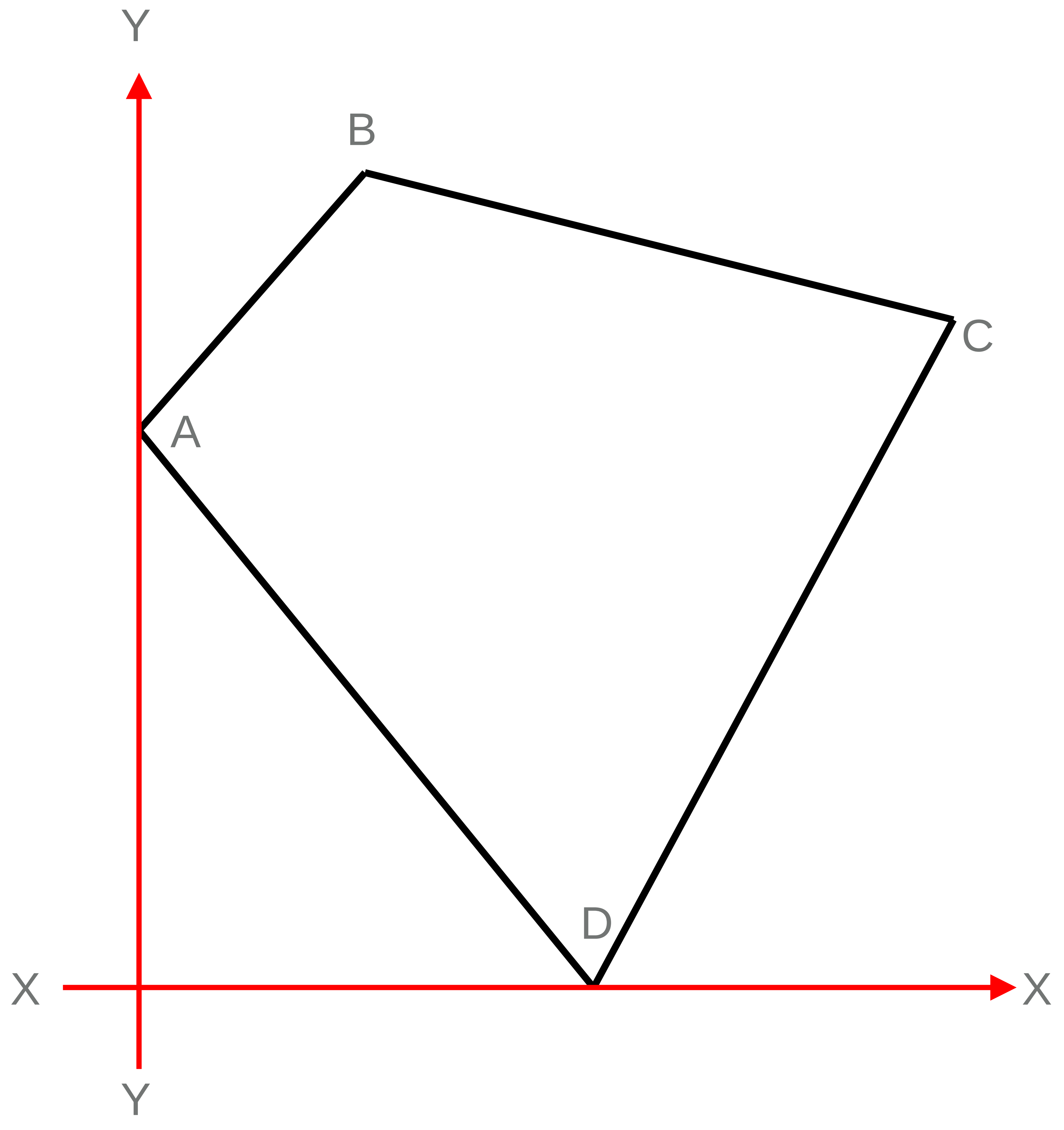
Using a Base Line to Subdivide Land Areas

- When you are choosing a base line, remember that it should:
- be easily accessible along its entire length;
- provide good sights to most of the summits of the polygon;
- be laid out along the longest side of the land area to keep the offsets as short as possible;
- join two polygon summits.



Area by Coordinates

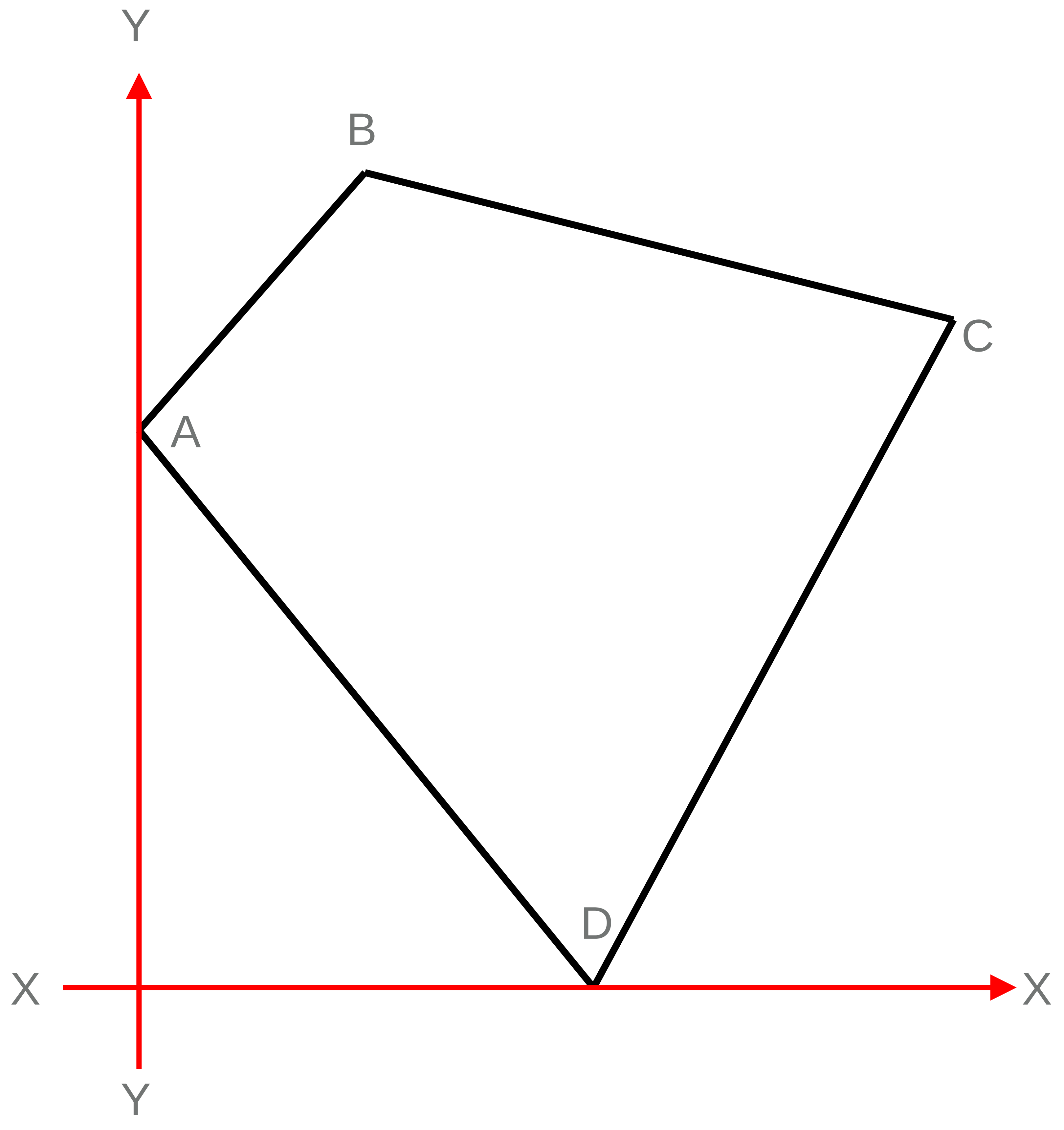
- Set coordinates for the corners of the traverse we are working on.
- To avoid using negative coordinates, measure Y coordinates from an X axis passing through the most southerly station and X coordinates from a Y axis passing through the most westerly station.



