College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1446 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Introduction to Partial Differential Equations**		اسم المقرر
Course Code	**425 Math**		رمز المقرر
Exam Date	**2024-12-17**	**1446-06-16**	تاريخ الامتحان
Exam Time	**08: 00 AM**		وقت الامتحان
Exam Duration	**3 hours**	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 1-1 (1 point)	QII(A)(a)		
2	C.L.O 2-1 (4+4+4 marks)	QII(A)(b) QII(B) QIV(B)		
3	C.L.O 2-2 ((4+4)+2 marks)	QI QII(A)(c)		
4	C.L.O 2-3 ((3.5+3.5+4)+6 marks)	QIII QIV(A)		
5				
6				
7				
8				

EXAM COVER PAGE

Question I: (4+4 points)

A) Find the general solution of the PDE :

$$u_x + u_y + u_z + u_t = 0$$

B) Find the integral surface of

$$(u^2 - 2yu - y^2)u_x + (xy + xu)u_y = xy - xu$$

Question II: ((1+4+2)+4 points)

A) Consider the following second-order PDE:

$$u_{xx} + 2u_{xy} + u_{yy} = 0 \text{ ....(1)}$$

a) Classify the equation (1).

b) Use the chain rule to write the differential equation (1) in the coordinates

$$\xi = x, \qquad \eta = x - y.$$

c) Find the general solution of the given equation (1).

B) What are the conditions on the constants a, b, c, d and the function f so that the function

$$u(x, y, z) = f(ax + by + cz + d)$$

is harmonic function.

Question III: (3.5+3.5+4 points)

A) Solve the following Cauchy problem:

$$\begin{cases} e^{-u}(u_x + xu_y) = \dfrac{1}{1+x}, \\ \Gamma: x = 0, \quad y = t, \quad u = t. \end{cases}$$

B) Find the integral surface of

$x^2u_x + y^2u_y + u^2 = 0$,

which passes through the hyperbola:

$$xy = x + y, u = 1.$$

C) Find the solution of the following initial-problem:

$$u_{xx} - 4u_{yy} = \sin(x + y),$$
$$u(0, y) = 0, \qquad u_x(0, y) = 0.$$

Question IV: (6+4 points)

A) Find the solution of the following initial-boundary value problem

$$u_t = 2u_{xx}, \quad 0 < x < \pi, \quad t > 0$$
$$u(0, t) = 5, u(\pi, t) = 10, \qquad t > 0$$
$$u(x, 0) = \sin 3x - \sin 5x, \qquad 0 < x < \pi$$

(Hint: $\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$)

B) Prove that the following initial-boundary value problem

$$\frac{\partial u}{\partial t} = \beta \frac{\partial^2 u}{\partial x^2}, 0 < x < L, \quad t > 0$$
$$u(0, t) = u(L, t) = 0, \quad t > 0$$
$$u(x, 0) = F(x), \quad 0 < x < L$$

has a unique solution.

Good Luck