King Saud University

College of Sciences

Department of Mathematics

Second Examination	Math 244	Semester I, (1441),	Duration: 1hr. 30 mn

Calculators are not allowed

Question 1 [6 pts]

1. Let E be the subspace of $\mathbb{R}^4$ spanned by the following vectors: $v_1 = (1, 1, 2, -1)$, $v_2 = (2, -2, 1, 3)$, $v_3 = (3, 0, 5, 1), v_4 = (-1, 3, 1, -4), v_5 = (1, 2, 4, -2)$.
   Find a basis of E contained in $\{v_1, v_2, v_3, v_4, v_5\}$.
2. Consider the basis $S = \{P_1 = 2x - 1, P_2 = 1 - x, P_3 = 2 + x^2\}$ of $\mathcal{P}_2$.
   Find $[P]_S$ (the coordinates of P on the basis S), where $P = 3 + 8x + x^2$.

Question 2 [4 pts]
Let V be a vector space of dimension 3 and $B = \{u_1, u_2, u_3\}$, $C = \{v_1, v_2, v_3\}$ are basis for V such that

$$\begin{aligned} u_1 + v_2 &= v_1 + v_3 \\ u_2 - v_1 &= 2v_2 + 3v_3 \\ u_3 + 2v_3 &= -2v_1 + v_2 \end{aligned}$$

1. Find ${}_CP_B$ (the transition matrix from base B to C).
2. If $v = -3u_1 + u_2 + 2u_3$, then find $[v]_B$ and $[v]_C$.

Question 3 [5 pts]

Let $A = \begin{pmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ -1 & -2 & 0 & -1 & -2 \\ 1 & 2 & -1 & 0 & 1 \end{pmatrix}$.

1. Find $\text{Rank}(A)$.
2. Find a basis of the null space of A.

Question 4 [6 pts]
Consider the following inner product on $\mathbb{R}^3$

$$\langle u, v\rangle = x_1y_1 + 2x_2y_2 + x_3y_3 - x_1y_2 - x_2y_1,$$

for $u = (x_1, x_2, x_3)$ and $v = (y_1, y_2, y_3)$.
Consider $B = \{u_1 = (1, 1, 0), u_2 = (0, 1, 1), u_3 = (1, 0, 1)\}$ a basis of $\mathbb{R}^3$.

1. Find $\|u_1\|$.
2. Use Gram-Schmidt algorithm on $\{u_1, u_2, u_3\}$ to obtain an orthonormal basis of $\mathbb{R}^3$.

Question 5 [4 pts]

1. Let $\mathbb{R}^3$ be the Euclidean space, $u = (1, -2, 2)$ and $v = (-2, 1, -2)$.
   Find $\cos\theta$, where θ is the angle between the vectors u and v.
2. Consider the mapping $T\colon \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ such that $T(1, -1, 0, 1) = (2, 3, 5)$ and $T(3, -3, 0, 3) = (-3, 5, 2)$.
   Explain why T is not a linear transformation.