Area by Coordinates

- Given the coordinates of the corners of the area ABCDA (in m) as in Table below, compute the area enclosed by corners ABCDA:

STATION	A	B	C	D
X	0	125.66	716.31	523.62
Y	591.64	847.60	694.07	0



Area by Coordinates

- First, multiply pairs of diagonally opposite X and Y coordinates and determine the sum of the products.
- Then, multiply pairs diagonally in the opposite direction and determine the sum of the products.
- The difference between the sums is the double area or
- 1,044,918.76 – 397,011.37 = 647,907.39 square m.
- ➔ Actual area = 323,953.695 square m.

Area by Coordinates

STATION	COORDINATES Y	X
A	591.64	0
B	847.60	125.66
C	694.07	716.31
D	0	523.62
A	591.64	0

591.64 × 125.66 = 74,345.48

847.60 × 716.31 = 607,144.35

694.07 × 523.62 = 363,428.93

0 × 0 = 0

Σ → 1,044,918.76

STATION	COORDINATES Y	X
A	591.64	0
B	847.60	125.66
C	694.07	716.31
D	0	523.62
A	591.64	0

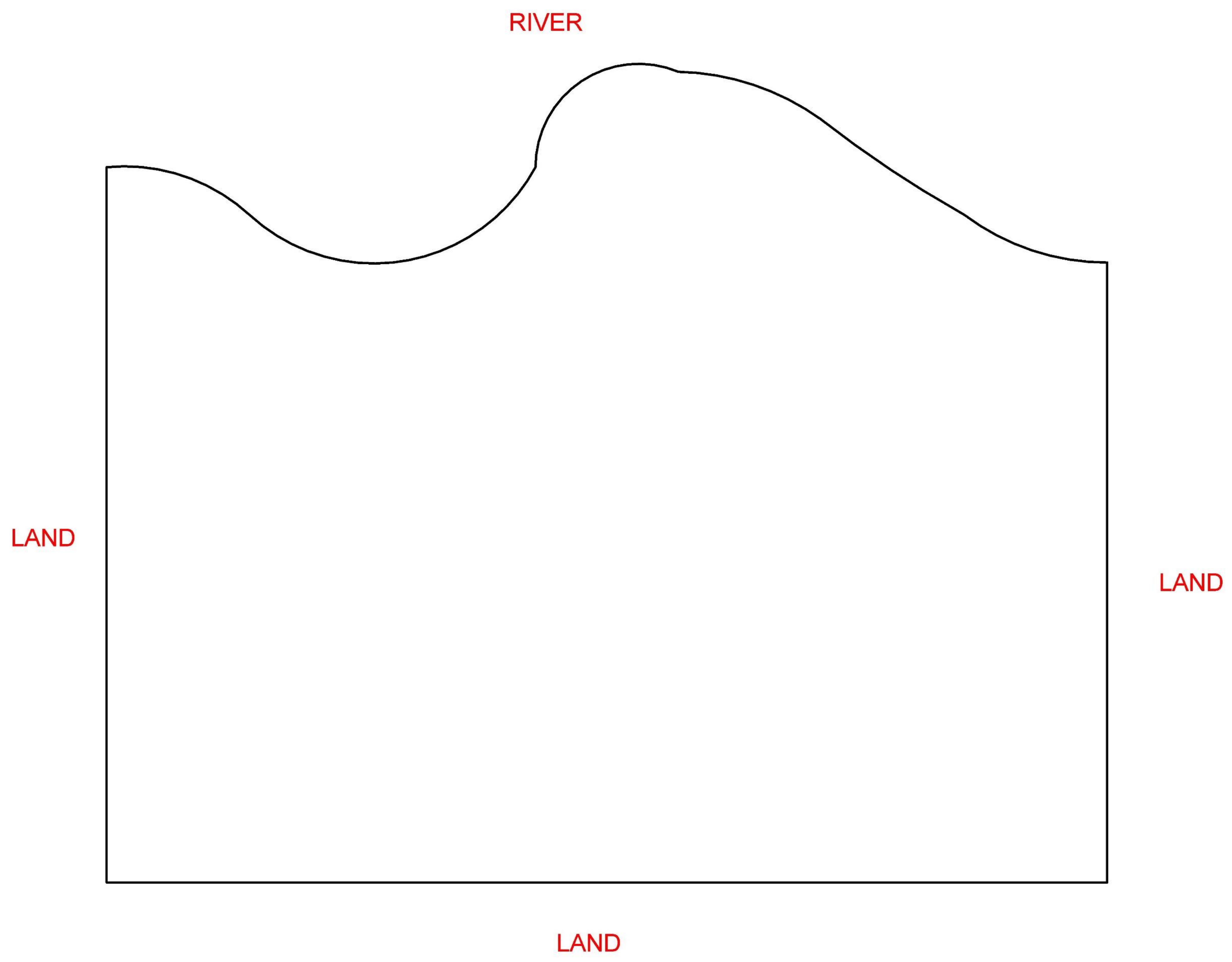
0 × 847.60 = 0

125.66 × 694.07 = 87,216.84

716.31 × 0 = 0

523.62 × 591.64 = 309794.53

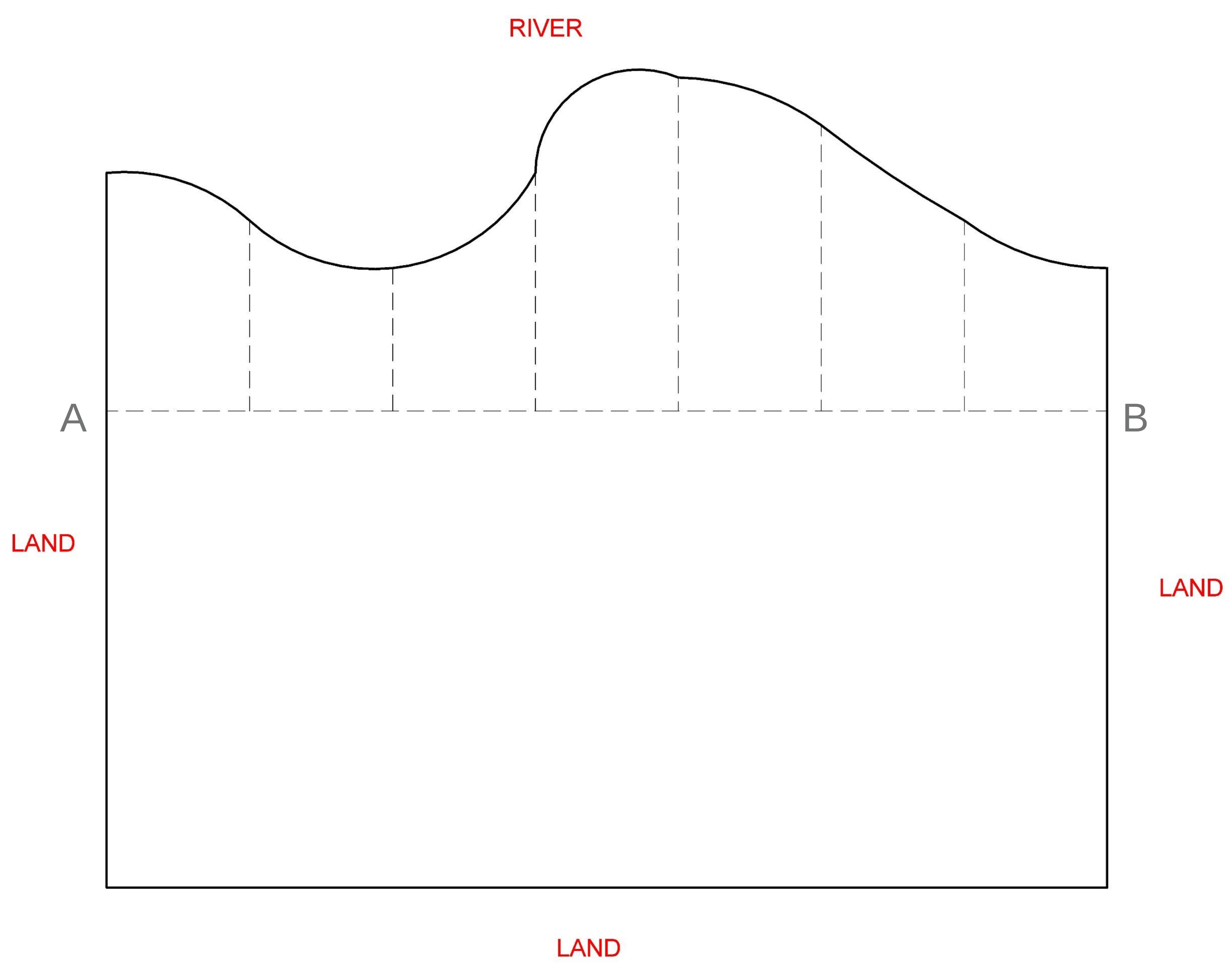
Σ ← 397,011.37



Areas Bounded by a Curve

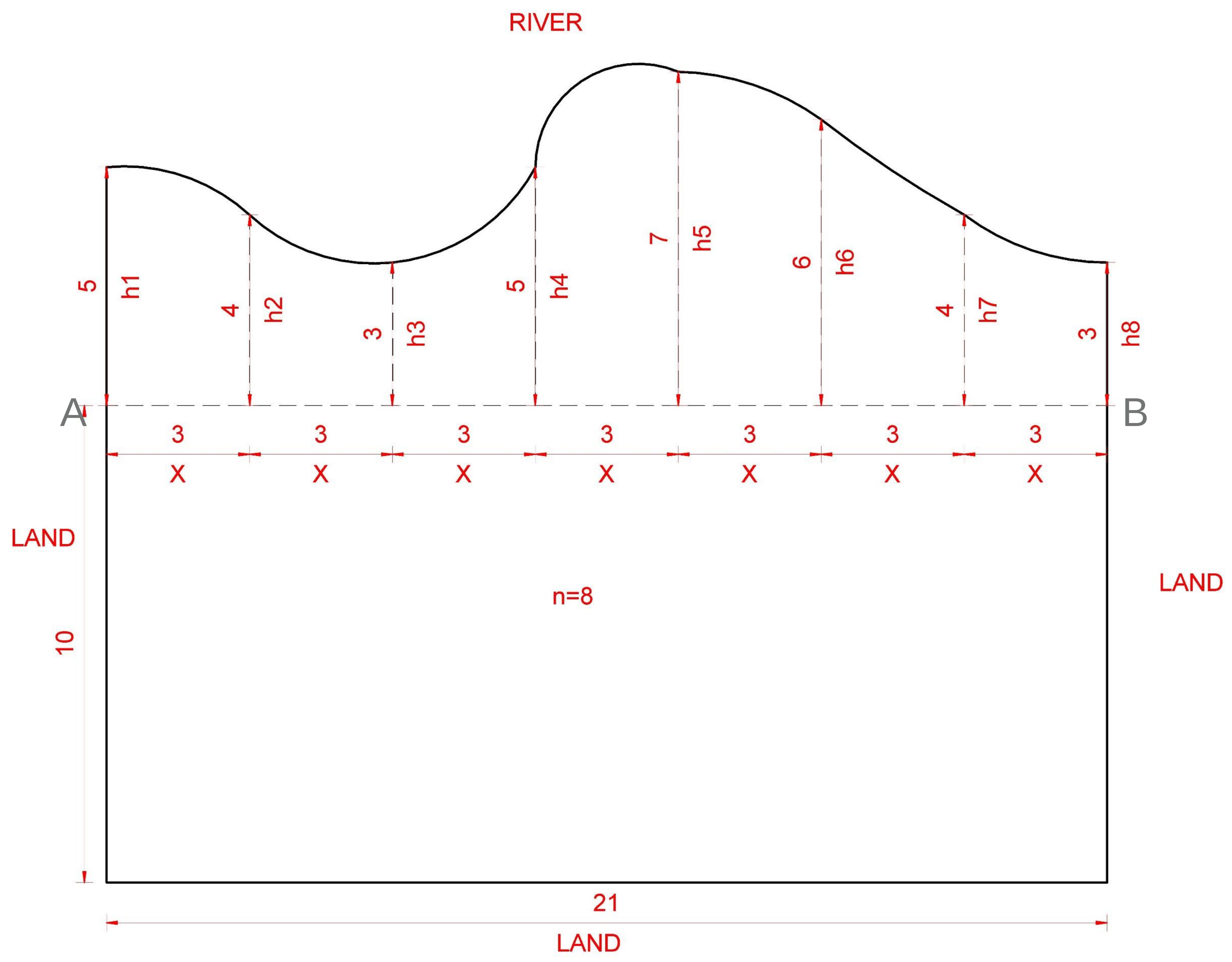
- Average height area method.
- Trapezoidal Rule.
- Simpson's Rule.

