Chapter 7

Ultimate Bearing Capacity of Shallow Foundations: Special Cases

Omitted parts:

Sections 7.6 , 7.10 , 7.12

Ultimate Bearing Capacity of Shallow Foundations

The ultimate bearing capacity problems described in Chapter 6 assume that :

- The soil supporting the foundation is homogeneous and extends to a great depth below the bottom of the foundation.
- The ground surface is horizontal.

However, that is not true in all cases:

- It is possible to encounter a rigid layer at a shallow depth.
- The soil may be layered and have different shear strength parameters.
- It may be necessary to construct foundations on or near a slope.
- It may be required to design a foundation subjected to uplifting load.

This chapter discusses bearing capacity problems related to these special cases.

Foundation Supported by a Soil with a Rigid Base at Shallow Depth

For shallow, rough *continuous* foundation supported by a soil that extends to a great depth

$$q_u = c'N_c + qN_q + \frac{1}{2}\gamma B N_\gamma$$

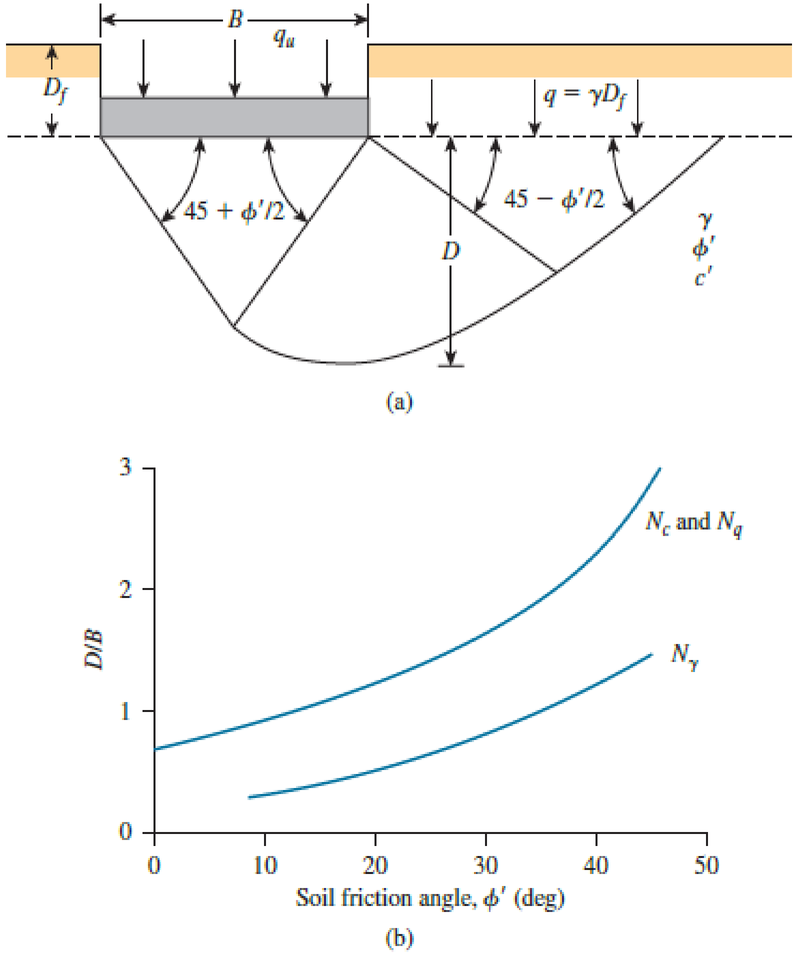


FIGURE 7.1 (a) Failure surface under a rough continuous foundation; (b) variation of D/B with soil friction angle ϕ'

Foundation Supported by a Soil with a Rigid Base at Shallow Depth

If a rigid, rough base is located at a depth of $H < D$ below the bottom of the foundation, full development of the failure surface in soil will be restricted. In such a case, the soil failure zone and the development of slip lines at ultimate load will be as shown in the Figure

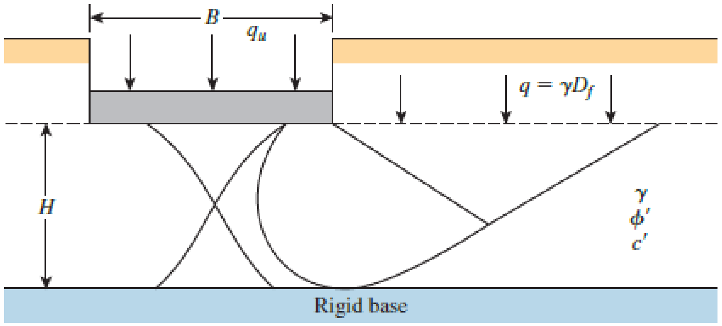


FIGURE 7.2 Failure surface under a rough continuous foundation with a rigid rough base located at a shallow depth

$$q_u = c'N_c^* + qN_q^* + \frac{1}{2}\gamma BN_\gamma^*$$

N_c^*, N_q^*, N_γ^* = modified bearing capacity factors

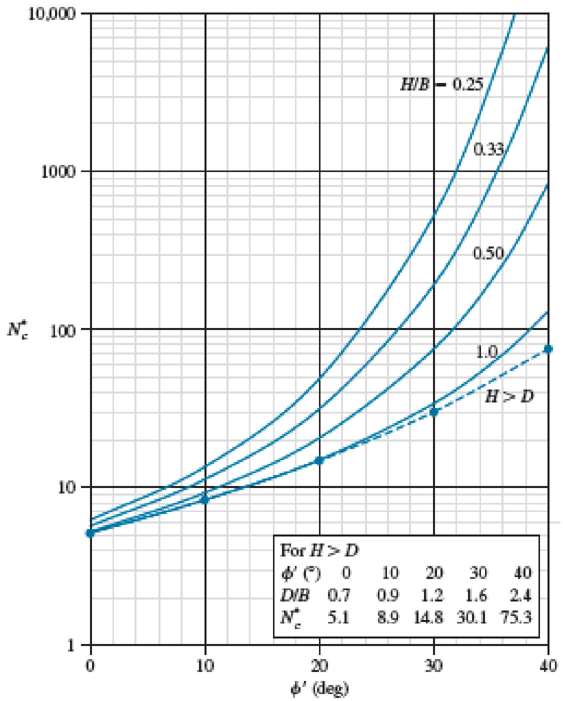
B = width of foundation

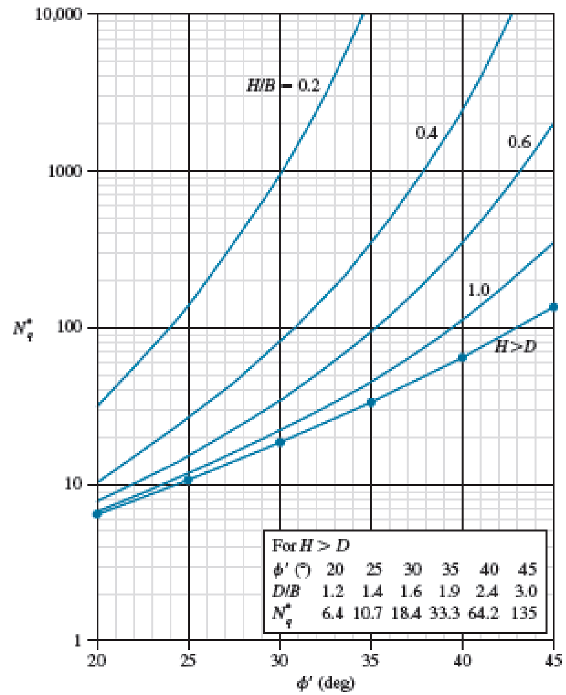
γ = unit weight of soil

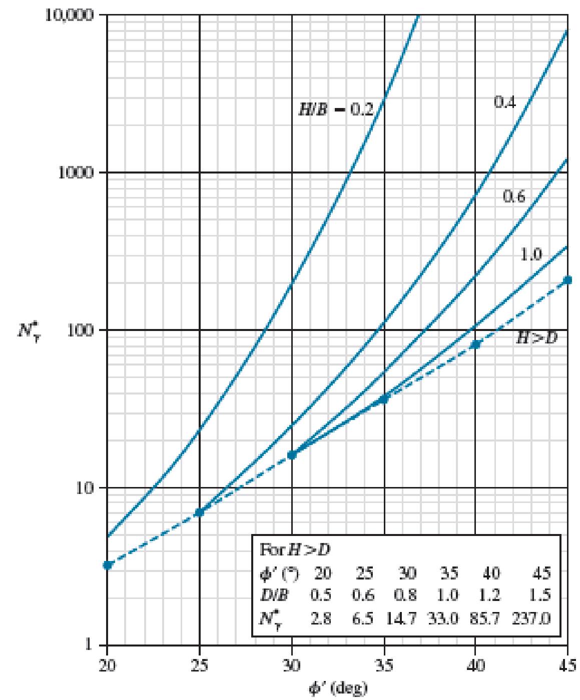
for $H \geq D$, $N_c^* = N_c$, $N_q^* = N_q$, and $N_\gamma^* = N_\gamma$

The variations of N_c^*, N_q^*, and N_γ^* with H/B and the soil friction angle ϕ' are given in Figures 7.3, 7.4, and 7.5, respectively.

Foundation Supported by a Soil with a Rigid Base at Shallow Depth







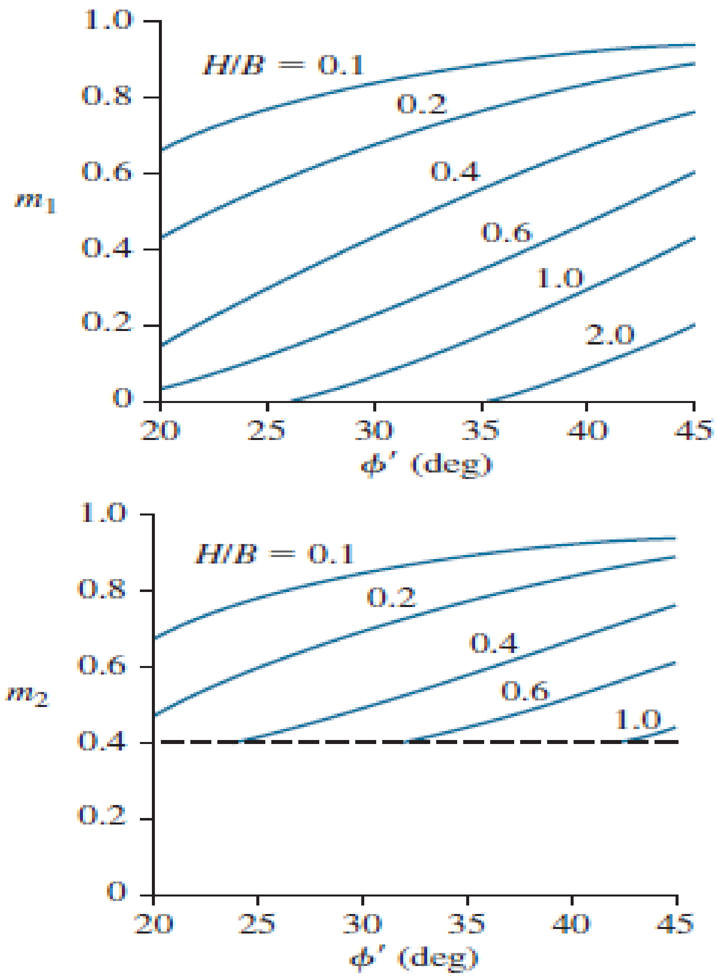
Rectangular Foundation on Granular Soil

$$q_u = qN_q^* F_{qs}^* + \frac{1}{2}\gamma B N_\gamma^* F_{\gamma s}^*$$

F_{qs}^*, $F_{\gamma s}^*$ are modified shape factors

$$F_{qs}^* \approx 1 - m_1\left(\frac{B}{L}\right)$$

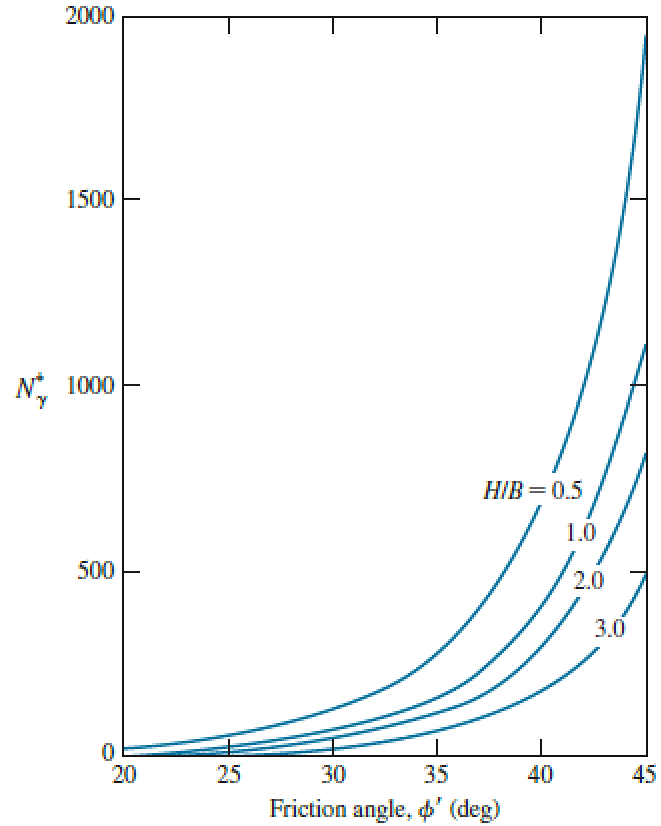
$$F_{\gamma s}^* \approx 1 - m_2\left(\frac{B}{L}\right)$$



