QI : $\boxed{7}$

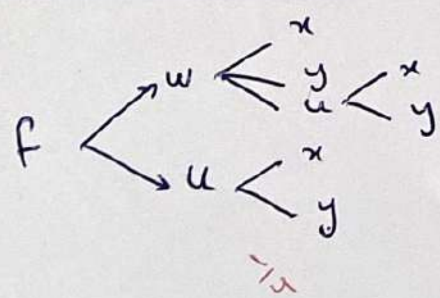
4Ps $\boxed{1}$ (a) linear, second-order - nonhomogenous.

(b) linear, third order - nonhomogenous.

(c) quasilinear, third order - homogenous.

(d) quasilinear, not linear, fourth order - homogenous.

3Ps $\boxed{2}$ $f(x^2+y^2+u^2, u) = 0$



let $w = x^2+y^2+u^2$ $\frac{1}{4}$

$$f_w[w_x + w_u \cdot u_x] + f_u \cdot u_x = 0$$

$$f_w[w_y + w_u \cdot u_y] + f_u \cdot u_y = 0$$

$$f_w w_x + f_w w_u u_x + f_u u_x = 0 \rightarrow (1) \quad \tfrac{3}{4}$$

$$f_w w_y + f_w w_u \cdot u_y + f_u u_y = 0 \rightarrow (2) \quad \tfrac{3}{4}$$

multiplay (1),(2) by u_y and u_x

$$f_w w_x u_y + f_w w_u u_x u_y + f_u u_x u_y = 0 \quad \tfrac{1}{2}$$

$$f_w w_y u_x + f_w w_u u_y u_x + f_u u_y u_x = 0$$

substract two equations we have.

$$f_w[w_x u_y + w_y u_x] = 0 \Rightarrow w_x u_y + w_y u_x = 0$$

$$\Rightarrow 2x u_y + 2y u_x = 0 \quad \tfrac{1}{2} \Rightarrow \boxed{x u_y + y u_x = 0}$$

Q II $\boxed{6}$

3B $\boxed{1}$ Constant coefficients.

$A = 2\,,\quad B = 3\,,\quad C = 0$

$u \begin{cases} \xi \begin{cases} x \\ y \end{cases} \\ \eta \begin{cases} x \\ y \end{cases} \end{cases}$

let $\xi = x\,,\quad \eta = 3x - 2y$

$\xi_y = 0\,,\ \xi_x = 1 \qquad \eta_x = 3\,,\ \eta_y = -2$

$$u_x = u_\xi \xi_x + u_\eta \eta_x = u_\xi + 3u_\eta$$

$$u_y = u_\xi \xi_y + u_\eta \eta_y = -2u_\eta$$

substitute in $2u_x + 3u_y = 3x + 2y$

$$\Rightarrow 2u_\xi + 6u_\eta - 6u_\eta = \eta$$

$$\Rightarrow 2u_\xi = \eta$$

$$\Rightarrow u_\xi = \tfrac{1}{2}\eta$$

integrate w.r.t ξ, we have

$$u(\xi,\eta) = \tfrac{1}{2}\eta\xi + f(\eta)$$

$$\Rightarrow u(x,y) = \tfrac{1}{2}(3x-2y)\cdot x + f(3x-2y)$$

$$\boxed{u(x,y) = \frac{3}{2}x^2 - xy + f(3x-2y)}$$

B) non constant coefficients:

$A = 5, \; B = 4y, \; C = 0$

let $\xi = x$ and η is the solution of $\frac{dy}{dx} = \frac{B}{A} = \frac{4y}{5}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{5} y$

$\Rightarrow 5 \frac{dy}{y} = 4\, dx \Rightarrow 5 \ln y = 4x + C_1 \Rightarrow y^5 = Ce^{4x}, \quad C = e^{C_1}$

$\Rightarrow y^5 e^{-4x} = C$

$\Rightarrow \eta = y^5 e^{-4x}$ $\quad \begin{vmatrix} \xi_x & \xi_y \\ \eta_x & \eta_y \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -4y^5 e^{-4x} & 5y^4 e^{-4x} \end{vmatrix} = 5y^4 e^{-4x} \neq 0$

$\xi_x = 1, \; \xi_y = 0, \; \eta_x = -4y^5 e^{-4x}, \; \eta_y = 5y^4 e^{-4x}$

$u_x = u_\xi \xi_x + u_\eta \eta_x = u_\xi + (-4) y^5 e^{-4x} u_\eta$

$u_y = u_\xi \xi_y + u_\eta \eta_y = 5y^4 e^{-4x} u_\eta$

substitute in $5u_x + 4y u_y = 1 + e^{2x}$

$5u_\xi - \cancel{20y^5 e^{-4x} u_\eta} + \cancel{20y^5 e^{-4x} u_\eta} = 1 + e^{2x}$

$$\boxed{u(x,y) = \frac{1}{5}x + \frac{1}{10} e^{2x} + f(y^5 e^{-4x})}$$

$5u_\xi = 1 + e^{2\xi}$

$u_\xi = \frac{1}{5} + \frac{1}{5} e^{2\xi}$

integrate w.r.t ξ, we have

$u(\xi, \eta) = \frac{1}{5}\xi + \frac{1}{5(2)} e^{2\xi} + f(\eta)$

III ① 4B $x^2 u_x + y^2 u_y = u^2$, $x=t$, $y=2t$, $u=1$

$P = x^2$, $Q = y^2$, $R = u^2$

The subsidiary equations:

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{du}{u^2}$$

① $\frac{dx}{x^2} = \frac{dy}{y^2} \Rightarrow x^{-2}dx = y^{-2}dy \Rightarrow -\frac{1}{x} = -\frac{1}{y} + C_1 \Rightarrow \frac{1}{y} - \frac{1}{x} = C_1$

$\Rightarrow v = \frac{1}{y} - \frac{1}{x}$

② $\frac{dx}{x^2} = \frac{du}{u^2} \Rightarrow x^{-2}dx = u^{-2}du \Rightarrow -\frac{1}{x} = -\frac{1}{u} + C_2 \Rightarrow \frac{1}{u} - \frac{1}{x} = C_2$

$\Rightarrow w = \frac{1}{u} - \frac{1}{x}$

The general solution is given by:

$$F\left(\frac{1}{y} - \frac{1}{x}, \frac{1}{u} - \frac{1}{x}\right) = 0$$

$\Rightarrow \frac{1}{u} - \frac{1}{x} = f\left(\frac{1}{y} - \frac{1}{x}\right) \rightarrow (*)$

apply the conditions:

$1 - \frac{1}{t} = f\left(\frac{1}{2t} - \frac{1}{t}\right)$

$1 - \frac{1}{t} = f\left(\frac{-1}{2t}\right)$

let $w = \frac{-1}{2t} \Rightarrow t = \frac{-1}{2w}$

$\Rightarrow f(w) = 1 + \frac{2w}{1}$

substitute in (*)

$\frac{1}{u} - \frac{1}{x} = 1 + 2\left(\frac{1}{y} - \frac{1}{x}\right)$

$\frac{1}{u} = 1 + \frac{2}{y} - \frac{2}{x} + \frac{1}{x} = 1 + \frac{2}{y} - \frac{1}{x}$

$\frac{1}{u} = \frac{xy + 2x - y}{xy} \Rightarrow \boxed{u = \frac{xy}{xy + 2x - y}}$

1) $u_x + 3u_y = u^2$, $u(x,0) = \cos x$.

$$\frac{dx}{1} = \frac{dy}{3} = \frac{du}{u^2}$$

1) $dx = \frac{dy}{3}$

$3dx = dy$

$y - 3x = C_1$

$\boxed{v = y - 3x}$

2) $dx = u^{-2} du$

$x = \frac{-1}{u} + C$

$\frac{1}{u} + x = C_2$ $\qquad C_2 = -C$

$\boxed{w = \frac{1}{u} + x}$

The general solution

$$F(y-3x, \frac{1}{u} + x) = 0$$

$\frac{1}{u} + x = f(y-3x) \rightarrow (*)$

apply the condition:

$\frac{1}{\cos x} + x = f(-3x)$

let $w = -3x \Rightarrow x = -\frac{1}{3} w$

$\Rightarrow f(w) = \frac{1}{\cos(-\frac{1}{3}w)} + (-\frac{1}{3}w)$

$f(w) = \frac{1}{\cos(\frac{w}{3})} - \frac{w}{3}$

substitute in $(*)$

$\frac{1}{u} + x = \frac{1}{\cos \frac{y-3x}{3}} - \frac{y-3x}{3}$

$\frac{1}{u} = \frac{3}{3\cos(\frac{y-3x}{3})} - \frac{(y-3x)\cos(\frac{y-3x}{3})}{3\cos(\frac{y-3x}{3})} - \frac{3x\cos(\frac{y-3x}{3})}{3\cos(\frac{y-3x}{3})}$

$\frac{1}{u} = \frac{3 - y\cos(\bullet) + \cancel{3x\cos(\bullet)} - \cancel{3x\cos(\bullet)}}{3\cos(\frac{y-3x}{3})} = \frac{3 - y\cos(\frac{y-3x}{3})}{3\cos(\frac{y-3x}{3})}$

$$\boxed{u = \frac{3\cos(\frac{y-3x}{3})}{3 - y\cos(\frac{y-3x}{3})}}$$

IV :-

6

$\boxed{1}$ 1B

$P = 1 \quad , Q = -1 \quad , \quad R = 2$

$\frac{\partial x_0}{\partial t} = 2 \qquad \frac{\partial y_0}{\partial t} = -2 \quad , \frac{\partial u_0}{\partial t} = 2$

$$\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} = (-2)(1) - (-1)(2) = -2 + 2 = 0$$

$$\frac{P}{\frac{\partial x_0}{\partial t}} = \frac{1}{2} \quad , \quad \frac{Q}{\frac{\partial y_0}{\partial t}} = \frac{-1}{-2} = \frac{1}{2} \quad , \quad \frac{R}{\frac{\partial u_0}{\partial t}} = \frac{2}{2} = 1$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} \neq 1$$

no solution.

$\boxed{2}$ 3Ps

$x u_x + y u_y = 2xy\, e^{-u}$

$P = x \quad , Q = y \quad , R = 2xy e^{-u}$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{2xy e^{-u}}$$

① $\frac{dx}{x} = \frac{dy}{y}$

$\Rightarrow \ln x - \ln y = C$

$\ln \frac{x}{y} = C$

$\boxed{\frac{x}{y} = C_1} \quad , C_1 = e^C$

$\frac{dy}{y} = \frac{du}{2xy e^{-u}}$

$2x\,dy = e^{u} du$

$2C_1 y \, dy = e^{u} du$

$\cancel{2} C_1 \frac{y^2}{\cancel{2}} = e^{u} + C_2$

$C_1 y^2 = e^u + C_2$

$\frac{x}{y} y^2 = e^u + C_2$

$xy = e^u + C_2$

$\boxed{xy - e^u = C_2}$