Areas Bounded by a Curve



- Set out straight line AB joining the sides of the tract of land and running as closely as possible to the curved boundary.
- Measure distance AB and subdivide it into a number of regular intervals.
- The shorter these intervals are, the more accurate your area estimate will be.
- At each of these marked points, set out a perpendicular line joining AB to the curved boundary. Measure each of these offsets.

Average Height Method



$$Average\ Hieght, H = \frac{h_1 + h_2 + h_3 \ldots + h_n}{n}$$

$$Area = [(n-1)X] \times H$$

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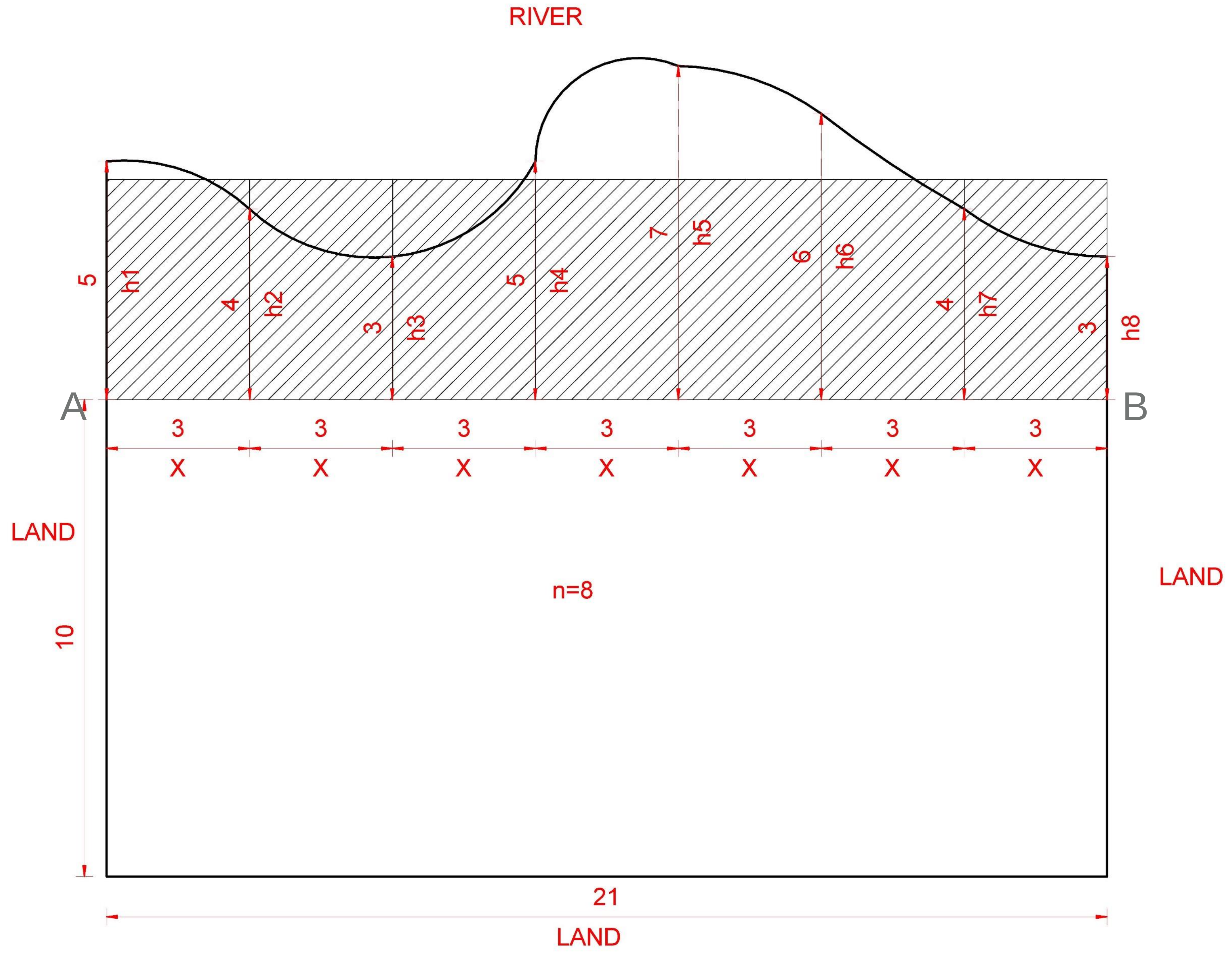
Average Height Method