Square and Circular Foundations on Granular Soil

$$q_u = qN_q^* + 0.4\gamma BN_\gamma^* \text{ (square foundation)}$$

$$q_u = qN_q^* + 0.3\gamma BN_\gamma^* \text{ (circular foundation)}$$



Foundations on Saturated Clay

For a continuous foundation on saturated clay (i.e., under the undrained condition, or $\phi = 0$)

$$q_u = c_u N_c^* + q$$

Buisman (1940) gave the following relationship for obtaining the ultimate bearing capacity of square foundations:

$$q_{u(\text{square})} = \left(\pi + 2 + \frac{B}{2H} - \frac{\sqrt{2}}{2}\right) c_u + q \qquad (\text{for } H < 0.707B)$$

$$q_{u(\text{square})} = \underbrace{5.14\left(1 + \frac{0.5\dfrac{B}{H} - 0.707}{5.14}\right)}_{N^*_{c(\text{square})}} c_u + q$$

TABLE 7.1 Values of N_c^* for Continuous and Square Foundations ($\phi = 0$)

$\frac{B}{H}$	N_c^*	
	Square[a]	Continuous[b]
2	5.43	5.24
3	5.93	5.71
4	6.44	6.22
5	6.94	6.68
6	7.43	7.20
8	8.43	8.17
10	9.43	9.05

[a]Buisman's analysis (1940)
[b]Mandel and Salencon's analysis (1972)

EXAMPLE 7.1

EXAMPLE 7.1

A square foundation measuring 1.2 m × 1.2 m is constructed on a layer of sand. We are given that $D_f = 1$ m, $\gamma = 15.5$ kN/m^3, $\phi' = 35°$, and $c' = 0$. A rock layer is located at a depth of 0.48 m below the bottom of the foundation. Using a factor of safety of 4, determine the gross allowable load the foundation can carry.

$$q_u = qN_q^* F_{qs}^* + \frac{1}{2}\gamma B N_\gamma^* F_{\gamma s}^*$$

$$q = 15.5 \times 1 = 15.5 \text{ kN/m}^3$$

For $\phi' = 35°$, $H/B = 0.48/1.2 = 0.4$, $N_q^* \approx 336$ (Figure 7.4), and $N_\gamma^* \approx 138$ (Figure 7.5), and we have

$$F_{qs}^* = 1 - m_1\left(\frac{B}{L}\right)$$

From Figure 7.6a for $\phi' = 35°$, $H/B = 0.4$. The value of $m_1 \approx 0.58$, so

$$F_{qs}^* = 1 - (0.58)(1.2/1.2) = 0.42$$

Similarly,

$$F_{\gamma s}^* = 1 - m_2(B/L)$$

From Figure 7.6b, $m_2 = 0.6$, so

$$F_{\gamma s}^* = 1 - (0.6)(1.2/1.2) = 0.4$$

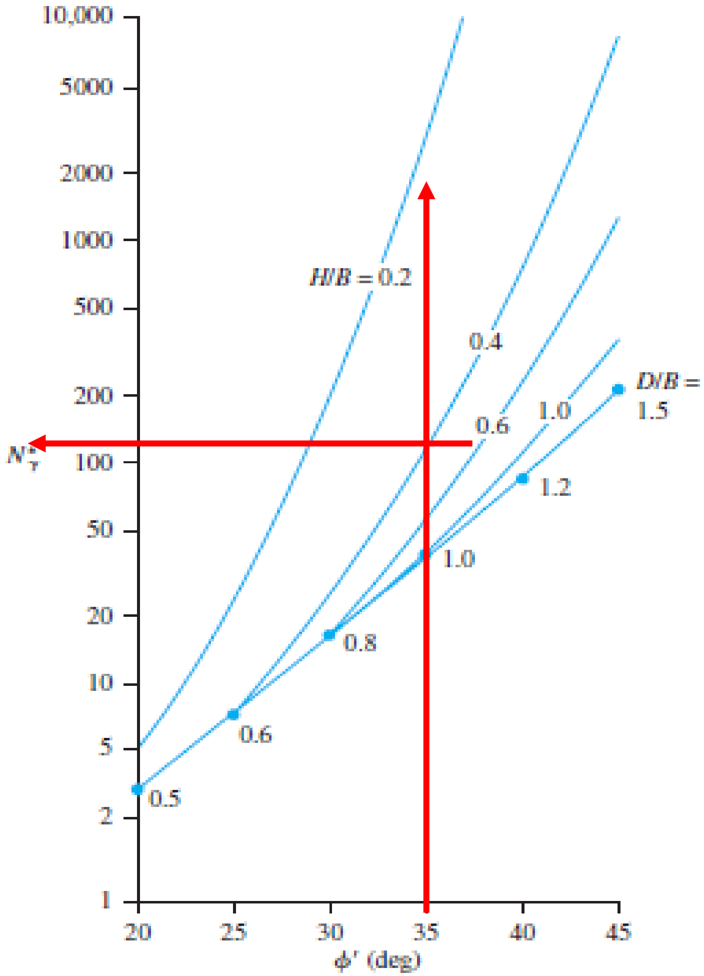
Hence,

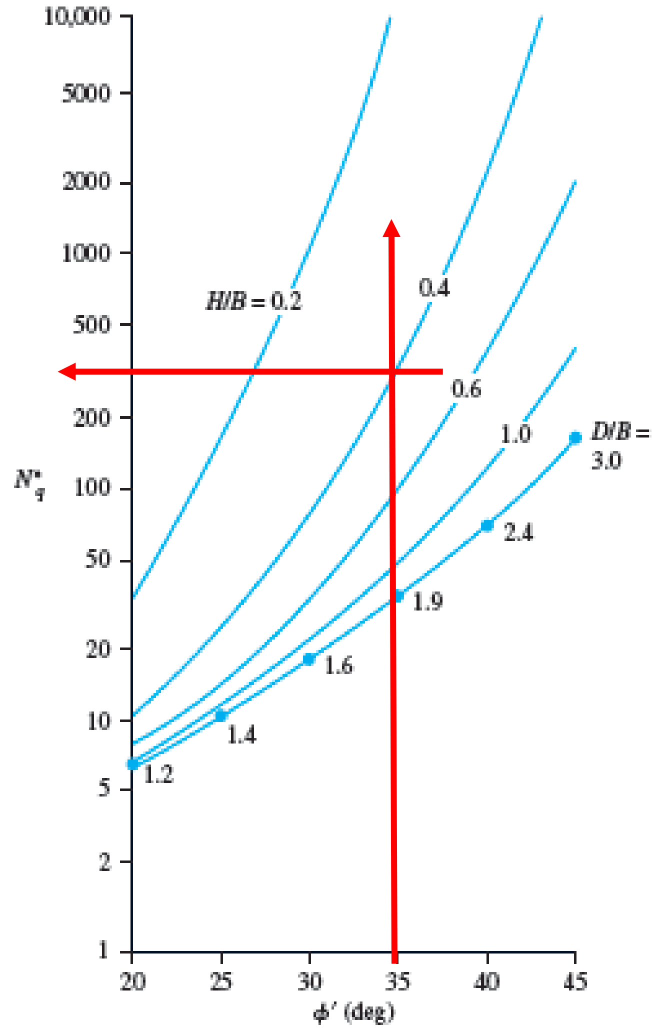
$$q_u = (15.5)(336)(0.42) + (1/2)(15.5)(1.2)(138)(0.4) = 2700.72 \text{ kN/m}^2$$

and

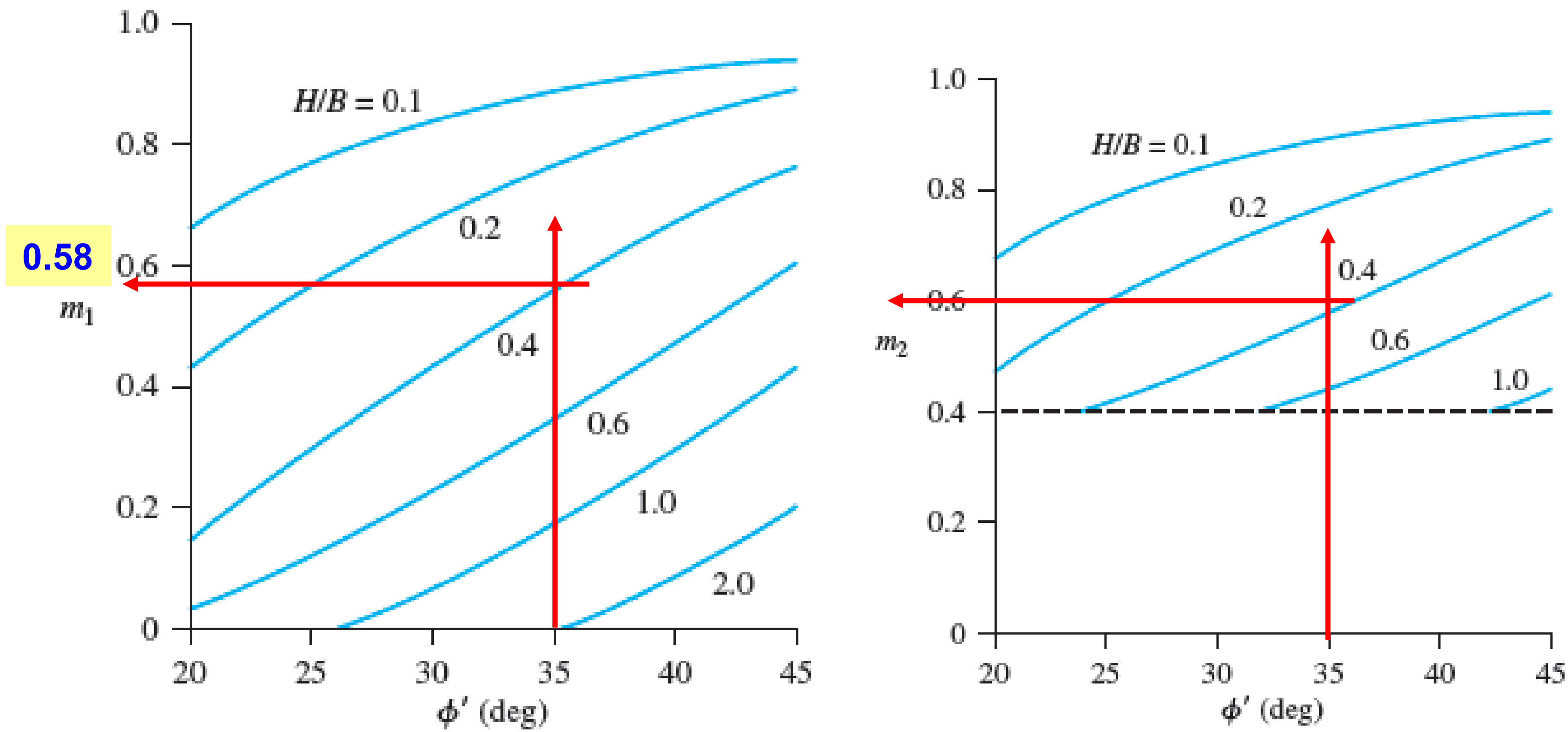
$$Q_{all} = \frac{q_u B^2}{FS} = \frac{(2700.72)(1.2 \times 1.2)}{4} = 972.3 \text{ kN}$$

EXAMPLE 7.1


10,000
5000
2000
1000
500
200
100
50
20
10
5
2
1
N_γ^*
H/B = 0.2
0.4
0.6
1.0
D/B =
1.5
1.2
1.0
0.8
0.6
0.5
20
25
30
35
40
45
φ′ (deg)


10,000
5000
2000
1000
500
200
100
50
20
10
5
2
1
N_q^*
H/B = 0.2
0.4
0.6
1.0
D/B =
3.0
2.4
1.9
1.6
1.4
1.2
20
25
30
35
40
45
φ′ (deg)

EXAMPLE 7.1



0.58
1.0
0.8
0.6
0.4
0.2
0
m_1
H/B = 0.1
0.2
0.4
0.6
1.0
2.0
20
25
30
35
40
45
φ′ (deg)
1.0
0.8
0.6
0.4
0.2
0
m_2
H/B = 0.1
0.2
0.4
0.6
1.0
20
25
30
35
40
45
φ′ (deg)

EXAMPLE 7.2

EXAMPLE 7.2

Consider a square foundation 1 m × 1 m in plan located on a saturated clay layer underlain by a layer of rock. Given:

Clay: $c_u = 72\ \text{kN/m}^2$
Unit weight: $\gamma = 18\ \text{kN/m}^3$
Distance between the bottom of foundation and the rock layer = 0.25 m
$D_f = 1$ m

Estimate the gross allowable bearing capacity of the foundation. Use FS = 3.

