Options

Mhamed Eddahbi
KSU-COS-DM

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1 Introduction and motivations

We have seen that forward and futures contract are binding contract, a forward/futures contract obligates the buyer (the holder of the long position) to pay the forward price at expiration, even if the value of the underlying asset at expiration is less than the forward price. Because losses are possible with a forward/futures contract, it is natural to wonder: Could there be a contract where the buyer has the right to walk away from the deal?

The answer is yes; a **call option** is a contract where the buyer has the right to buy, but not the obligation to buy.

1.1 Learning objectives

- Understand how call and put options are used and how they are priced

- Examine the instruments traded on the options market

- Understand how options can be used for either risk management or for speculative purposes

2 Call options

2.1 Definitions and examples

Definition. A financial option contract gives its owner the right (but not the obligation) to purchase or sell an asset at a fixed price at some future date.

An option is a financial instrument like a stock or bond, an option is a derivative security. It is also a non–binding contract with strictly defined terms and properties.

Here is an example illustrating how a call option works at expiration.

Example 1. *Suppose that the call buyer agrees to pay $1020 for the ABC index in 6 months but is not obligated to do so. (The buyer has purchased a call option.)If in 6 months the ABC price is $1100, the buyer will pay $1020 and receive the index. This leads to a payoff of $80 per unit of the index. If the ABC price is $900, the buyer walks away, hence his payoff is 0.*

Remark 2. *From the seller's point of view the buyer is in control of the option, deciding when to buy the index by paying $1020. Thus, the rights of the option buyer are obligations for the option seller.*

Example 3. *If in 6 months the ABC price is $1100, the seller will receive $1020 and give up an index worth more, for a loss of $80 per unit of the index. If the ABC price is less than $1020, the buyer will not buy, so the seller has no obligation. Thus, at expiration, the seller will have a payoff that is zero (if the ABC price is less than $1020) or negative (if the ABC price is greater than $1020).*

3   Different type of options

Two distinct kinds of option contracts exist: **call options and put options**. A **call option** gives the owner the right to **buy** the asset;a **put option** gives the owner the right to **sell** the asset. Because an option is a contract between two parties, for every owner of a financial option, there is also an option writer, the person who takes the other side of the contract.

Initial payment is necessary Because the buyer can decide whether to buy, the seller cannot make money at expiration. This situation suggests that the seller must get something to enter into the contract in the first place. At the time the buyer and seller agree to the contract, the buyer must pay the seller an initial price, the **premium**. This initial payment compensates the seller for being at a disadvantage at expiration .Contrast this with a forward/futures contract, for which the initial premium is **zero**

Example 4. *The most commonly encountered option contracts are options on shares of stock. A stock option gives the holder the option to buy or sell a share of stock on or before a given date for a given price. For example, a call option on 3M Corporation stock might give the holder the right to purchase a share of 3M for $75 per share at any time up to, for example, July 19, 2016.Similarly, a put option on 3M stock might give the holder the right to sell a share of 3M stock for $50 per share at any time up to, say, June 16, 2016.*

4   Option Terminology

Here are some key terms used to describe options:

1. **Strike price:** The strike price, or exercise price, of a call option is what the buyer pays for the asset. In the example above, the strike price was $1020. The strike price can be set at any value.

2. **Exercise:** The exercise of a call option is the act of paying the strike price to receive the asset. In the abode example, the buyer decided after 6 months whether to exercise the option–that is, whether to pay $1020 (the strike price) to receive the ABC index.

3. **Expiration:** The expiration of the option is the date by which the option must either be exercised or it becomes worthless. The option in previous example had an expiration of 6 months.

4. The (**owner**) buyer of a call option is called the **option call holder**. The holder of a call option is said to have a **long call position**.

5. The seller of a call option is called the **option call writer**.

6. The writer of a call is said to have a **short call position**. Assets used in call options are in *commodities, currency exchange, stock shares and stock indices.*

7. A call option needs to specify the **type** and **quality** of the **underlying**. The asset used in the call option is called the **underlier** or **underlying asset**.

8. **Notional**: The amount of the underlying asset to which the call option applies is called the **notional** amount.