$$Average\ Hieght, H = \frac{h_1 + h_2 + h_3 \ldots + h_n}{n}$$

$$Area = [(n-1)X] \times H$$

$$Average\ Hieght, H = \frac{5+4+3+5+7+6+4+3}{8} = 4.625\ m$$

$$Area = [(8-1)3] \times 4.625 = 97.125\ m^2$$

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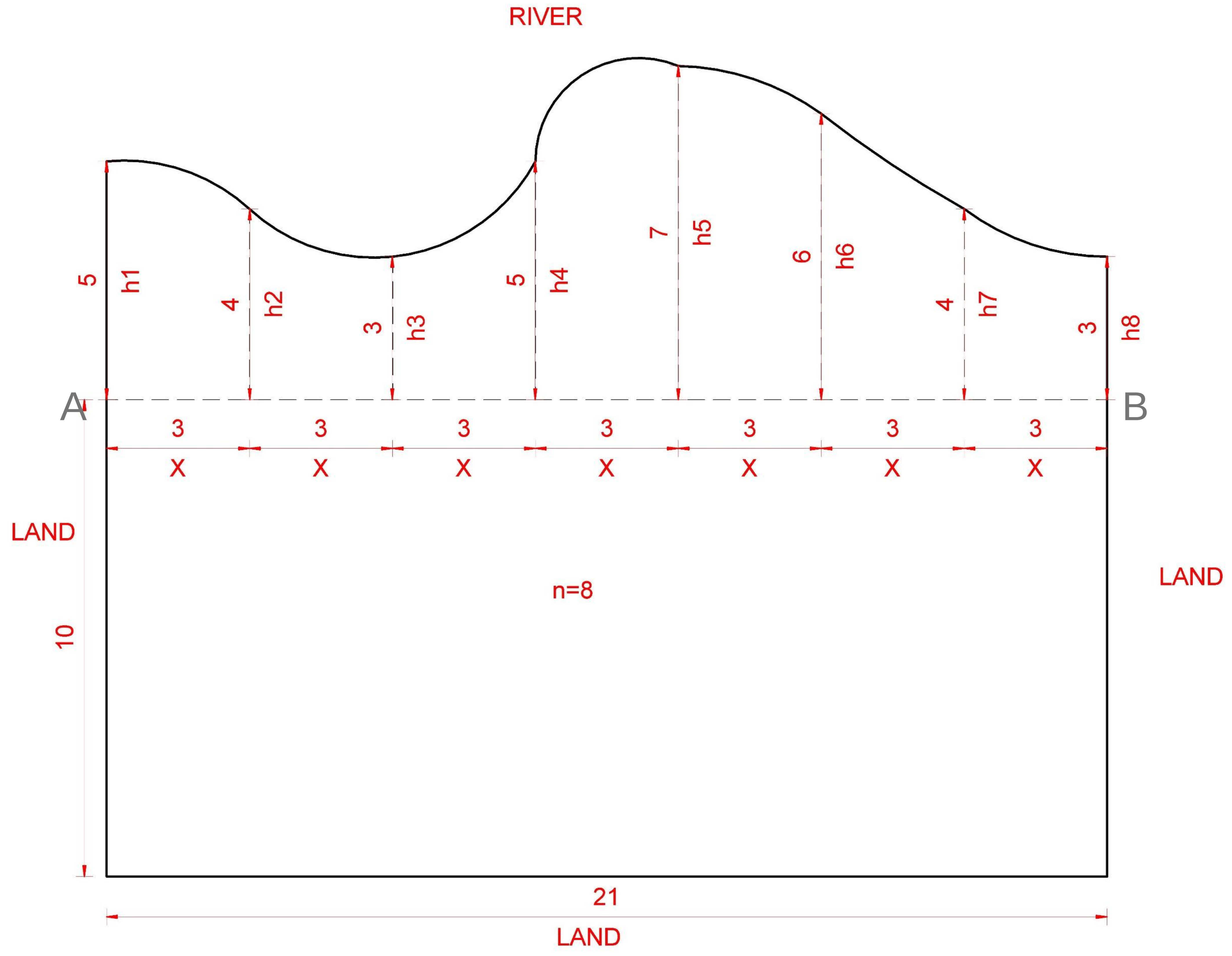
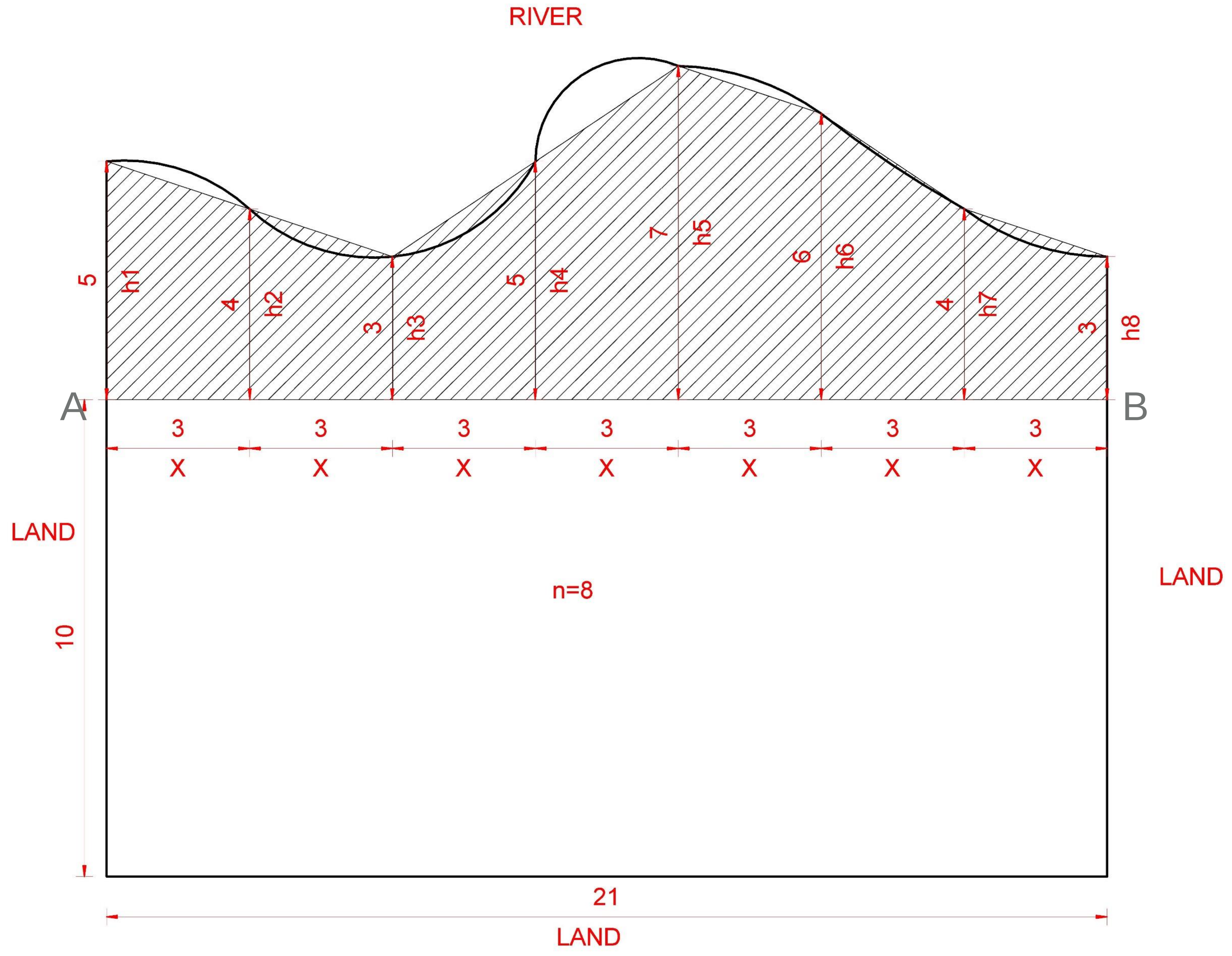
Trapezoidal Rule Method



