Question 1

1. Let $P = \begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ 1 & -2 & 0 & 3 & 2 \\ 2 & 1 & 5 & 1 & 4 \\ -1 & 3 & 1 & -4 & -2 \end{pmatrix}$. A row equivalent matrix to P is $\begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$.
   Then $\{v_1, v_2, v_3\}$ is a basis of E contained in $\{v_1, v_2, v_3, v_4, v_5\}$.

2. If C is the standard basis, ${}_CP_S = \begin{pmatrix} -1 & 1 & 2 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. ${}_SP_C = \begin{pmatrix} 1 & 1 & -2 \\ 2 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$ $[P]_S =$ ${}_SP_C[P]_C = \begin{pmatrix} 9 \\ 10 \\ 1 \end{pmatrix}$.

Question 2

1. ${}_CP_B = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 1 & 3 & -2 \end{pmatrix}$

2. If $v = -3u_1 + u_2 + 2u_3$, then find $[v]_B = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$ and $[v]_C = {}_CP_B[v]_B = \begin{pmatrix} -6 \\ 7 \\ -4 \end{pmatrix}$.

Question 3

the matrix $\begin{pmatrix} 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ is row equivalent to A.

1. $\text{Rank}(A) = 2$.

2. The set of solutions is $S = \{y(-2, 1, 0, 0, 0) + t(-1, 0, -1, 1, 0) + u(-2, 0, -1, 0, 1),\ y, t, u \in \mathbb{R}\}$.
   The set $\{(-2, 1, 0, 0, 0), (-1, 0, -1, 1, 0), (-2, 0, -1, 0, 1)\}$ is a basis of the null space of A.

Question 4

1. $\|u_1\| = 1$.

2. $v_1 = u_1$.
   $\langle u_2, v_1\rangle = 1$, $u_2 - v_1 = (-1, 0, 1)$, $v_2 = \frac{1}{\sqrt{2}}(-1, 0, 1)$.
   $\langle u_3, v_1\rangle = 0$ and $\langle u_3, v_2\rangle = 0$, then $v_3 = \frac{1}{\sqrt{2}}(1, 0, 1)$.
   $\{v_1, v_2, v_3\}$ is an orthonormal basis of $\mathbb{R}^3$.

Question 5

1. $\langle u, v\rangle = -8$. $\|u\| = \|v\| = 3$, and $\cos\theta = -\frac{8}{9}$.

2. $(3, -3, 0, 3) = 3(1, -1, 0, 1)$, but $T(3, -3, 0, 3) \neq 3T(1, -1, 0, 1)$