SOLUTION

From Eq. (7.10),

MEYERHOF?

$$q_u = 5.14\left(1 + \frac{0.5\dfrac{B}{H} - 0.707}{5.14}\right)c_u + q$$

For $B/H = 1/0.25 = 4$; $c_u = 72\ \text{kN/m}^2$; and $q = \gamma D_f = (18)(1) = 18\ \text{kN/m}^3$.

$$q_u = 5.14\left[1 + \frac{(0.5)(4) - 0.707}{5.14}\right]72 + 18 = 481.2\ \text{kN/m}^2$$

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{481.2}{3} = \mathbf{160.4\ kN/m^2}$$

EXAMPLE 7.2

TERZAGHI

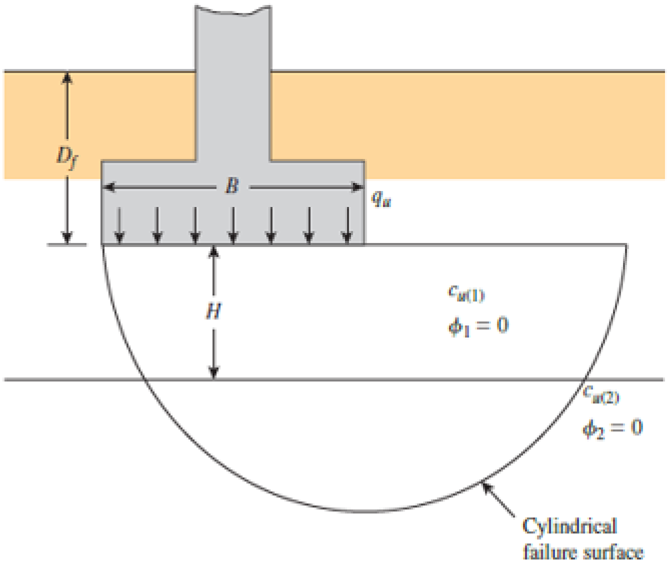
$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma \quad \text{(square foundation)}$$

$N_{C} = 5.7, \quad N_q = 1$

$$q_u = 1.3 x 72 x 5.7 + 18 x 1 = 551.5 \text{ kPa}$$

Terzaghi's equation is conservative

Foundations on Layered Clay ($\phi = 0$)



<u>1. Reddy and Srinivasan (1967)</u>

For undrained loading ($\phi = 0$ condition) :

let $c_{u(1)}$ = shear strength of the upper clay layer

$c_{u(2)}$ = shear strength of the lower clay layer

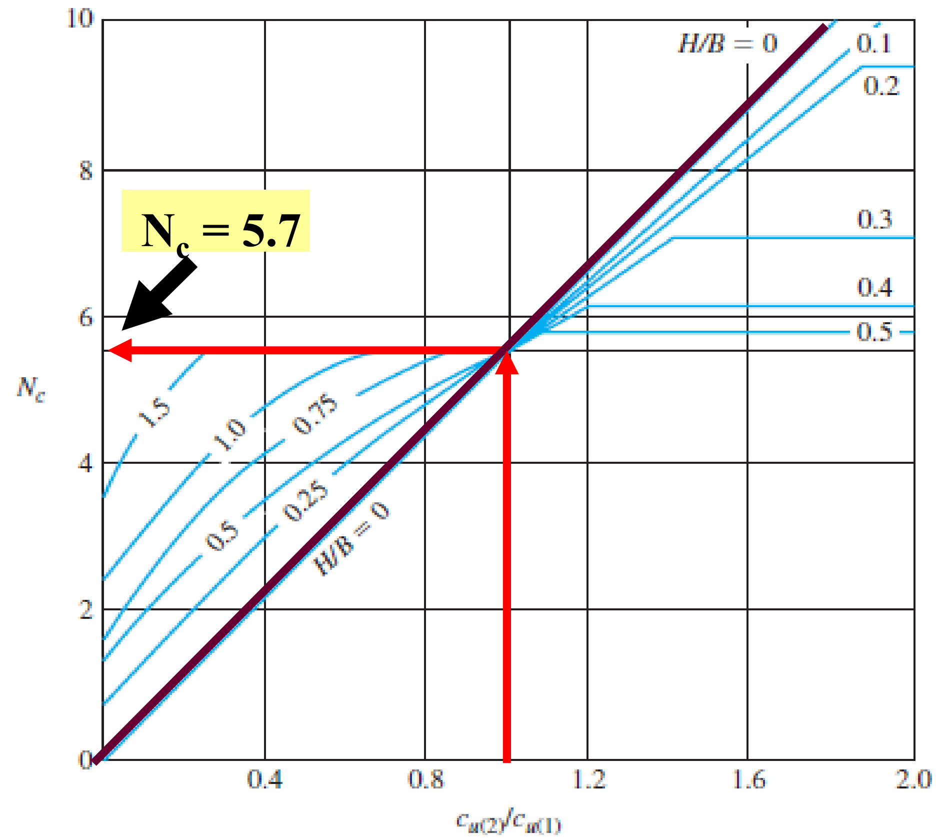
$$q_u = c_{u(1)} N_c F_{cs} F_{cd} + q$$

The relationships for F_{cs} and F_{cd} given in Table 6.3

Foundations on Layered Clay ($\phi = 0$)

- For layered soils, the value of the bearing capacity factor, N_c, is not a constant.
- It is a function of $c_{u(2)}/c_{u(1)}$ and H/B (*note*: H = depth measured from the bottom of the foundation to the interface of the two clay layers).

- If the lower layer of clay is softer than the top one ($c_{u(2)}/c_{u(1)} < 1$), the value of (N_c) is lower than when the soil is not layered ($c_{u(2)}/c_{u(1)} = 1$).
- This means that the ultimate bearing capacity is reduced by the presence of a softer clay layer below the top layer.



Weaker Layer underlain by Stronger Layer ($\phi = 0$)

2. Vesic (1975)

Ultimate bearing capacity of a foundation supported by a weaker clay layer [$c_{u(1)}$] underlain by a stronger clay layer [$c_{u(2)}$] i.e ($c_{u(1)} / c_{u(2)} < 1$) :

$$q_u = c_{u(1)} m N_c F_{cs} F_{cd} + q$$

where

$$N_c = \begin{cases} 5.14 \text{ for continous foundation} \\ 6.17 \text{ for square or circular foundation} \end{cases}$$

F_{cs} = shape factor

F_{cd} = depth factor

$$m = f\left[\frac{c_{u(1)}}{c_{u(2)}}, \frac{H}{B}, \text{and } \frac{B}{L}\right]$$

TABLE 7.2 Variation of m [Eq. (7.12)] for Continuous Foundation ($B/L \le 0.2$)

$c_{u(1)}/c_{u(2)}$	H/B ≥ 0.5	0.25	0.167	0.125	0.1
1	1	1	1	1	1
0.667	1	1.033	1.064	1.088	1.109
0.5	1	1.056	1.107	1.152	1.193
0.333	1	1.088	1.167	1.241	1.311
0.25	1	1.107	1.208	1.302	1.389
0.2	1	1.121	1.235	1.342	1.444
0.1	1	1.154	1.302	1.446	1.584

Based on Vesic (1975)

TABLE 7.3 Variation of m [Eq. (7.12)] for Square Foundation ($B/L = 1$)

$c_{u(1)}/c_{u(2)}$	H/B ≥ 0.25	0.125	0.083	0.063	0.05
1	1	1	1	1	1
0.667	1	1.028	1.052	1.075	1.096
0.5	1	1.047	1.091	1.131	1.167
0.333	1	1.075	1.143	1.207	1.267
0.25	1	1.091	1.177	1.256	1.334
0.2	1	1.102	1.199	1.292	1.379
0.1	1	1.128	1.254	1.376	1.494

Based on Vesic (1975)

m

EXAMPLE 7.3

EXAMPLE 7.3

Refer to Figure 7.8a. A foundation 1.5 m × 1 m is located at a depth (D_f) of 1 m in a clay. A softer clay layer is located at a depth (H) of 1 m measured from the bottom of the foundation. Given:

For the top clay layer,
Undrained shear strength = 120 kN/m²
Unit weight = 16.8 kN/m³

For the bottom clay layer,
Undrained shear strength = 48 kN/m²
Unit weight = 16.2 kN/m³

Determine the gross allowable load for the foundation with a factor of safety of 4. Use Eq. (7.11).

SOLUTION

From Eq. (7.11),

$$q_u = c_{u(1)} N_c F_{cs} F_{cd} + q$$

$$c_{u(1)} = 120 \text{ kN/m}^2$$

$$q = \gamma D_f = (16.8)(1) = 16.8 \text{ kN/m}^2$$

$$\frac{c_{u(2)}}{c_{u(1)}} = \frac{48}{120} = 0.4; \frac{H}{B} = \frac{1}{1} = 1$$

From Figure 7.8b, for $H/B = 1$ and $c_{u(2)}/c_{u(1)} = 0.4$, the value of N_c is equal to 4.6. From Table 6.3,

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1}{1.5}\right)\left(\frac{1}{4.6}\right) = 1.145$$

$$F_{cd} = 1 + 0.4\frac{D_f}{B} = 1 + 0.4\left(\frac{1}{1}\right) = 1.4$$

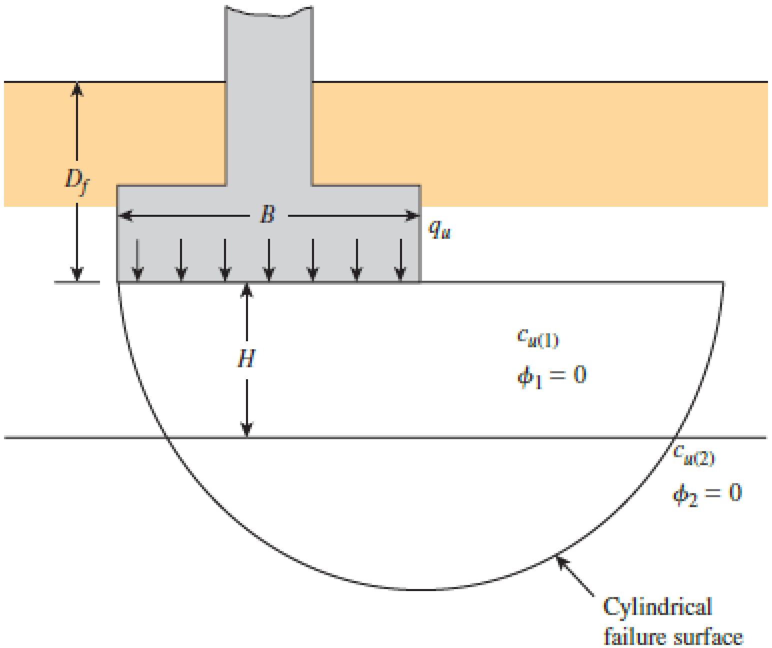
Thus,

$$q_u = (120)(4.6)(1.145)(1.4) + 16.8 = 884.8 + 16.8 = 901.6 \text{ kN/m}^2$$

So

$$q_{all} = \frac{q_u}{FS} = \frac{901.6}{4} = 225.4 \text{ kN/m}^2$$

Total allowable load = $(q_{all})(B \times L) = (225.4)(1 \times 1.5) = 338.1$ kN



