College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1446 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	introduction to Partial Differential Equations		اسم المقرر
Course Code	425 Math		رمز المقرر
Exam Date	2025-05-22	1446-11-24	تاريخ الامتحان
Exam Time	08: 00 AM		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.	[illegible]		رقم قاعة الاختبار
Instructor Name	[illegible]		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان ١ صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	CLO 1-1	QIV(1)	1	
2	CLO 2-1	QII(A),QIV(2)	2+4	
3	CLO 2-2	QI(A),QIV(3)	7+3	
4	CLO 2-3	QI(B,C), QII(B), QIII	9+2+12	
5				
6				40
7				
8				

EXAM COVER PAGE

Question I: [(3+4) +5+4=16 points]

A) Solve the following:

1- $\frac{y^2 z}{x} z_x + zx\, z_y - y^2 = 0$

2- $3u_x - 2u_y + u = x.(use\ the\ change\ of\ veriables)$

B) Find the solution of the partial differential equation subject to the given condition

$$\begin{cases} xu_x + yu_y = x^2 - y \\ u(1,y) = y. \end{cases}$$

C) Solve the following PDE (determine if it has a unique, infinity many solutions or no solution]:

$$u_x + u_y = 1$$

1- $u(x,x) = x$

2- $u(x,0) = h(x)$

3- $u(x,x) = 1$

Question II: [2+2=4 points]

A) Obtain the partial differential equation of lower derivatives by eliminating the arbitrary functions $u(x,y) = e^x G(2x - y)$.

B) Use D'Alembert solution to solve the problem:

$$u_{tt} - 4u_{xx} = 0, -\infty < x < \infty, t > 0$$

$$u(x,0) = e^{-x}, u_t(x,0) = 0, -\infty < x < \infty.$$

Hint: D'Alembert formula is $u(x,t) = \frac{1}{2}[f(x - ct) + f(x + ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)ds$

Question III: [5+7=12 points]

A) Find the surface passing through the parabolas,

$$y^2 = 4ax, \quad u = 0. \qquad and \quad y^2 = -4ax, \quad u = 1$$

and satisfying the equation $xu_{xx} + 2u_x = 0$.

B) Show that the solution of the equation $u_t = u_{xx}$ satisfying the condition

$$u = 0 \text{ for } x = 0 \text{ and } x = a \text{ for all } t > 0$$

$$u = x \text{ for } t = 0 \text{ and } 0 < x < a$$

is given by

$$u(x,t) = \frac{2a}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\sin\frac{n\pi}{a}xe^{-\left(\frac{n\pi}{a}\right)^2 t}.$$

[Hint: use $\alpha > 0$ is as a separation constant]

Question IV: [1+4+3=8 points]

Consider the following equation:

$$u_{xx} - 4u_{xy} + 4u_{yy} = \cos(2x + y)$$

1- Classify the equation.

2- Reduce it to the appropriate canoncial form.

3- Obtain the general solution.

Good Luck