9. **Exercise style:** The exercise style of the option governs the time at which exercise can occur. In the above example, exercise could occur only at expiration. Such an option is said to be a **European–style option**.

10. If the buyer has the right to exercise at any time during the life of the option, it is an **American–style option**.

11. If the buyer can only exercise during specified periods, but not for the entire life of the option, the option is a **Bermudan–style option**.

The terms "European" and "American", have nothing to do with geography. European, American, and Bermudan options are bought and sold worldwide.

An European call option gives the owner of the call the right, but not the obligation, to buy the underlying asset on the expiration date by paying the strike price. The option described in two examples above is a 6–month European–style ABC call with a strike price of $1020.The buyer of the call can also be described as having a **long position** in the call.

5 Payoff and Profit for options

5.1 Payoff and Profit for a purchased call option

We can graph call options as we did forward contracts. The buyer is not obligated to buy the index, and hence will only exercise the option if the payoff is greater than zero. The algebraic expression for the payoff to a purchased call is therefore

$$\text{Purchased call payoff} = \max(\text{spot price at expiration} - \text{strike price}; 0) \tag{1}$$

The expression $\max(a, b)$ means take the greater of the two values a and b.

Example 5. *Consider a call option on the ABC index with 6 months to expiration and a strike price of $1000.Suppose the index in 6 months is $1100. Clearly it is worthwhile to pay the $1000 strike price to acquire the index worth $1100.Using the equation (1), the call payoff is*

$$\max(\$1100 - \$1000, 0) = \$100$$

If the index is $900 at expiration, it is not worthwhile paying the $1000 strike price to buy the index worth $900. The payoff is then

$$\max(\$900 - \$1000, 0) = \$0$$

See the link for more information:

http://www.cboe.com/delayedquote/quotetable.aspx?ticker=SPX

Notice that the payoff does not take account of the initial cost of acquiring the position. For a purchased option, the premium is paid at the time the option is acquired. In computing profit at expiration, suppose we defer the premium payment; then by the time of expiration we accrue 6 months' interest on the premium. The option profit is computed as

$$\begin{aligned} \text{Purchased call profit} \quad = \quad & \max(\text{spot price at expiration} - \text{strike price}, 0) \\ & -\text{future value of option premium} \end{aligned} \tag{2}$$

Example illustrating the computation of the profit Use the same option as before, and suppose that the risk–free rate is 2% over 6 months. Assume that the index spot price is $1000 and that the premium for this call is $93.81.Hence, the future value of the call premium is

$$\$93.81 \times 1.02 = \$95.68.$$

If the ABC index price at expiration is $1100, the owner will exercise the option. Using equation (2), the call profit is

$$\max(\$1100 - \$1000, 0) - \$95.68 = \$4.32.$$

If the index is $900 at expiration, the owner does not exercise the option. It is not worthwhile paying the $1000 strike price to buy the index worth $900.The Profit is then

$$\max(\$900 - \$1000, 0) - \$95.68 = -\$95.68$$

reflecting the loss of the premium.

We graph the call payoff by computing, for any index price at expiration, the payoff on the option position as a function of the price. We graph the call profit by subtracting from this the future value of the option premium.

Payoff and profit after 6 months from a purchased $1000–strike ABC call option with a future value of premium of $95.68. The option premium is assumed to be $93.81 and the effective interest rate is 2% over 6 months. The payoff is computed using equation (1), and the profit using equation (2).

S&R Index in 6 Months	Call Payoff	Future Value of Premium	Call Profit
800	$0	−$95.68	−$95.68
850	0	−95.68	−95.68
900	0	−95.68	−95.68
950	0	−95.68	−95.68
1000	0	−95.68	−95.68
1050	50	−95.68	−45.68
1100	100	−95.68	4.32
1150	150	−95.68	54.32
1200	200	−95.68	104.32

