College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

First Midterm Exam
Academic Year 1446 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Introduction to Partial Differential Equations**		اسم المقرر
Course Code	**425 Math**		رمز المقرر
Exam Date	**2024-11-25**	**1446-05-23**	تاريخ الامتحان
Exam Time	**10: 00 AM**		وقت الامتحان
Exam Duration	**2 hours**	ساعتان	مدة الامتحان
Classroom No.	**G14**		رقم قاعة الاختبار
Instructor Name	د. هدى الرشيدي		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 7 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان ٧ صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 2-1(2 marks)	QII(b)		
2	C.L.O 2-2 (4+2 marks)	QI QII(a)		
3	C.L.O 2-3 (6+3+3 marks)	QIII QIV		
4				
5				
6				
7				
8				

EXAM COVER PAGE

<u>Question I</u>: Obtain the general solution of the given equation:

$$u_{xx} - 2u_{xy} - 3u_{yy} = e^{2x+3y} + \sin(2x + y)$$

Question II:

(a) Solve the PDE $xyu_{xy} = 1$.

(b) Determine the condition under which the function

$$u(x, y) = \cos(ax - by)e^{(ax-by)}$$

is a harmonic.

Question III: Solve the Cauchy problem $yu_{xy} + 2u_x = x^2, y \neq 0,$ in $\mathbb{R}^2$, that satisfies:

$$u(s, 2s)_{|\Gamma_0} = \frac{s^3}{6}, \qquad u_n(s, 2s)_{|\Gamma_0} = -\frac{s^2}{\sqrt{5}}$$

where

$\Gamma_0 = \{(x, y) | x = s, y = 2s, s \in \mathbb{R}^*\}$, and u_n is a derivative of u in the direction of Γ_0.

<u>Question IV:</u>

(a) Consider the series solution

$$u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(\frac{r}{\boldsymbol{a}}\right)^n (a_n \cos n\theta + b_n \sin n\theta),$$

for Dirichlet problem for circle of radius $\boldsymbol{a}$, if the boundary condition is $u(\boldsymbol{a},\theta) = \sin\theta,\ \ 0 \le \theta \le 2\pi$. Obtain $u(r,\theta)$?

(b) Consider the Laplace equation in spherical coordinate

$$u_{rr} + \frac{2}{r}u_r + \frac{1}{r^2 sin^2\phi}u_{\theta\theta} + \frac{1}{r^2}u_{\phi\phi} + \frac{\cot\phi}{r^2}u_\phi = 0.$$

If u is a symmetry around z axis, then the solution is given by

$$u(r,\theta) = \sum_{n=0}^{\infty}(a_n r^n + b_n r^{-n-1})P_n(\cos\phi).$$

Find the solution at the boundary condition

$u(1,\phi) = f(\phi), 0 \leq r < 1\ where$

$$f(\phi) = \begin{cases} 110\ \ if\ \ 0 \leq \phi < \frac{\pi}{2}. \\ 0\quad if\quad \frac{\pi}{2} < \phi \leq \pi. \end{cases}$$

(Hint: $\|P_n\|^2 = \frac{2}{2n+1}$, $P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1), P_3(x) = \frac{1}{2}(5x^3 - 3x).$)

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Good Luck