

First Midterm Exam  
Academic Year 1446 Hijri- First Semester

Exam Information معلومات الامتحان		
Course name	Introduction to Partial Differential Equations	
Course Code	425 Math	
Exam Date	2024-10-14	1446-04-11
Exam Time	10: 00 AM	
Exam Duration	2 hours	ساعتان
Classroom No.	G14	
Instructor Name	د. هدى الرشيدى	

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

**General Instructions:**

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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- عدد صفحات الامتحان 6 صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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**تعليمات عامة:**

هذا الجزء خاص بأستاذ المادة

*This section is ONLY for instructor*

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 1-1 (3 marks)	QI(1(a)) QI(2) QIV(1)		
2	C.L.O 2-1 (6 marks)	QI(1(b,c)) QI(3) QIV(2)		
3	C.L.O 2-2 (6 marks)	QI(1(d,e)) QII QIV(3)		
4	C.L.O 2-3 (5 marks)	QI(1(f)) QIII		
5				
6				
7				
8				



**Question I:**[3 points]

1. Choose the correct answer:

- (a) The equation  $u_{xx} = x^2 u_{yy}$  is
- Hyperbolic for all  $x$ .
  - Hyperbolic for  $x > 0$ .
  - Hyperbolic for  $x \neq 0$ .
  - None of previous.
- (b) A partial differential equation of the family surfaces of  $z = x^2 + y^2$  is
- $yz_y - xz_x = 0$ .
  - $yz_x - xz_y = 0$ .
  - $yz_x + xz_y = 0$ .
  - None of previous.
- (c) The subsidiary equation of  $x^2 u_x + u - 1 = \frac{y-1}{3} u_y$  is given by
- $\frac{dx}{x^2} = \frac{3dy}{1-y} = \frac{du}{1-u}$ .
  - $\frac{dx}{x^2} = \frac{3dy}{y-1} = \frac{du}{1-u}$ .
  - $\frac{dx}{x^2} = \frac{3dy}{1-y} = \frac{du}{u-1}$ .
  - None of previous.
- (d) The general solution of the partial differential equation  $5\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + u = 0$  equals
- $u(x,y) = f(4x - 5y)e^{-\frac{x}{5}}$ .
  - $u(x,y) = f(4x - 5y)e^{\frac{x}{5}}$ .
  - $u(x,y) = f(4x - 5y)$ .
  - None of previous.
- (e) A particular solution of PDE  $3u_x + 3u_y = x^3$  is
- $\frac{x^4}{4}$ .
  - $\frac{x^4}{3}$ .
  - $\frac{x^4}{12}$ .
  - None of previous.
- (f) The Cauchy problem  $u_x + u_y = u$  with initial condition  $x_0 = t, y_0 = t, u_0 = sint$ , has
- One solution.
  - No solution.
  - Infinitely many solutions.

2. Classify each of the following PDEs as linear, quasilinear, or nonlinear and state its order and homogeneity:[2 points]

(a)  $\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{\frac{1}{2}}$ .

(b)  $u_y u_{xxy}^2 + u_x u_{yy} - u_z^2 = (z + xy^2)u$ .

3. Prove that the PDE which has integral surface  $F(v, w) = 0$  where  $v = x + y + u$  and  $w = x^2 + y^2 - u^2$  such that  $u = u(x, y)$ , can be written as [2 points]

$$(y + u)u_x - (x + u)u_y = x - y.$$

**Question II:**

Find the integral surface of  $x^2u_x - xyu_y + u = 0$ . [3 points]

**Question III:**

Find the explicit solution of the following partial differential equation

$$xu_x + yu_y = x^2 - y$$

which passes through the curve  $u(1, y) = y$ . [4.5 points]

**Question IV:**

1. Show that the one-dimensional wave equation

$$u_{tt} - c^2 u_{xx} = 0$$

is hyperbolic?[0.5 point]

2. Find an equivalent canonical form.[3 points]
3. Obtain the general solution.[2 points]

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Good Luck