$$Area = \frac{X}{2}\left[h_1 + h_n + 2\sum(h_2 + h_3 \ldots h_{n-1})\right]$$

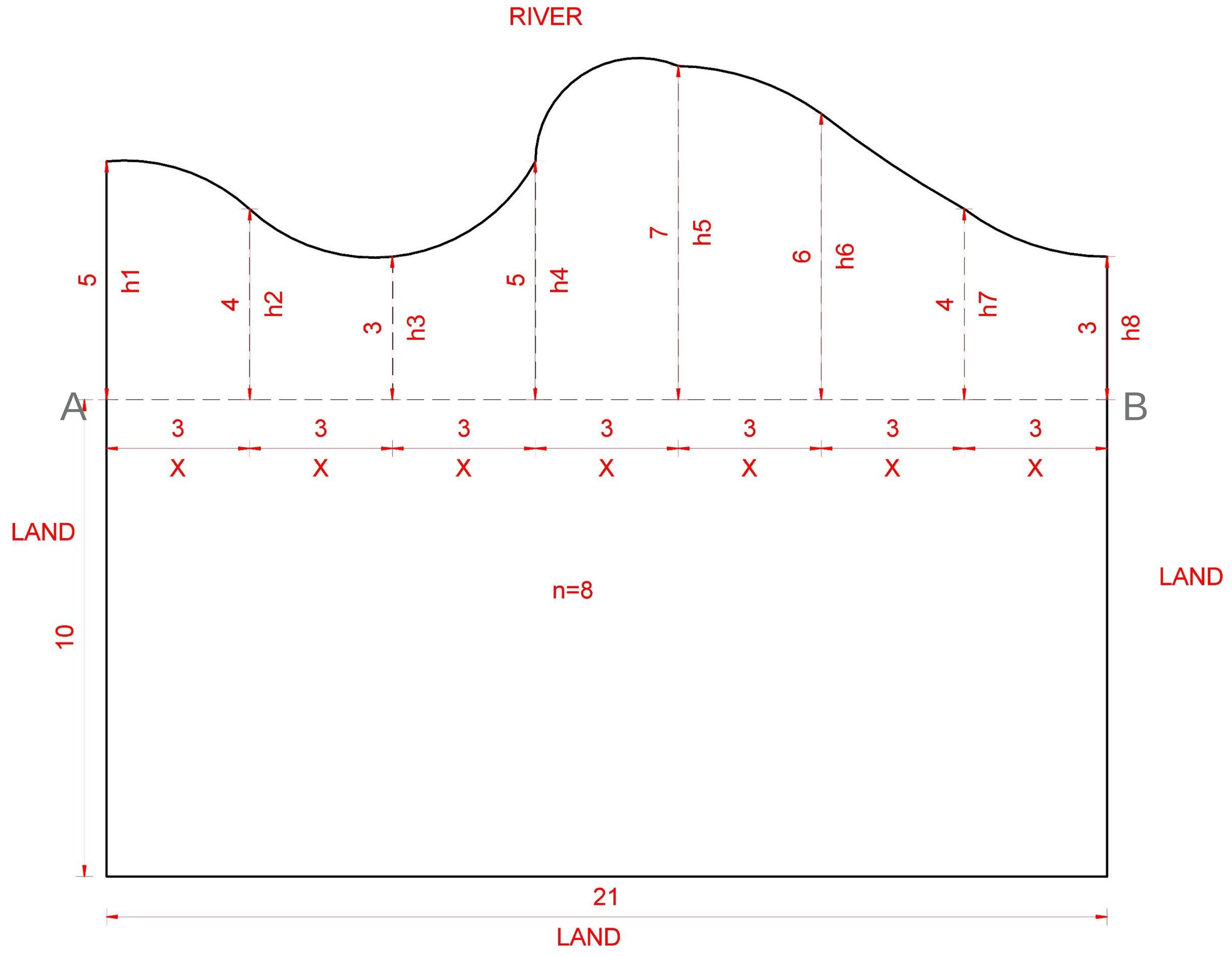
Trapezoidal Rule Method



$$Area = \frac{X}{2}\left[h_1 + h_n + 2\sum(h_2 + h_3 \ldots h_{n-1})\right]$$

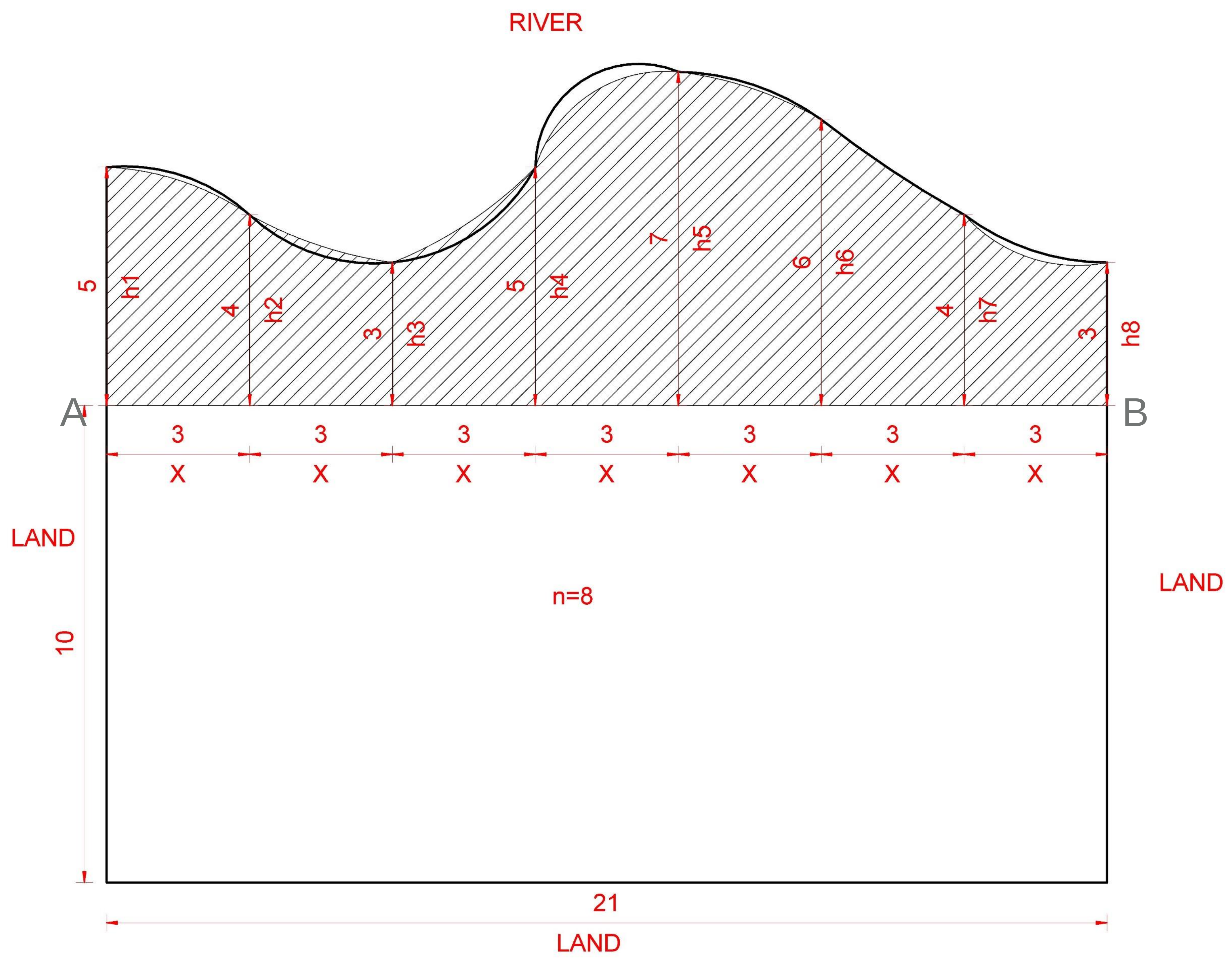
$$Area = \frac{3}{2}\left[5 + 3 + 2\sum(4 + 3 + 5 + 7 + 6 + 4)\right] = 99\ m^2$$