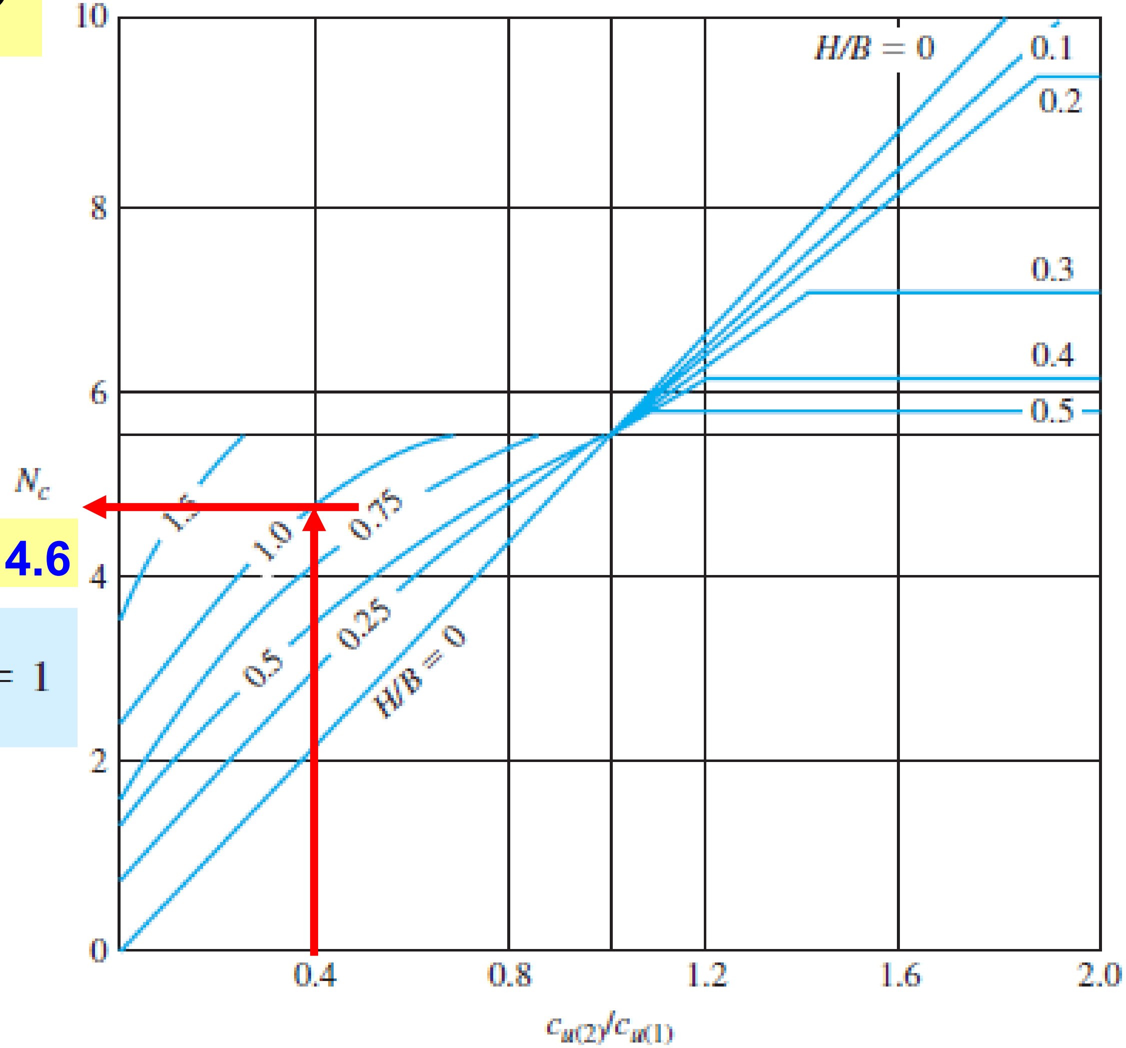
Reddy and Srinivasan, 1967) Solution

EXAMPLE 7.3

Reddy and Srinivasan (1967) Solution

$N_c = 4.6$

$$\frac{c_{u(2)}}{c_{u(1)}} = \frac{48}{120} = 0.4; \frac{H}{B} = \frac{1}{1} = 1$$



EXAMPLE 7.3

Vesic (1975)

$(c_{u(1)}/c_{u(2)} = 120/48 = 2.5$

Weaker Layer Underlain by Stronger Layer ($\phi = 0$)

TABLE 7.2 Variation of m [Eq. (7.12)] for Continuous Foundation ($B/L \leq 0.2$)

$c_{u(1)}/c_{u(2)}$	H/B ≥ 0.5	0.25	0.167	0.125	0.1
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Based on Vesic (1975)

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0.2	1	1.102	1.199	1.292	1.379
0.1	1	1.128	1.254	1.376	1.494

Based on Vesic (1975)

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

$$q_u = q_b + \frac{2(C_a + P_p \sin \delta')}{B} - \gamma_1 H$$

B = width of the foundation

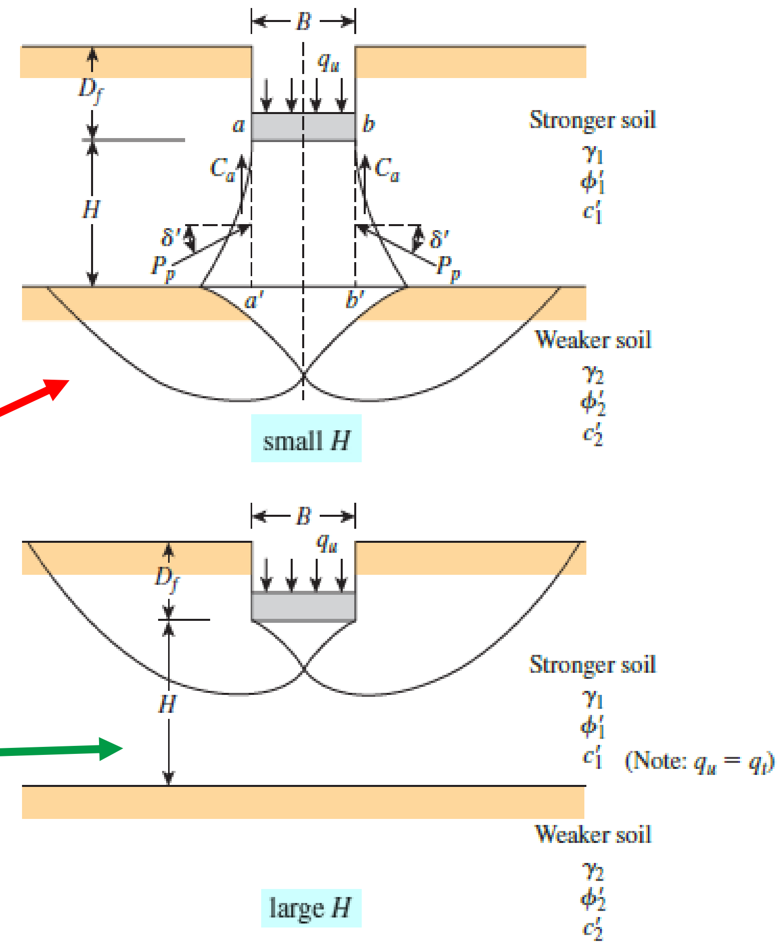
C_a = adhesive force $C_a = c'_a H$ ⟶ c'_a is the adhesion.

P_p = passive force per unit length of the faces aa' and bb'

q_b = bearing capacity of the bottom soil layer

δ' = inclination of the passive force P_p with the horizontal

Continuous Foundation



Bearing capacity of a continuous foundation on layered soil:

If the depth *H* is relatively small compared with the foundation width *B*, a punching shear failure will occur in the top soil layer, followed by a general shear failure In the bottom soil layer.

If the depth *H* is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity.

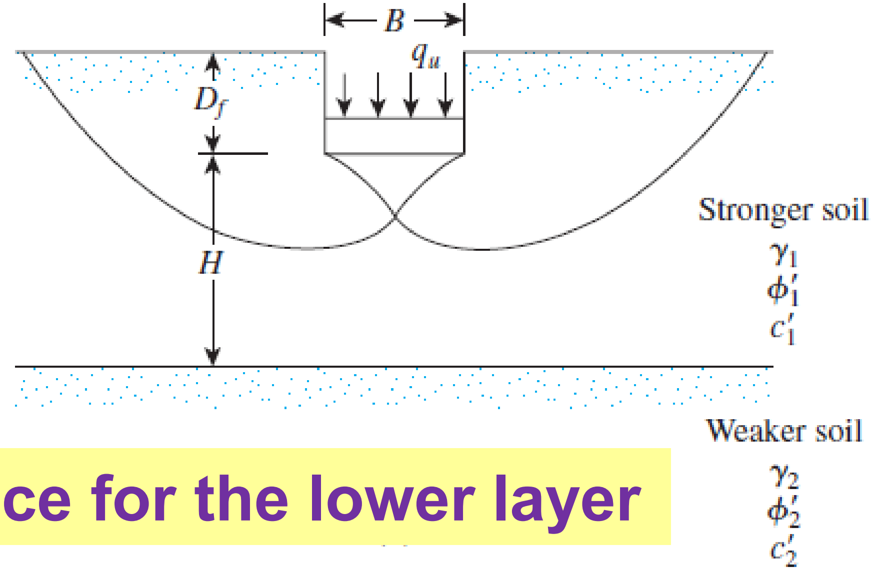
Stronger Layer underlain by Weaker Layer (c' - ϕ' soil)

Continuous Foundation

Layer	Soil properties: Unit weight	Friction angle	Cohesion
Top	γ_1	ϕ'_1	c'_1
Bottom	γ_2	ϕ'_2	c'_2

a. H is relatively large

- If the depth H is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity.



No relevance for the lower layer

$$q_u = q_t = c'_1 N_{c(1)} + q N_{q(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}.$$

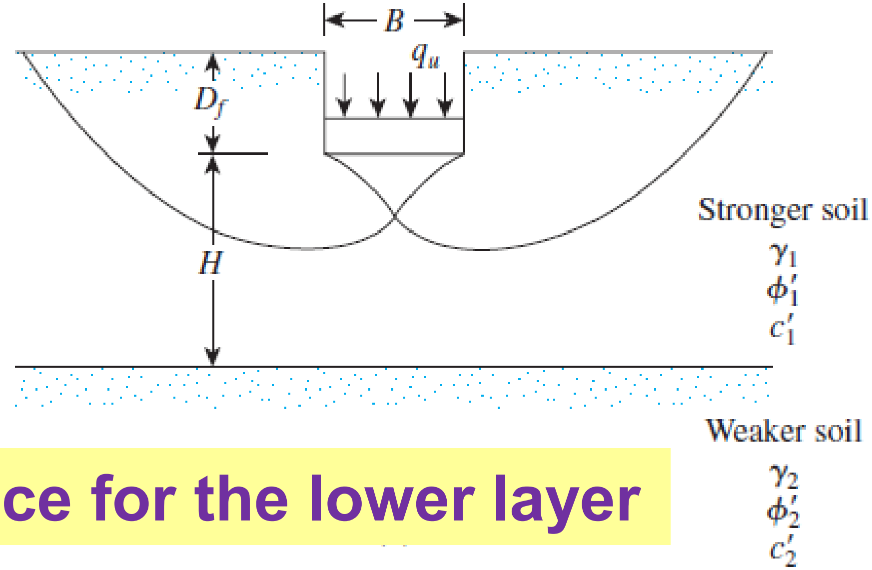
Stronger Layer underlain by Weaker Layer (c' - ϕ' soil)

Continuous Foundation

Layer	Soil properties		
	Unit weight	Friction angle	Cohesion
Top	γ_1	ϕ'_1	c'_1
Bottom	γ_2	ϕ'_2	c'_2

a. H is relatively large

- If the depth H is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity.