The payoff at expiration of a purchased ABC call with a $1000 strike price.
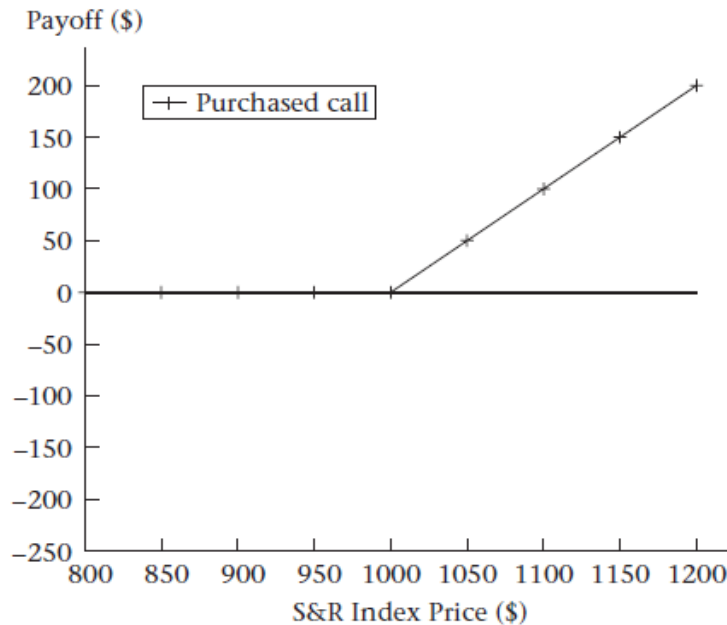
Profit at expiration for purchase of a 6-month ABC index call with a strike price of $1000 versus profit on a long ABC index forward position.
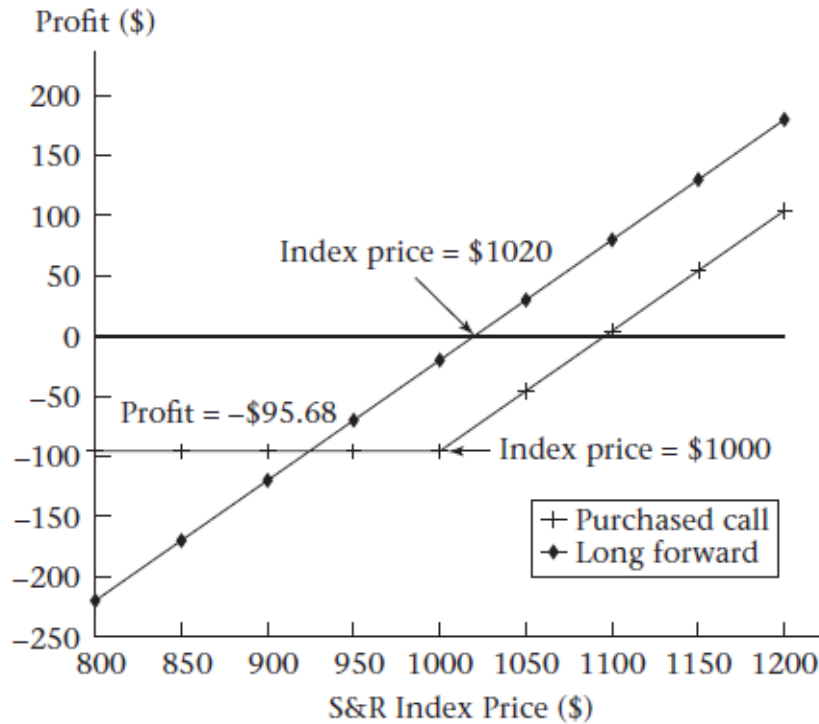
Profit ($)

5.2 Payoff and Profit for a Written Call Option

Now, let us look at the option from the point of view of the seller.
The seller is said to be the **option writer**, or to have a **short position** in a call option.

The option writer is the counterparty to the option buyer.

The writer receives the premium for the option and then has an obligation to sell the underlying security in exchange for the strike price if the option buyer exercises the option.

The payoff and profit to a written call are just the opposite of those for a purchased call:

$$\text{Written call payoff} = -\max(\text{spot price at expiration } - \text{ strike price}; 0)$$

and

$$
\begin{aligned}
\text{Written call profit} \quad &= \quad -\max(\text{spot price at expiration} - \text{strike price}, 0) \\
&\quad +\text{future value of option premium}
\end{aligned}
\tag{3}
$$

The following example illustrates the option writer's payoff and profit. Just as a call buyer is long in the call, the call seller has a short position in the call.