Simpson's Rule Method



$$Area = \frac{X}{3}\left[h_1 + h_n + 4\sum(h_2 + h_4 + h_6 \ldots) + 2\sum(h_3 + h_5 + h_7 \ldots)\right]$$

Simpson's Rule Method

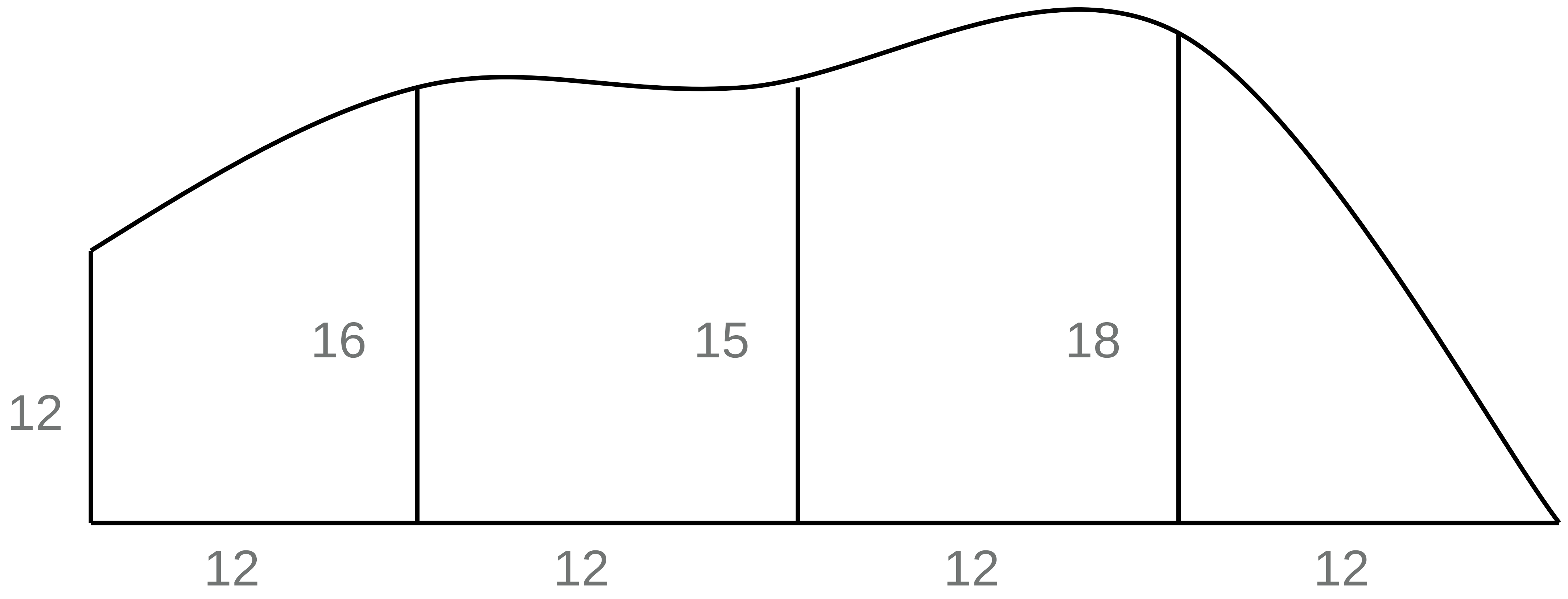


$$Area = \frac{X}{3}\left[h_1 + h_n + 4\sum(h_2 + h_4 + h_6 \dots) + 2\sum(h_3 + h_5 + h_7 \dots)\right]$$