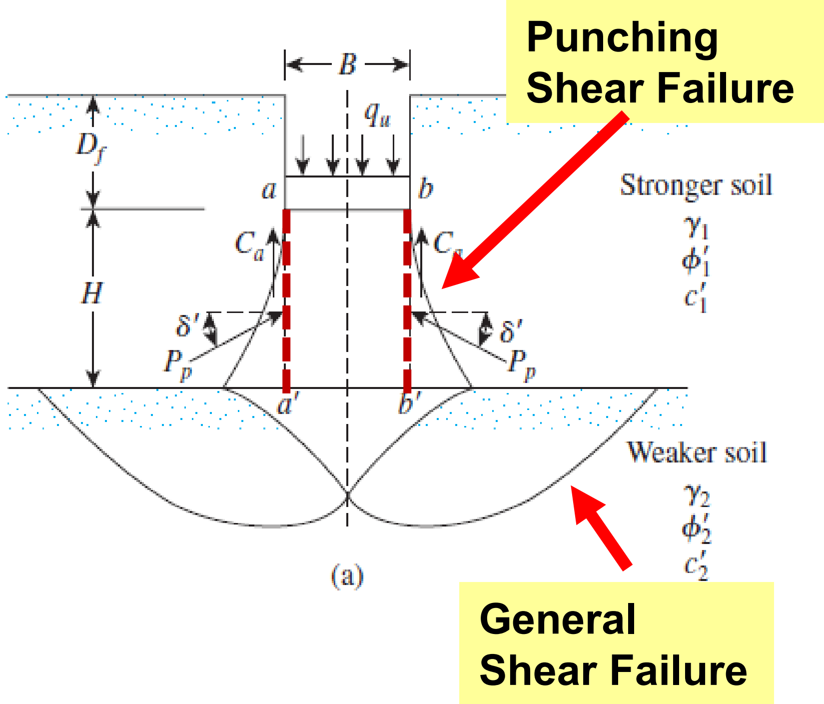
No relevance for the lower layer

$$q_u = q_t = c'_1 N_{c(1)} + qN_{q(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}.$$

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

b. H is relatively small

- If the depth H is relatively small compared with the foundation width B, a punching shear failure will occur in the top soil layer, followed by a general shear failure In the bottom soil layer.
- The failure of the footing may be considered due to pushing of soil with in the boundary aa' and bb' through the top layer into the weaker layer.
- The resisting force for punching may be assumed to develop on the faces of aa' and bb' passing through the edges of the footing.
- The forces that act on these surfaces are (per unit length of footing)



Adhesive force, $C_a = c_a H$

Frictional force, $F_f = P_p \sin \delta$

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

- The equation for the ultimate bearing capacity q_u for the two layer soil system may now be expressed as

$$q_u = q_b + \frac{2(C_a + P_p \sin\delta')}{B} - \gamma_1 H$$

B = width of the foundation
C_a = adhesive force
P_p = passive force per unit length of the faces aa' and bb'
q_b = bearing capacity of the bottom soil layer
δ' = inclination of the passive force P_p with the horizontal

$$P_p = \frac{\gamma_1 H^2}{2\cos\delta}\left(1+\frac{2D_f}{H}\right)K_p$$

$$C_a = c'_a H$$

Substituting for P_p and C_a, the equation for q_u may be written as

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_{pH}\tan\delta'}{B} - \gamma_1 H$$

where K_{pH} = horizontal component of passive earth pressure coefficient.

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

We need to know c_a and k_s . The rest are geometric parameters

$$K_{pH} \tan \delta' = K_s \tan \phi'_1$$

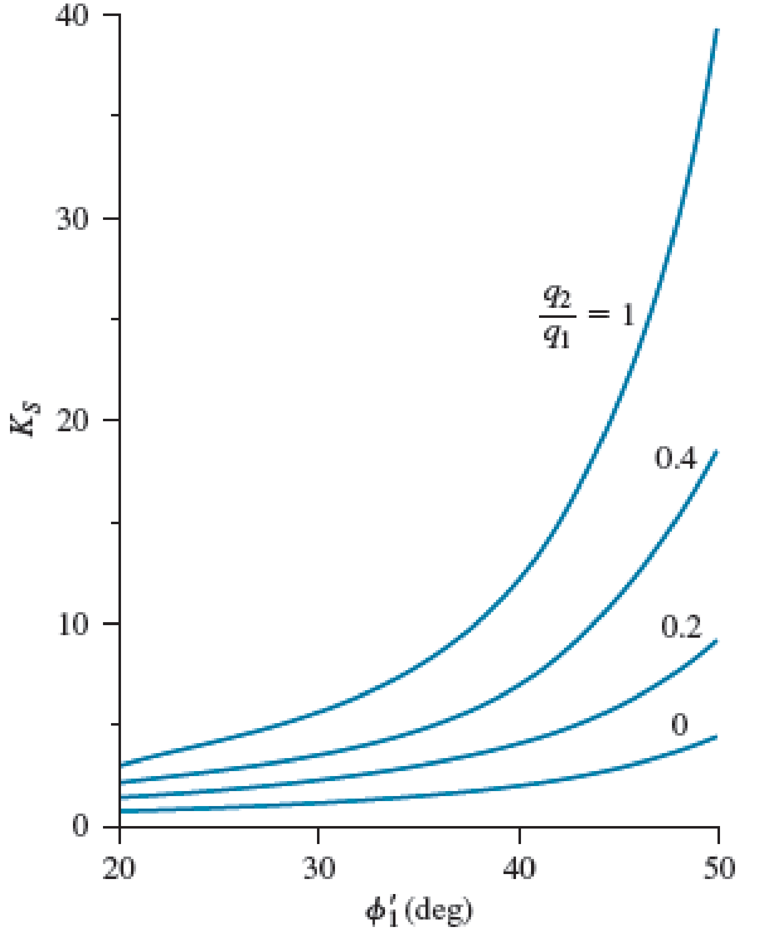
where K_s = punching shear coefficient. Then,

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan \phi'_1}{B} - \gamma_1 H$$

$$K_s = f\left(\frac{q_2}{q_1}, \phi'_1\right)$$

$$q_1 = c'_1 N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

$$q_2 = c'_2 N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$



Note that q_1 and q_2 are the ultimate bearing capacities of a continuous foundation of width B under vertical load on the surfaces of homogeneous thick beds of upper and lower soil.

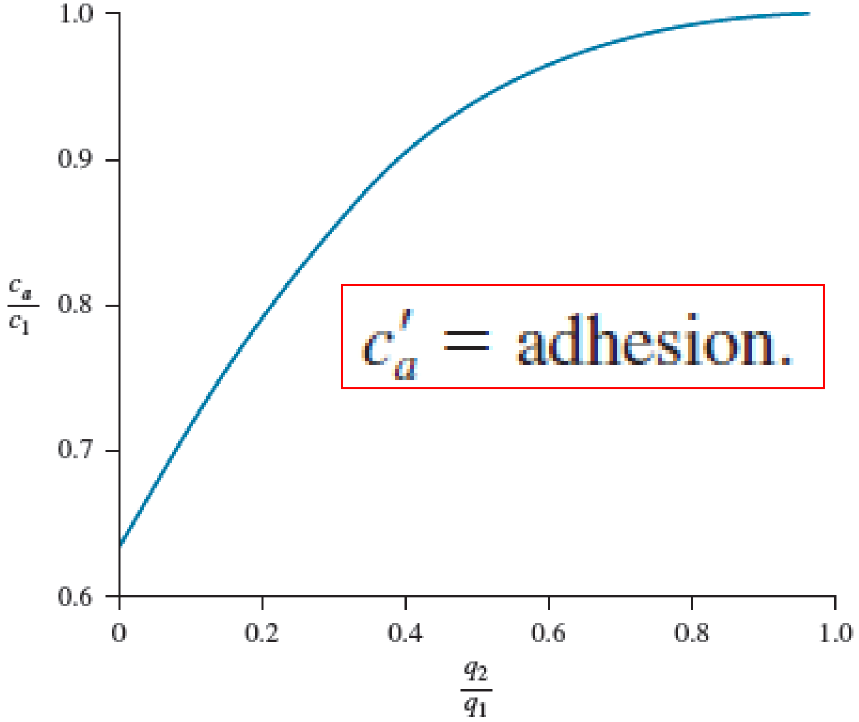
Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Where q_1 is the ultimate bearing capacity of the top layer and q_2 is the ultimate bearing capacity of the bottom layer with a fictitious footing of the same size and shape but resting on the surface of the bottom layer.

Very important

q_1 and q_2 are different from q_t and q_b.

q_1 and q_2 are for surface footings.



Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Continuous Foundation

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_{pH}\tan\delta'}{B} - \gamma_1 H$$

K_{pH} is the horizontal component of passive earth pressure coefficient.

$$K_{pH}\tan\delta' = K_s\tan\phi'_1$$

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s\tan\phi'_1}{B} - \gamma_1 H \quad \text{(a)}$$

$$K_s = f\left(\frac{q_2}{q_1}, \phi'_1\right)$$

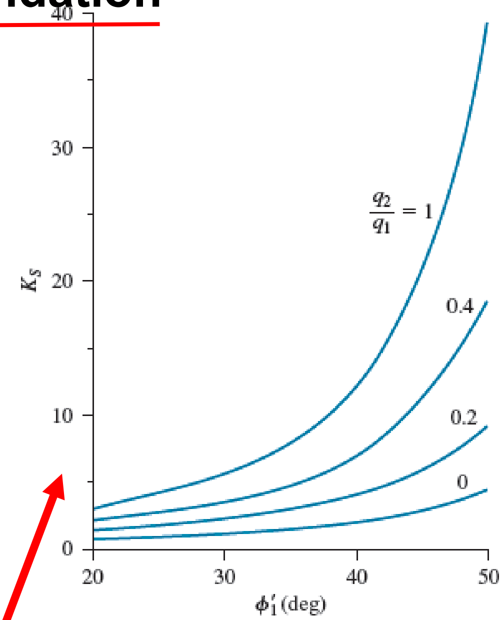
$$q_1 = c'_1 N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

$$q_2 = c'_2 N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$

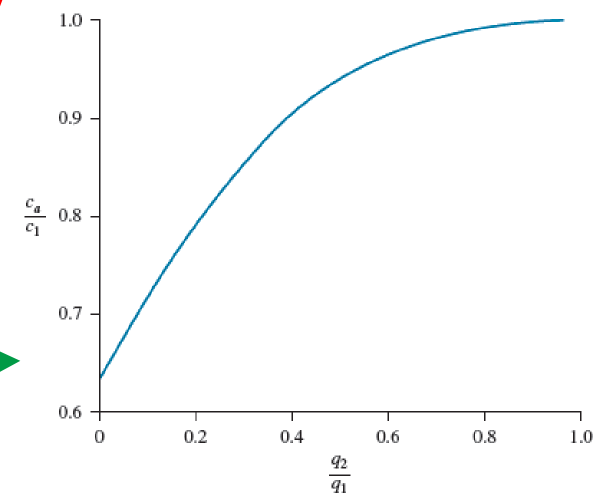
$N_{c(1)}$, $N_{\gamma(1)}$ are the bearing capacity factors for friction angle ϕ'_1 (Table 6.2)

$N_{c(2)}$, $N_{\gamma(2)}$ are the bearing capacity factors for friction angle ϕ'_2 (Table 6.2)

the top layer to be a stronger soil, q_2/q_1 should be less than unity.



The variation of K_s with q_2/q_1 and ϕ_1 is shown in Figure.



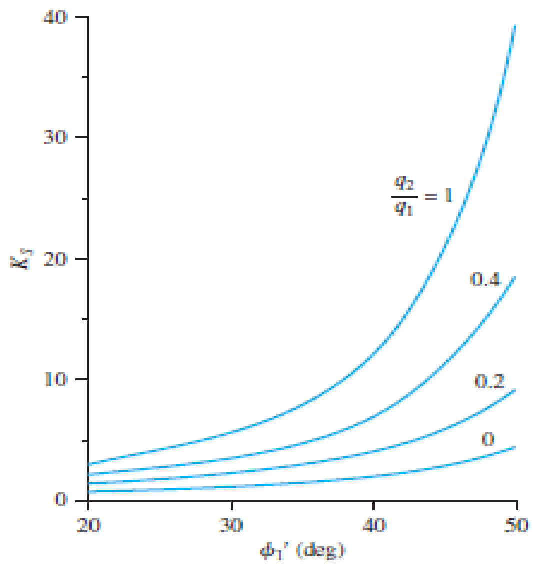
The variation of c_a/c_1 with q_2/q_1 is shown in Figure .

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Continuous Foundation

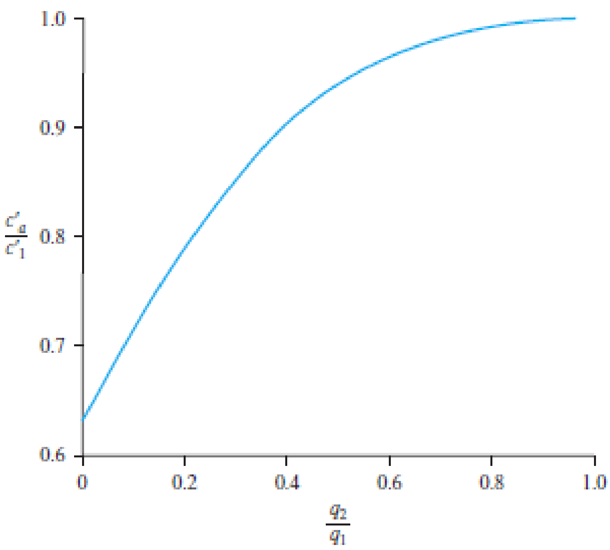
$N_{c(1)}$, $N_{q(1)}$, and $N_{\gamma(1)}$ are the bearing capacity factors for $\phi' = \phi_1'$ (Table 6.2)

$$q_u = q_b + \frac{2c_a'H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} - \gamma_1 H \le q_t$$



If the height H is relatively large, then the failure surface in soil will be completely located in the stronger upper-soil Layer. For this case

$$q_u = q_t = c_1' N_{c(1)} + qN_{q(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}. \quad \text{(b)}$$



Combining Eqs. (a) and (b) yields

$$q_u = q_b + \frac{2c_a'H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} - \gamma_1 H \le q_t$$

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

For rectangular foundations

$$q_u = q_b + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a'H}{B}\right) + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan \phi_1'}{B}\right) - \gamma_1 H \leq q_t$$

where

$$q_b = c_2' N_{c(2)} F_{cs(2)} + \gamma_1 (D_f + H) N_{q(2)} F_{qs(2)} + \frac{1}{2} \gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}$$

and

$$q_t = c_1' N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2} \gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

in which

$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors with respect to top soil layer (Table 4.3)
$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors with respect to bottom soil layer (Table 4.3)

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

A. H is relatively large

$$q_u = q_t = c_1' N_{c(1)} + q N_{q(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}.$$

B. H is relatively small

$$q_u = q_b + \frac{2c_a' H}{B} + \gamma_1 H^2 \left(1 + \frac{2D_f}{H}\right) \frac{K_s \tan \phi_1'}{B} - \gamma_1 H$$

$$q_u \leq q_t$$

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Top layer is strong sand and bottom layer is saturated soft clay $\phi_2 = 0$

$$q_b = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \gamma_1(D_f + H)$$ **ϕ =0**

and

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$ **c =0**

Hence,

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} + \gamma_1 D_f \leq \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

where $c_{u(2)}$ = undrained cohesion.