Example 6. *Consider a $1000–strike call option on the ABC index with 6 months to expiration. At the time the option is written, the option seller receives the premium of $93.81.*
Suppose the index in 6 months is $1100. It is worthwhile for the option buyer to pay the $1000 strike price to acquire the index worth $1100.
Thus, the option writer will have to sell the index, worth $1100, for the strike price of $1000. Using equation (??), the written call payoff is

$$-\max(\$1100 - \$1000, 0) = -\$100$$

The premium has earned 2% interest for 6 months and is now worth $95.68. Profit for the written call is
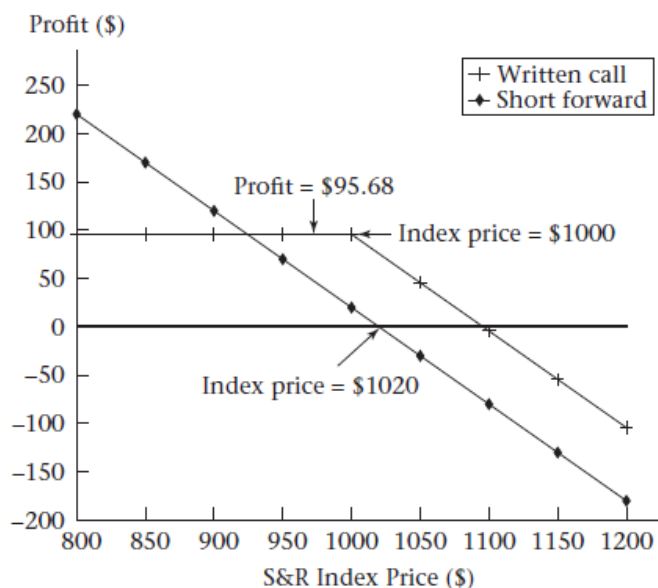
$$-\$100 + \$95.68 = -\$4.32$$

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If the index is $900 at expiration, it is not worthwhile for the option buyer to pay the $1000 strike price to buy the index worth $900. The payoff is then

$$-\max(\$900 - \$1000, 0) = \$0.$$

The option writer keeps the premium, for a profit after 6 months of

$$\$95.68.$$

Profit for the writer of a 6-month ABC call with a strike of $1000 versus profit for a short ABC forward.

6 Put options

6.1 Definitions and examples

We introduced a call option by comparing it to a forward contract in which the buyer need not buy the underlying asset if it is worth less than the agreed-to purchase price. Perhaps you wondered if there could also be a contract in which the seller could walk away if it is not in his or her interest to sell. The answer is yes. A put option is a contract where the seller has the right to sell, but not the obligation. Here is an example to illustrate how a put option works.

Example 7. *Suppose that the seller agrees to sell the ABC index for $1020 in 6 months but is not obligated to do so. (The seller has purchased a put option.) If in 6 months the ABC price is $1100, the seller will not sell for $1020 and will walk away. If the ABC price is $900, the seller will sell for $1020 and will earn $120 at that time.*

Remark 8. *A put must have a premium for the same reason a call has a premium. The buyer of the put controls exercise; hence the seller of the put will never have a positive payoff at expiration. A premium paid by the put buyer at the time the option is purchased compensates the put seller for this no-win position.*

6.2 Payoff and Profit for a Purchased Put Option

We now see how to compute payoff and profit for a purchased put option. The put option gives the put buyer the right to sell the underlying asset for the strike price. The buyer does this only if the asset is less valuable than the strike price. Thus, the payoff on the put option is

$$\text{Put option payoff} = \max\left(\text{strike price} - \text{ spot price at expiration}, 0\right) \tag{4}$$

The put buyer has a long position in the put.

Example 9. *Consider a put option on the ABC index with 6 months to expiration and a strike price of $1000. Suppose the index in 6 months is $1100. It is not worthwhile to sell the index worth $1100 for the $1000 strike price. Using equation (4), the put payoff is*

$$\max(\$1000 - \$1100) = \$0$$

If the index were 900 at expiration, it is worthwhile selling the index for $1000. The payoff is then

$$\max(\$1000 - \$900, 0) = \$100$$

As with the call, the payoff does not take account of the initial cost of acquiring the position. At the time the option is acquired, the put buyer pays the option premium to the put seller; we need to account for this in computing profit. If we borrow the premium amount, we must pay 6 months' interest. The option profit is computed as

$$\text{Purchased put profit} \quad = \quad \max(\text{strike price} - \text{spot price at expiration}; 0) \qquad (5)$$
$$-\text{future value of option premium}$$

The following example illustrates the computation of profit on the put.

