Section 4.1:

In problems 6-16 determine whether the given functions are linearly independent or dependent on $(-\infty, \infty)$

9) $f_1(x) = \cos(2x)$, $f_2(x) = 3$, $f_3(x) = \cos^2(x)$.

(9) $f_1 = \cos(2x) = 2\cos^2(x) - 1$

$= 2f_3(x) - \frac{1}{3}(3) = 2f_3(x) - \frac{1}{3}f_2(x)$

$\Rightarrow (1) f_1(x) + \frac{1}{3} f_2(x) - 2 f_3(x) = 0 \quad \forall x \in (-\infty, \infty)$

$1 \neq 0$, $\frac{1}{3} \neq 0$, $-2 \neq 0$

$\Rightarrow$ f_1, f_2, and f_3 are linearly dependent on $\mathbb{R}$

16) $f_1(x) = x\,|x|$, $f_2(x) = x^2$.

scan me

In exercises 17-24 show by computing the *Wronskian* that the given functions are linearly independent or dependent on the indicated interval.

23) e^x , xe^x , x^2e^x ; $(0, \infty)$.

23) $y_1 = e^x$, $y_2 = xe^x$, $y_3 = x^2e^x$ on $(0, \infty)$

$$W(x, y_1, y_2, y_3) = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$$= \begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & xe^x + e^x & x^2e^x + 2xe^x \\ e^x & xe^x + 2e^x & x^2e^x + 3xe^x + 2e^x \end{vmatrix}$$

$$= e^x \cdot e^x \cdot e^x \begin{vmatrix} 1 & x & x^2 \\ 1 & x+1 & x^2+2x \\ 1 & x+2 & x^2+3x+2 \end{vmatrix}$$

$$= e^{3x} \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 2 & 3x+2 \end{vmatrix} = e^{3x} \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2-x \end{vmatrix}$$

$$= e^{3x}(2-x) \neq 0 \quad \text{for } x \neq 2 \text{ in } (0, \infty)$$

So $y_1 = e^x$, $y_2 = xe^x$, and $y_3 = x^2e^x$ are linearly independent on $(0, \infty)$

In exercises 25-32 verify that the given functions form a fundamental set solutions of the differential equation on the indicated interval .

25) $y'' - y' - 12y = 0$; e^{-3x} , e^{4x} , $(-\infty , \infty)$.

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In exercises 36-37 find an interval around $x = 0$ for which the given initial -value problem has a unique solution

36) $\begin{cases} (x-2)y'' + 3y = x \\ y(0) = 0 \ , \ y'(0) = 1 \end{cases}$.

36 $a_2(x) = (x-2)$ is continuous on $\mathbb{R}$

$a_1(x) = 0$ is // // //

$a_0(x) = 3$ is // // //

$R(x) = x$ is // // //

$\mathbb{R} \cap \mathbb{R} \cap \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

$a_2(x) = x - 2 \neq 0$ if $x \neq 2$

(D.E is normal) $\mathbb{R} - \{2\} = (-\infty, 2) \cup (2, \infty)$

$0 \in (-\infty, 2)$

So IVP has unique solution on $(-\infty, 2)$