K_s is determined from 7.10

$$\frac{q_2}{q_1} = \frac{c_{u(2)}N_{c(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{5.14c_{u(2)}}{0.5\gamma_1 B N_{\gamma(1)}}$$

Recall

Surface footings

$$q_1 = c_1' N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

$$q_2 = c_2' N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$

c_1 =0 ϕ_2=0

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Top layer is stronger sand and bottom layer is weaker sand $(c_1' = 0, c_2' = 0)$.

$$q_u = \left[\gamma_1(D_f + H)N_{q(2)}F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)}F_{\gamma s(2)}\right] + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} - \gamma_1 H \leq q_t$$

where

$$q_t = \gamma_1 D_f N_{q(1)}F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}F_{\gamma s(1)}$$

Then

$$\frac{q_2}{q_1} = \frac{\frac{1}{2}\gamma_2 B N_{\gamma(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{\gamma_2 N_{\gamma(2)}}{\gamma_1 N_{\gamma(1)}}$$

Recall **Surface footings**

$$q_1 = c_1' N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

$$q_2 = c_2' N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$

$(c_1' = 0, c_2' = 0)$.

Stronger Layer underlain by Weaker Layer (c'- ϕ' soil)

Top layer is stronger saturated clay and bottom layer is weaker saturated clay ($\phi_1 = \phi_2 = 0$)

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 D_f \leq q_t$$

where

$$q_t = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(1)} + \gamma_1 D_f$$

and $c_{u(1)}$ and $c_{u(2)}$ are undrained cohesions. For this case,

$$\frac{q_2}{q_1} = \frac{5.14c_{u(2)}}{5.14c_{u(1)}} = \frac{c_{u(2)}}{c_{u(1)}}$$

Recall **Surface footings**

$$q_1 = c_1' N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

$$q_2 = c_2' N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$

$\phi_1 = 0$ $\phi_2 = 0$

EXAMPLE 7.4

EXAMPLE 7.4

Refer to Figure 7.9a and consider the case of a continuous foundation with $B = 2$ m, $D_f = 1.2$ m, and $H = 1.5$ m. The following are given for the two soil layers:

Top sand layer:

Unit weight $\gamma_1 = 17.5$ kN/m³
$\phi'_1 = 40°$
$c'_1 = 0$

Bottom clay layer:

Unit weight $\gamma_2 = 16.5$ kN/m³
$\phi'_2 = 0$
$c_{u(2)} = 30$ kN/m²

Determine the gross ultimate load per unit length of the foundation.

SOLUTION

For this case, Eqs. (7.27) and (7.28) apply. For $\phi'_1 = 40°$, from Table 6.2, $N_\gamma = 109.41$ and

$$\frac{q_2}{q_1} = \frac{c_{u(2)}N_{c(2)}}{0.5\gamma_1 B N_{\gamma(1)}} = \frac{(30)(5.14)}{(0.5)(17.5)(2)(109.41)} = 0.081$$

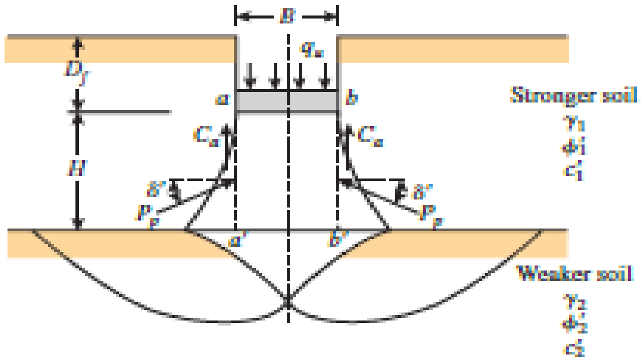
From Figure 7.10, for $c_{u(2)}N_{c(2)}/0.5\gamma_1 B N_{\gamma(1)} = 0.081$ and $\phi'_1 = 40°$, the value of $K_s \approx 2.5$. Equation (7.27) then gives

$$q_u = \left[1 + (0.2)\left(\frac{B}{L}\right)\right]5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)K_s\frac{\tan\phi'_1}{B} + \gamma_1 D_f$$

$$= [1 + (0.2)(0)](5.14)(30) + (1 + 0)(17.5)(1.5)^2$$

$$\times\left[1 + \frac{(2)(1.2)}{1.5}\right](2.5)\frac{\tan 40}{2.0} + (17.5)(1.2)$$

$$= 154.2 + 107.4 + 21 = 282.6 \text{ kN/m}^2$$



Again, from Eq. (7.26),

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

From Table 6.2, for $\phi'_1 = 40°$, $N_\gamma = 109.4$ and $N_q = 64.20$.
From Table 6.3,

$$F_{qs(1)} = 1 + \left(\frac{B}{L}\right)\tan\phi'_1 = 1 + (0)\tan 40 = 1$$

and

$$F_{\gamma s(1)} = 1 - 0.4\frac{B}{L} = 1 - (0.4)(0) = 1$$

so that

$$q_t = (17.5)(1.2)(64.20)(1) + \left(\frac{1}{2}\right)(17.5)(2)(109.4)(1) = 3262.7 \text{ kN/m}^2$$

Hence,

$$q_u = 282.6 \text{ kN/m}^2$$

$$Q_u = (282.6)(B) = (282.6)(2) = \mathbf{565.2 \text{ kN/m}}$$

EXAMPLE 7.4

Top layer is strong sand

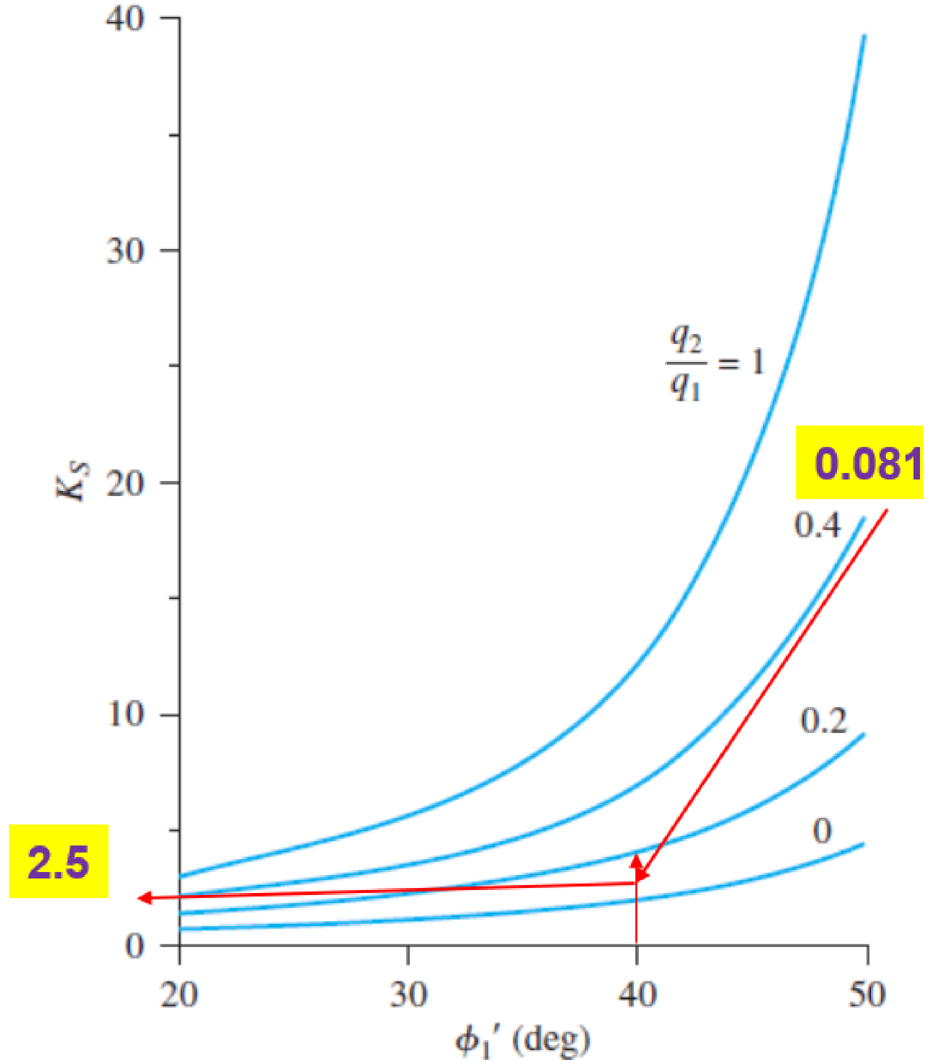
Bottom layer is saturated soft clay

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi'_1}{B} + \gamma_1 D_f \leq \underbrace{\gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}}_{q_t}$$

K_s is determined from 7.10

$$\frac{q_2}{q_1} = \frac{c_{u(2)} N_{c(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{5.14 c_{u(2)}}{0.5\gamma_1 B N_{\gamma(1)}}$$

EXAMPLE 7.4



EXAMPLE 7.5

EXAMPLE 7.5

A foundation 1.5 m × 1 m is located at a depth, D_f, of 1 m in a stronger clay. A softer clay layer is located at a depth, H, of 1 m measured from the bottom of the foundation. For the top clay layer,

Undrained shear strength = 120 kN/m²
Unit weight = 16.8 kN/m³

and for the bottom clay layer,

Undrained shear strength = 48 kN/m²
Unit weight = 16.2 kN/m³

Determine the gross allowable load for the foundation with an FS of 4. Use Eqs. (7.32), (7.33), and (7.34).

SOLUTION

For this problem, Eqs. (7.32), (7.33), and (7.34) will apply, or

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 D_f$$

$$\leq \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(1)} + \gamma_1 D_f$$

Given:

$B = 1$ m $H = 1$ m $D_f = 1$ m
$L = 1.5$ m $\gamma_1 = 16.8$ kN/m³

From Figure 7.11, $c_{u(2)}/c_{u(1)} = 48/120 = 0.4$, the value of $c_a/c_{u(1)} \approx 0.9$, so

$$c_a = (0.9)(120) = 108 \text{ kN/m}^2$$

$$q_u = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(48) + \left(1 + \frac{1}{1.5}\right)\left[\frac{(2)(108)(1)}{1}\right] + (16.8)(1)$$

$$= 279.6 + 360 + 16.8 = 656.4 \text{ kN/m}^2$$

Check: From Eq. (7.33),

$$q_t = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(120) + (16.8)(1)$$

$$= 699 + 16.8 = 715.8 \text{ kN/m}^2$$

Thus $q_u = 656.4$ kN/m² (q_t is always larger than q_u) and

$$q_{all} = \frac{q_u}{FS} = \frac{656.4}{4} = 164.1 \text{ kN/m}^2$$

The total allowable load is

$$(q_{all})(1 \times 1.5) = \mathbf{246.15 \text{ kN}}$$

Note: This is the same problem as in Example 7.3. The allowable load is about 40% lower than that calculated in Example 7.3. This is due to the different failure surface in the soil assumed at the ultimate load.