Example 10. *Use the same option as in Example above and suppose that the risk-free rate is 2% over 6 months. Assume that the premium for this put is $74.20.8 The future value of the put premium is $74.20 × 1.02 = $75.68. If the ABC index price at expiration is $1100, the put buyer will not exercise the option. Using equation (5), profit is*

$$\max(\$1000 - \$1100; 0) - \$75.68 = -\$75.68$$

reflecting the loss of the premium. If the index is $900 at expiration, the put buyer exercises the put, selling the index for $1000. Profit is then

$$\max(\$1000 - \$900; 0) - \$75.68 = \$24.32$$

Profit after 6 months from a purchased 1000-strike ABC put option with a future value of premium of $75.68.

S&R Index in 6 Months	Put Payoff	Future Value of Premium	Put Profit
$800	$200	−$75.68	$124.32
850	150	−75.68	74.32
900	100	−75.68	24.32
950	50	−75.68	−25.68
1000	0	−75.68	−75.68
1050	0	−75.68	−75.68
1100	0	−75.68	−75.68
1150	0	−75.68	−75.68
1200	0	−75.68	−75.68

Profit on a purchased ABC index put with a strike price of $1000 versus a short ABC index forward.
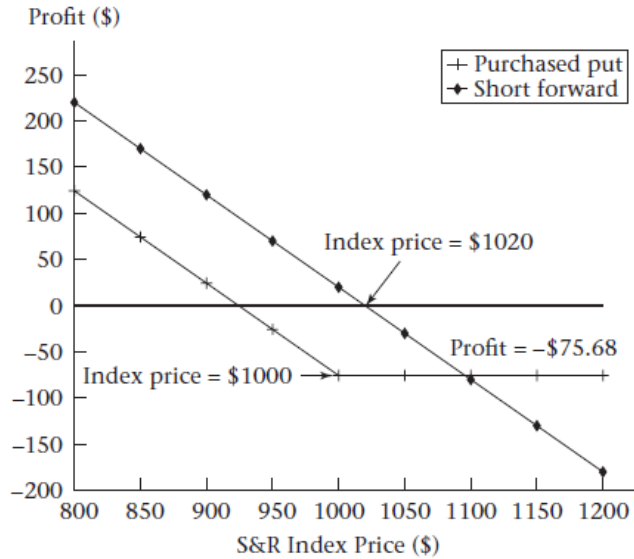
6.3 Payoff and Profit for a Written Put Option

Now we examine the put from the perspective of the put writer. The put writer is the counterparty to the buyer. Thus, when the contract is written, the put writer receives the premium. At expiration, if the put buyer elects to sell the underlying asset, the put writer must buy it.

The payoff and profit for a written put are the opposite of those for the purchased put:

$$\text{Written put payoff} = -\max(\text{strike price} - \text{spot price at expiration}; 0) \tag{6}$$

$$\begin{aligned}\text{Written put profit} \quad &= \quad -\max(\text{strike price} - \text{spot price at expiration}; 0) \\ &\quad +\text{future value of option premium}\end{aligned} \tag{7}$$

The put seller has a short position in the put.

Written ABC index put option with a strike of $1000 versus a long ABC index forward contract.
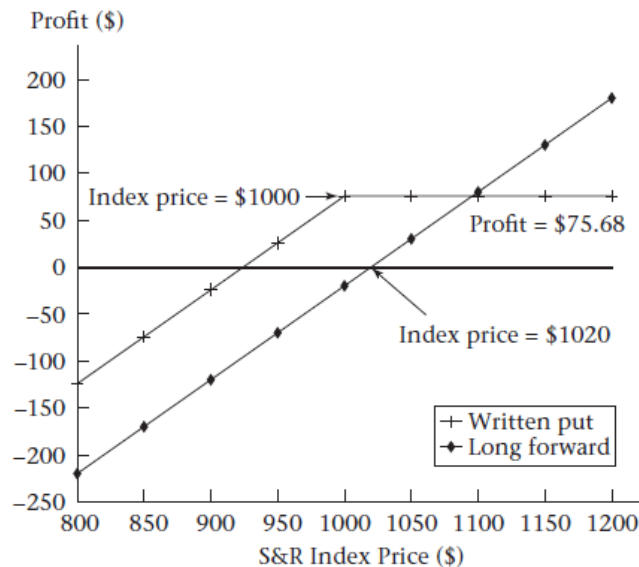
6.4 The "Moneyness" of an Option