$$Area = \frac{3}{3}\left[5 + 3 + 4\sum(4 + 5 + 6) + 2\sum(3 + 7 + 4)\right]$$

$$Area = \frac{3}{3}[5 + 3 + 4 \times 15 + 2 \times 14] = 96\ m^2$$

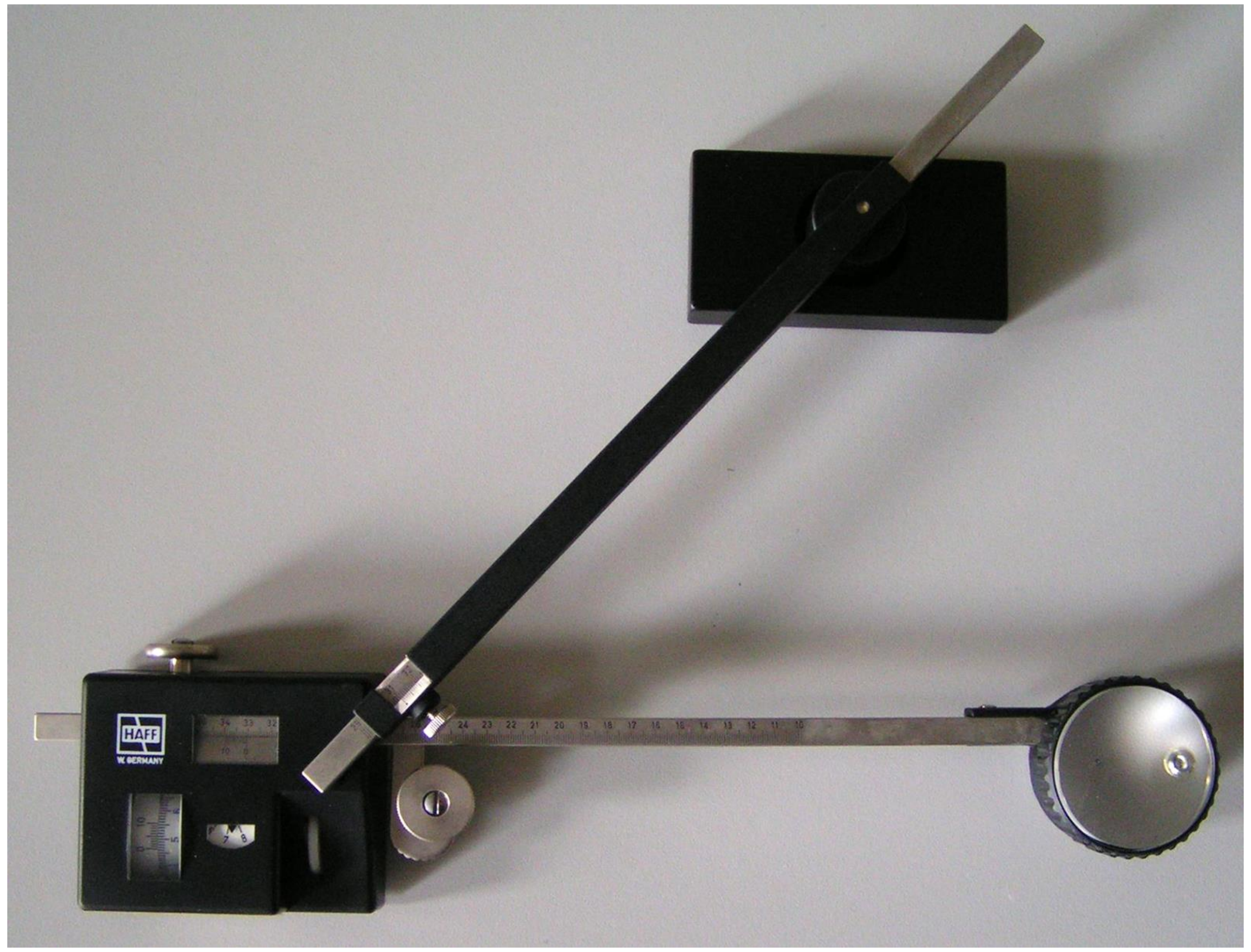
Example

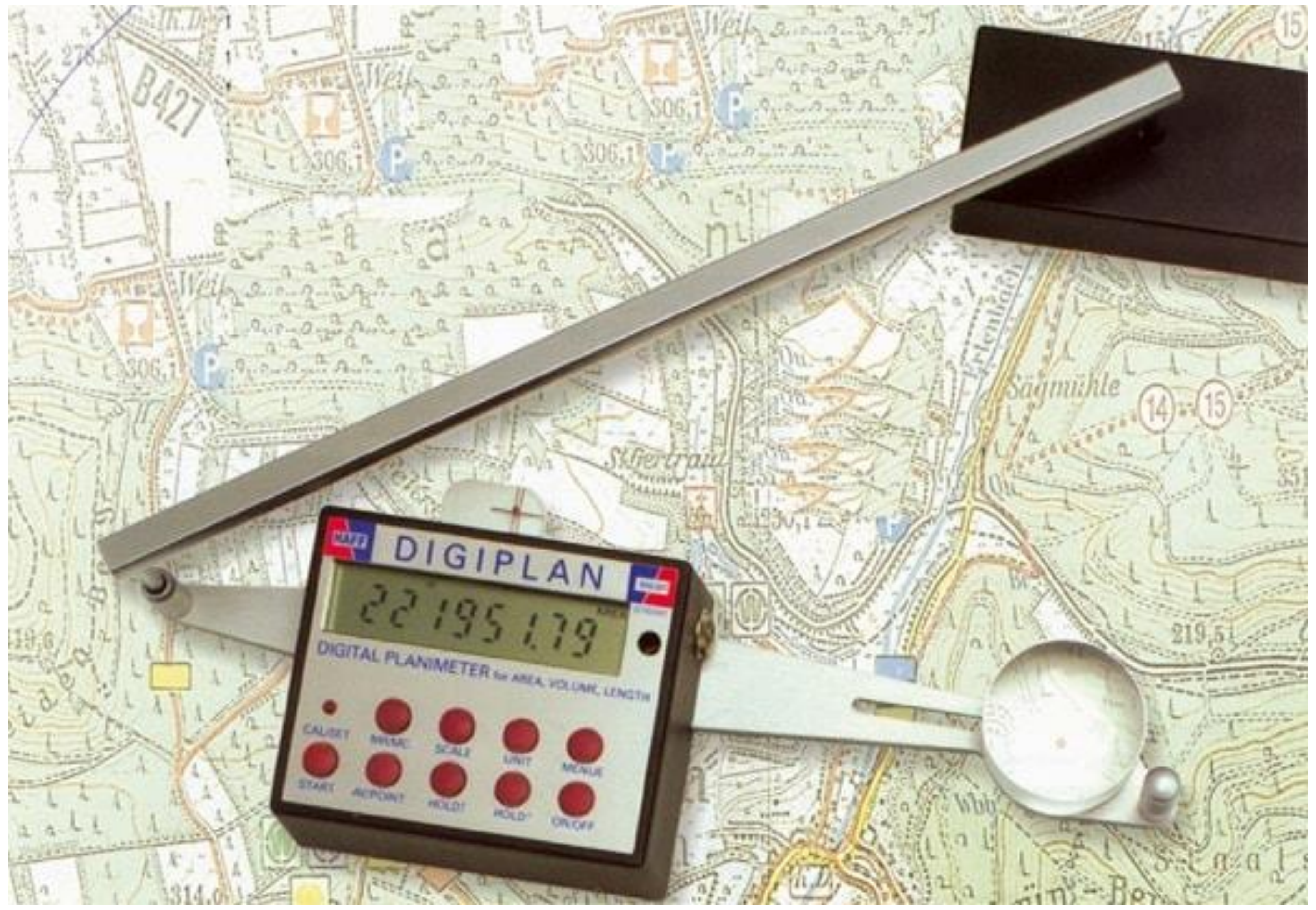


A land parcel is divided into 4 sections. If the dimensions are in meters, determine its area in square meters.

Solution:

- Area using Average height: 12x4 x (12 +16 +15 +18 +0)/5 = 585.6 m^2.
- Area using Trapezoidal rule: (12/2) x (12 + 0 + 2[16+15+18]) = 660 m^2.
- Area using Simpson's Rule: (12/3) x (12 + 0 + 4x[16+18] +2x15) = 712 m^2.





Instrumental: mechanical or digital planimeter

A Planimeter is composed of two joined road: a fixed rod with a weight at its end and a movable rod with a tracing mark at one end and a measuring unit at the other end. The tracing mark is set on a marked point on the boundary and traced the boundary clockwise until you come back to the same starting point. The reading of the measuring unit can be used to determine the bounded area.