EXAMPLE 7.5

Top layer is stronger saturated clay and bottom layer is weaker saturated clay ($\phi_1 = \phi_2 = 0$)

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_aH}{B}\right) + \gamma_1 D_f \leq q_t$$

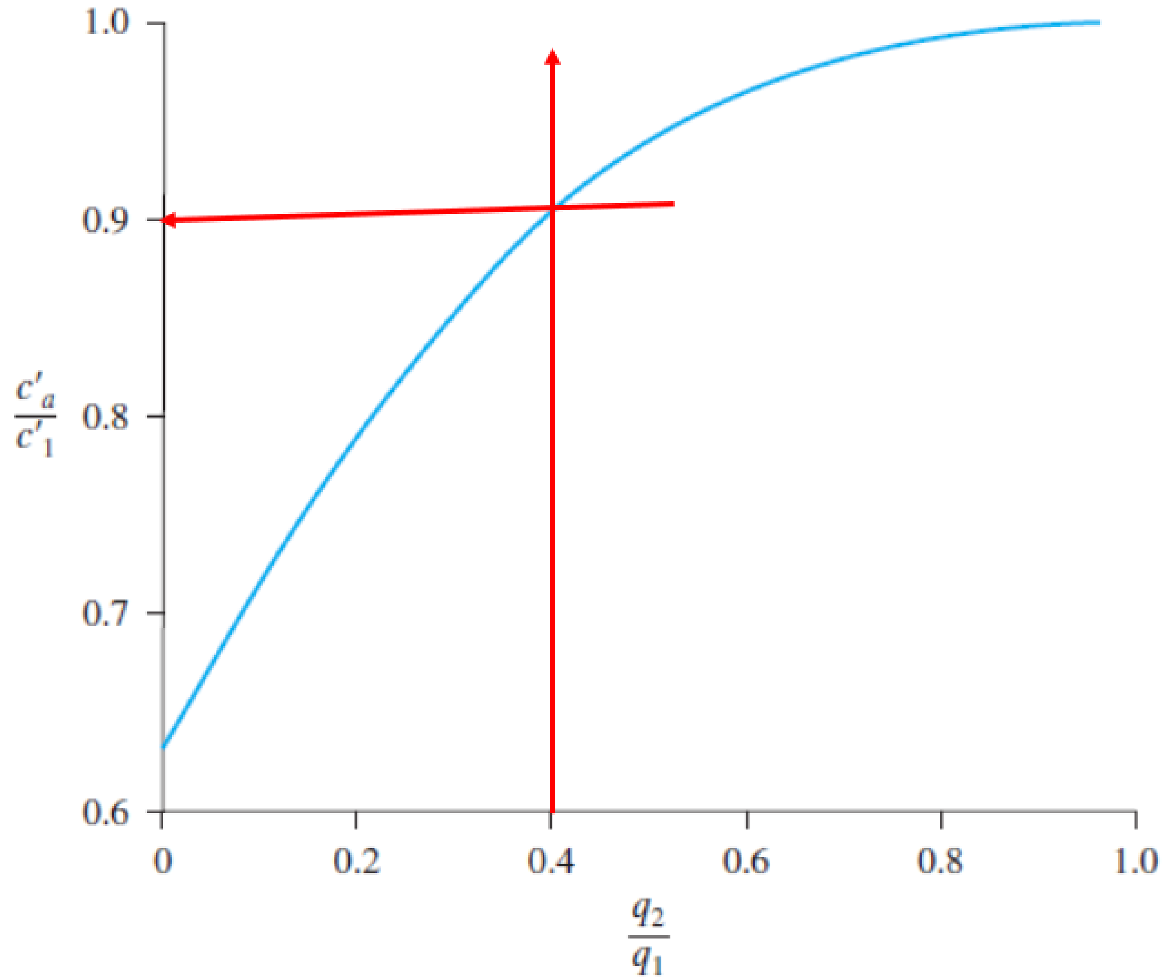
where

$$q_t = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(1)} + \gamma_1 D_f$$

and $c_{u(1)}$ and $c_{u(2)}$ are undrained cohesions. For this case,

$$\frac{q_2}{q_1} = \frac{5.14c_{u(2)}}{5.14c_{u(1)}} = \frac{c_{u(2)}}{c_{u(1)}}$$

EXAMPLE 7.5



Weaker Layer underlain by Stronger Layer (c' - ϕ' soil)

When a foundation is supported by a weaker soil layer underlain by a stronger layer, the ratio of q_2/q_1 will be greater than one.

If H/B is relatively small, the failure surface in soil at ultimate load will pass through both soil layers.

However, for larger H/B ratios, the failure surface will be fully located in the top, weaker soil layer.

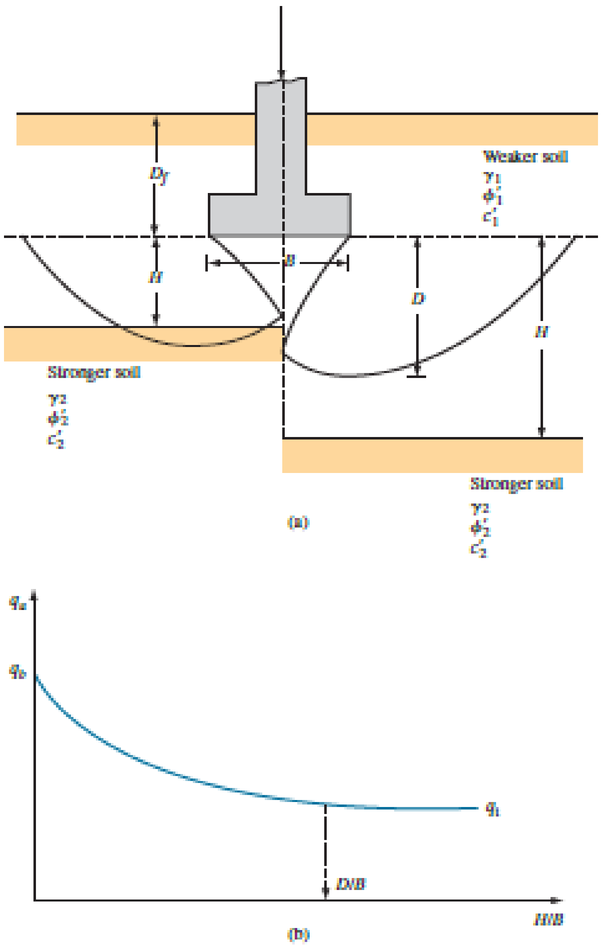


FIGURE 7.12 (a) Foundation on weaker soil layer underlain by stronger sand layer; (b) nature of variation of q_u with H/B

Weaker Layer underlain by Stronger Layer (c'- ϕ' soil)

The ultimate bearing capacity:

$$q_u = q_t + (q_b - q_t)\left(\frac{H}{D}\right)^2 \geq q_t$$

where

D = depth of failure surface beneath the foundation in the thick bed of the upper weaker soil layer

q_t = ultimate bearing capacity in a thick bed of the upper soil layer

q_b = ultimate bearing capacity in a thick bed of the lower soil layer

So

$$q_t = c'_1 N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

and

$$q_b = c'_2 N_{c(2)} F_{cs(2)} + \gamma_2 D_f N_{q(2)} F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}$$

where

$N_{c(1)}, N_{q(1)}, N_{\gamma(1)}$ = bearing capacity factors corresponding to the soil friction angle ϕ_1'

$N_{c(2)}, N_{q(2)}, N_{\gamma(2)}$ = bearing capacity factors corresponding to the soil friction angle ϕ_2'

$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors corresponding to the soil friction angle ϕ_1'

$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors corresponding to the soil friction angle ϕ_2'

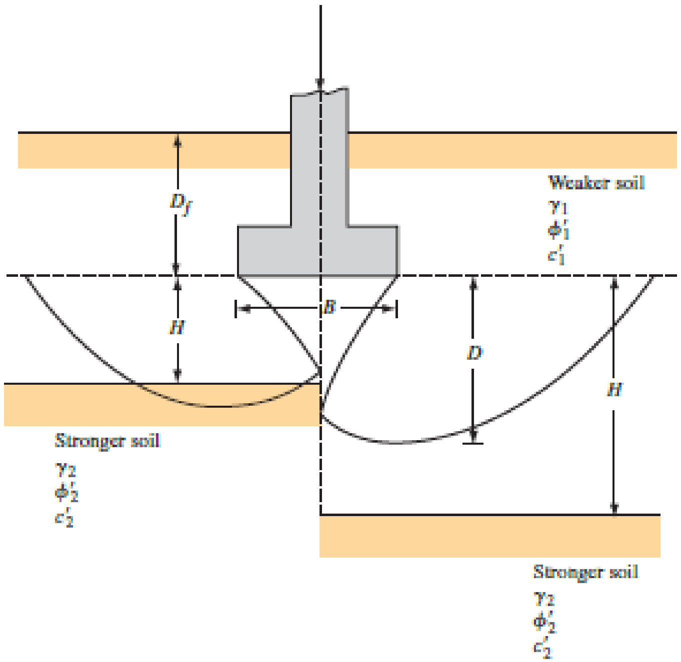
Meyerhof and Hanna (1978) suggested that

- $D \approx B$ for loose sand and clay
- $D \approx 2B$ for dense sand

EXAMPLE 7.6

EXAMPLE 7.6

Refer to Figure 7.12a. For a layered saturated-clay profile, given: $L = 1.83$ m, $B = 1.22$ m, $D_f = 0.91$ m, $H = 0.61$ m, $\gamma_1 = 17.29$ kN/m³, $\phi_1 = 0$, $c_{u(1)} = 57.5$ kN/m², $\gamma_2 = 19.65$ kN/m³, $\phi_2 = 0$, and $c_{u(2)} = 119.79$ kN/m². Determine the ultimate bearing capacity of the foundation.



SOLUTION

From Eqs. (7.18) and (7.19),

$$\frac{q_2}{q_1} = \frac{c_{u(2)}N_c}{c_{u(1)}N_c} = \frac{c_{u(2)}}{c_{u(1)}} = \frac{119.79}{57.5} = 2.08 > 1$$

So, Eq. (7.35) will apply.

From Eqs. (7.36) and (7.37) with $\phi_1 = \phi_2 = 0$,

$$q_t = \left(1 + 0.2\frac{B}{L}\right)N_c c_{u(1)} + \gamma_1 D_f$$

$$= \left[1 + (0.2)\left(\frac{1.22}{1.83}\right)\right](5.14)(57.5) + (0.91)(17.29) = 334.96 + 15.73 = 350.69 \text{ kN/m}^2$$

and

$$q_b = \left(1 + 0.2\frac{B}{L}\right)N_c c_{u(2)} + \gamma_2 D_f$$

$$= \left[1 + (0.2)\left(\frac{1.22}{1.83}\right)\right](5.14)(119.79) + (0.91)(19.65)$$

$$= 697.82 + 17.88 = 715.7 \text{ kN/m}^2$$

From Eq. (7.35),

$$q_u = q_t + (q_b - q_t)\left(\frac{H}{D}\right)^2$$

$$D \approx B$$

$$q_u = 350.69 + (715.7 - 350.69)\left(\frac{0.61}{1.22}\right)^2 \approx 442 \text{ kN/m}^2 > q_t$$

Hence,

$$q_u = \mathbf{442 \ kN/m^2}$$

EXAMPLE 7.7

EXAMPLE 7.7

Solve Example 7.6 using Vesic's theory [Eq. (7.12)]. For the value of m, use Table 7.3.

Refer to Figure 7.12a. For a layered saturated-clay profile, given: $L = 1.83$ m, $B = 1.22$ m, $D_f = 0.91$ m, $H = 0.61$ m, $\gamma_1 = 17.29$ kN/m^3, $\phi_1 = 0$, $c_{u(1)} = 57.5$ kN/m^2, $\gamma_2 = 19.65$ kN/m^3, $\phi_2 = 0$, and $c_{u(2)} = 119.79$ kN/m^2. Determine the ultimate bearing capacity of the foundation.

SOLUTION

From Eq. (7.12),

$$q_u = c_{u(1)} m N_c F_{cs} F_{cd} + q$$

From Table 6.3,

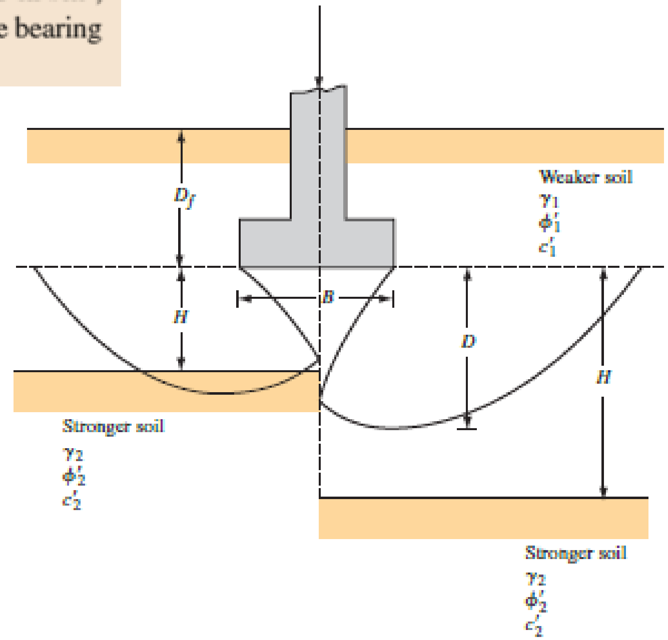
$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1.22}{1.83}\right)\left(\frac{1}{5.14}\right) = 1.13$$

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{0.91}{1.22}\right) = 1.3$$

From Table 7.3, for $c_{u(1)}/c_{u(2)} = 57.5/119.79 = 0.48$ and $H/B = 0.61/1.22 = 0.5$, the value of $m \approx 1$.