Options are often described by their degree of moneyness. This term describes whether the option payoff would be positive if the option were exercised immediately. (The term is used to describe both American and European options even though European options cannot be exercised until expiration.)

An **in-the-money option** is one which would have a positive payoff (but not necessarily positive profit) if exercised immediately.

A call with a strike price less than the asset price and a put with a strike price greater than the asset price are both **in-the-money**.

An **out-of-the-money option** is one that would have a negative payoff if exercised immediately.

A call with a strike price greater than the asset price and a put with a strike price less than the asset price are both **out-of-the-money**.

An **at-the-money option** is one for which the strike price is approximately equal to the asset price.

6.5 Positions Long with Respect to the Index

The following positions are long in the sense that there are circumstances in which they represent either a right or an obligation to buy the underlying asset:

Long forward: An obligation to buy at a fixed price.

Purchased call: The right to buy at a fixed price if it is advantageous to do so.

Written put: An obligation of the put writer to buy the underlying asset at a fixed price if it is advantageous to the option buyer to sell at that price.

(Recall that the option buyer decides whether or not to exercise.)

Maximum possible profit and loss at maturity for long and short forwards and purchased and written calls and puts. FV(premium) denotes the future value of the option premium.

Position	Maximum Loss	Maximum Gain
Long forward	-Forward price	Unlimited
Short forward	Unlimited	Forward price
Long call	-FV(premium)	Unlimited
Short call	Unlimited	FV(premium)
Long put	-FV(premium)	Strike price-FV(premium)
Short put	FV(premium)-Strike price	FV(premium)

Remark 11. *The terms "**long**" and "**short**" can be confusing, however, because they are often used more generally. In this more general usage, a position is long with respect to x if the value of the position goes up when x goes up, and it is short with respect to x if the value of the position goes down when x goes up.*

Profit diagrams for the three basic long positions: long forward, purchased call, and written put.
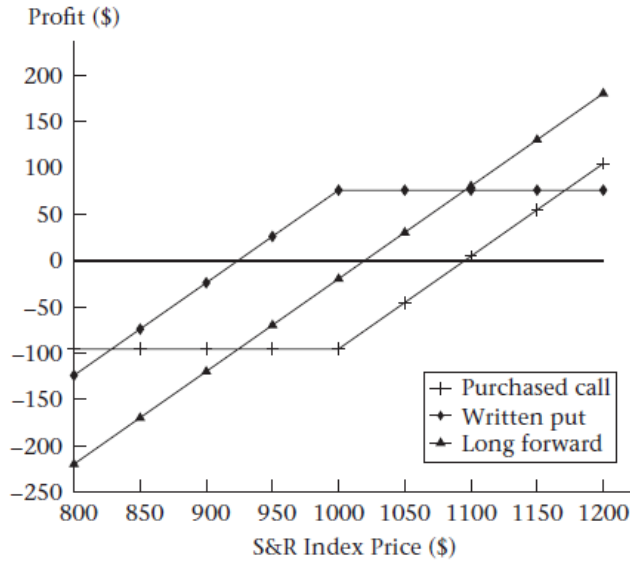
6.6   Positions Short with Respect to the Index

The following positions are short in the sense that there are circumstances in which they represent either a right or an obligation to sell the underlying asset:

1. **Short forward**: An obligation to sell at a fixed price.

2. **Written call**: An obligation of the call writer to sell the underlying asset at a fixed price if it is advantageous to the option holder to buy at that price (recall that the option buyer decides whether to exercise).

3. **Purchased put**: The right to sell at a fixed price if it is advantageous to do so.

Profit diagrams for the three basic short positions: short forward, written call, and purchased put.