College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1445 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	**Introduction to Partial Differential Equations**		اسم المقرر
Course Code	**425 MATH**		رمز المقرر
Exam Date	**2024-05-26**	**1445-11-18**	تاريخ الامتحان
Exam Time	**01: 00 PM**		وقت الامتحان
Exam Duration	**3 hours**	ثلاث ساعات	مدة الامتحان
Classroom No.	**F052**		رقم قاعة الاختبار
Instructor Name	د. هدى الرشيدي		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 2 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 2 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	CLO 1.1 (1 mark)	QII(A)		
2	CLO 2.1 (15 marks)	QI(A), QII(B, D) QIV(B)		
3	CLO 2.2 (7 marks)	QI(C) QII(C)		
4	CLO 2.3 (17 marks)	QI(B) QIII QIV(A)		
5				
6				
7				
8				

EXAM COVER PAGE

QI:

A) Assume that $\boldsymbol{P}$, $\boldsymbol{Q}$ and $\boldsymbol{R}$ are nonzero functions. The solution of the first-order quasilinear partial differential equation

$$\boldsymbol{P(x,y,u)u_x + Q(x,y,u)u_x = P(x,y,u)}$$

can be written as integral surfaces $\boldsymbol{F(v,w) = 0}$ where $\boldsymbol{v = v(x,y,z)}$ and $\boldsymbol{w = w(x,y,z)}$ are two linear known independent functions, and $\boldsymbol{F}$ is an arbitrary function in $\mathbf{C^1}$. **By using the subsidiary equations and the Jacobian,** prove that:

$$\frac{\partial(v,w)}{\partial(z,x)} = \frac{Q}{P}\frac{\partial(v,w)}{\partial(y,z)}, \qquad \frac{\partial(v,w)}{\partial(x,y)} = \frac{R}{P}\frac{\partial(v,w)}{\partial(y,z)}$$

B) Find the solution of partial differential equation

$$\boldsymbol{u(x+u)u_x - y(y+u)u_y = 0,}$$

where $\boldsymbol{u(1,t) = \sqrt{t}, t > 0.}$

C) Find the solution of the following equations:

1. $\boldsymbol{(u^2 - 2yu - y^2)u_x + (xy + xu)u_y = xy - xu}$
2. $\boldsymbol{(x-y)u_{xy} - u_x + u_y = 0}$, (Hint: $v = (x-y)u$).

QII:

A) Classify the following equation:

$$\boldsymbol{4u_{xx} + 12u_{xy} + 9u_{yy} = 0.} \quad \textbf{(1)}.$$

B) Reduce the equation **(1)** into a canonical form, then find its solution.

C) Solve the canonical form that you found it in Part (B).

D) Find a partial differential equation which has the following solution:

$$\boldsymbol{a(x^2 + y^2) + bz^2 = 1}$$

QIII:

A) By using the separation variables and appropriate separation constant, find the solution of heat equation

$$\boldsymbol{u_t = ku_{xx}, \quad 0 < x < 1, \qquad t > 0.}$$

B) By using Part (A), find its solution if

$$\boldsymbol{u(0,t) = u(1,t) = 0, \qquad t > 0,}$$

$$\boldsymbol{u(x,0) = f(x), \qquad 0 < x < 1.}$$

QIV:

A) Find the solution of wave equation:

$$(2)\quad u_{tt} = 4u_{xx}, \qquad 0 < x < \pi, \qquad t > 0$$

$$(3)\quad \begin{cases} u(x,0) = 0,\, u_t(x,0) = x, & 0 < x < \pi, \\ u(0,t) = u(\pi,t) = 0, & t > 0. \end{cases}$$

B) Prove the uniqueness of equation (2) – (3).

<u>Good Luck.</u>