Thus,

$$q_u = (57.5)(1)(5.14)(1.13)(1.3) + (17.29 \text{ kN/m}^3)(0.91 \text{ m}) = \mathbf{449.9\ kN/m^2}$$



Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

Stuart (1962)

Assumptions for the failure surface in granular soil under two closely spaced rough continuous foundations

(*Note:* $\alpha_1 = \phi'$, $\alpha_2 = 45 - \phi'/2$, $\alpha_3 = 180 - 2\phi'$)

Case I $x \geq x_1$

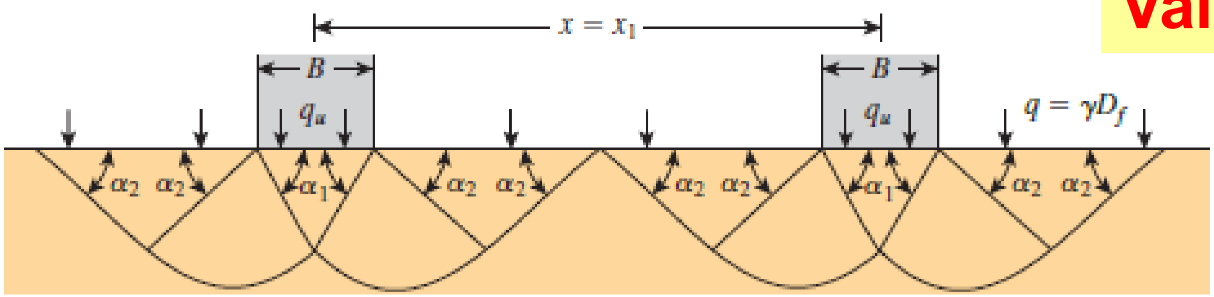
If the center-to-center spacing of the two foundations is $x \geq x_1$, the rupture surface in the soil under each foundation **will not overlap**.

So the ultimate bearing capacity of each continuous foundation can be given by Terzaghi For ($c' = 0$)

$$q_u = qN_q + \frac{1}{2}\gamma B N_\gamma$$

Where N_q, N_γ = Terzaghi's bearing capacity factors (Table 6.1).

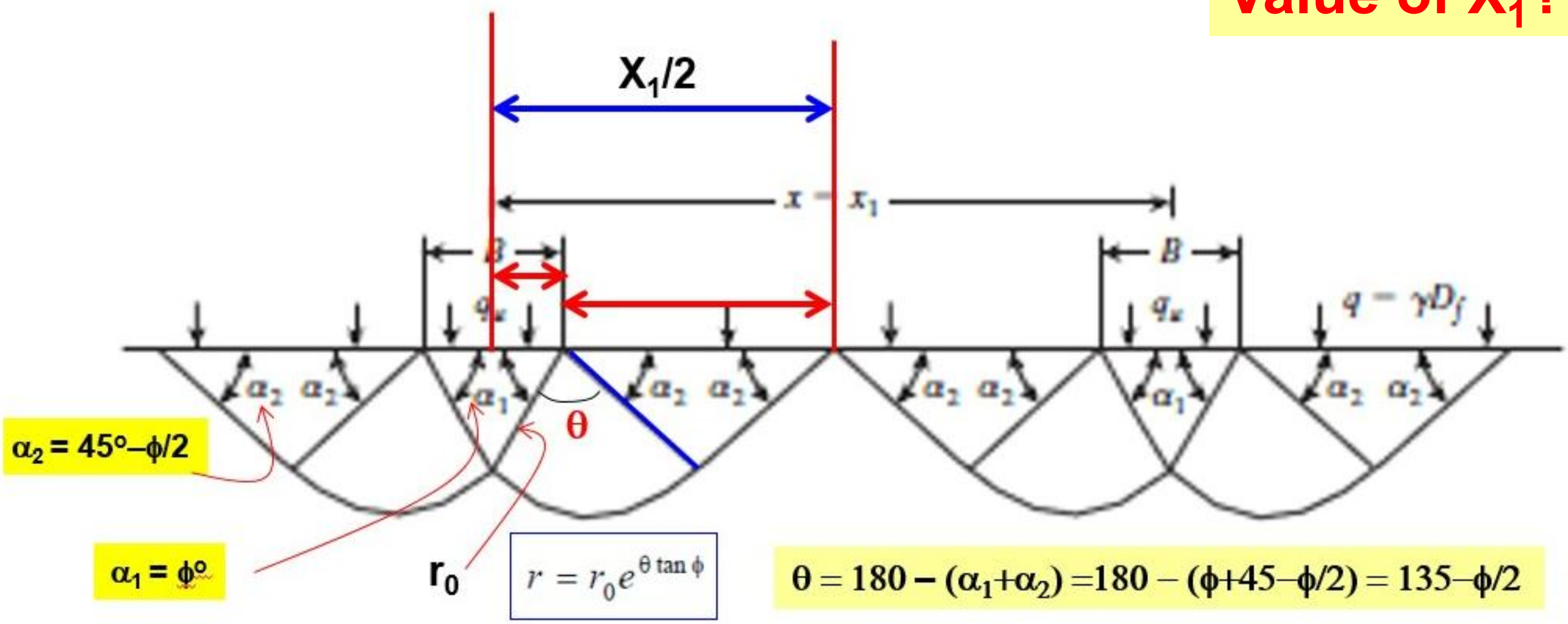
Value of X_1?



Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

From geometry , based on B, α_1, α_2, θ , we can find $x_1/2$

Value of X_1?

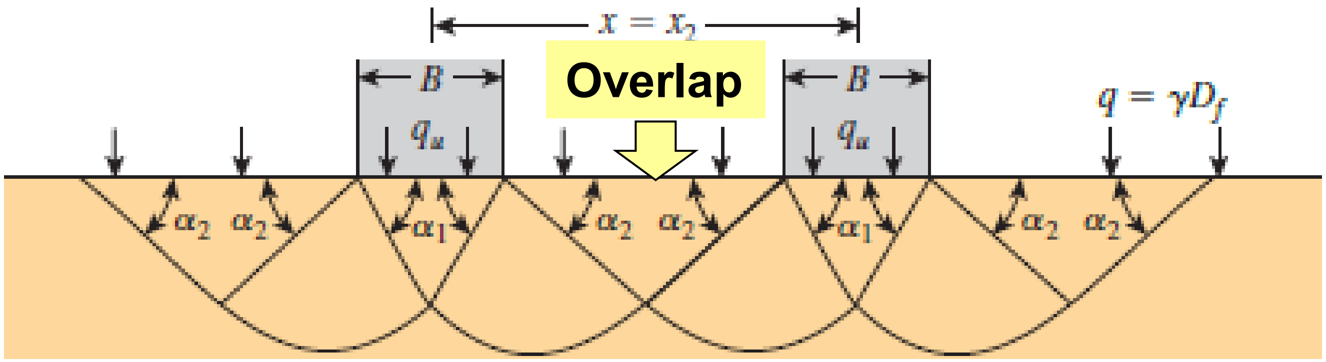


Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

Case II. $(x = x_2 < x_1)$

If the center-to-center spacing of the two foundations $(x = x_2 < x_1)$ are such that the Rankine passive zones **just overlap**, then the magnitude of q_u will still be given by Eq. of **Case I** .However, the foundation **settlement** at ultimate load will change (compared to the case of an **isolated** foundation).

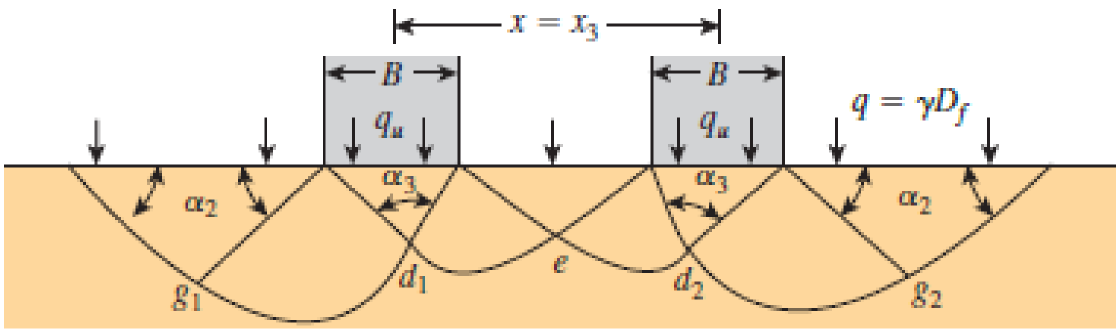
$$q_u = qN_q + \frac{1}{2}\gamma B N_\gamma$$



Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

Case III $x = x_3 < x_2$

- This is the case where the center-to-center spacing of the two continuous foundations $x = x_3 < x_2$.
- Note that the triangular wedges in the soil under the foundations make angles of **180 – 2ϕ'** at points d_1 and d_2.
- The arcs of the logarithmic spirals d_1g_1 and d_1e are tangent to each other at d_1. Similarly, the arcs of the logarithmic spirals d_2g_2 and d_2e are tangent to each other at d_2.



- For this case, the ultimate bearing capacity of **each** foundation can be given as

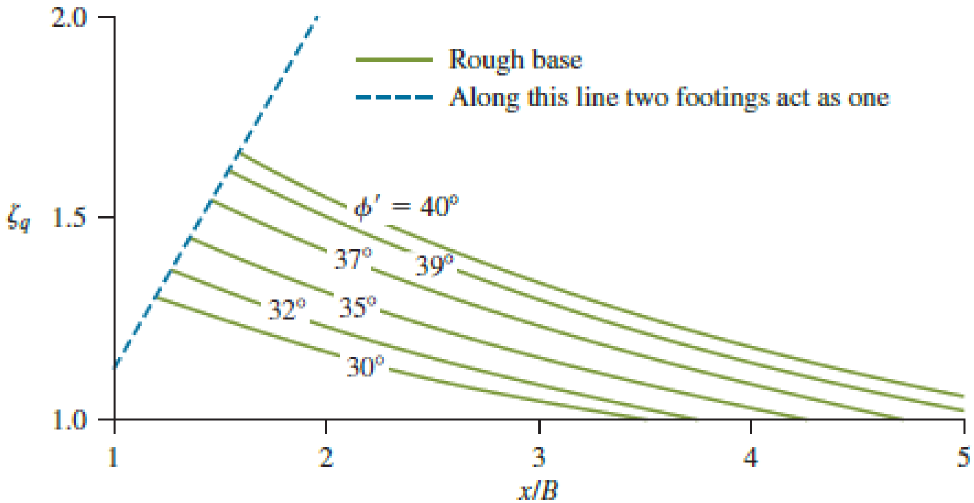
$$q_u = qN_q\zeta_q + \frac{1}{2}\gamma BN_\gamma \zeta_\gamma$$

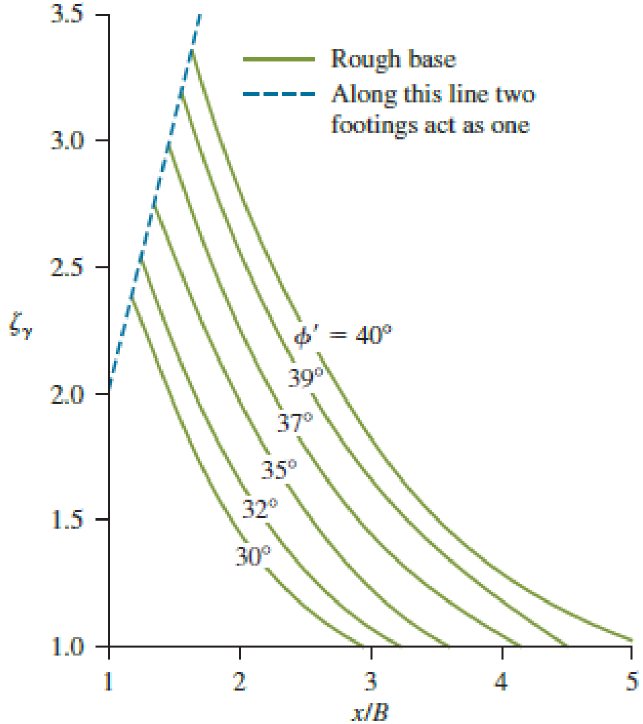
where ζ_q, ζ_γ = efficiency ratios

Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

$$q_u = qN_q\zeta_q + \frac{1}{2}\gamma BN_\gamma\zeta_\gamma$$

where ζ_q, ζ_γ = efficiency ratios





Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

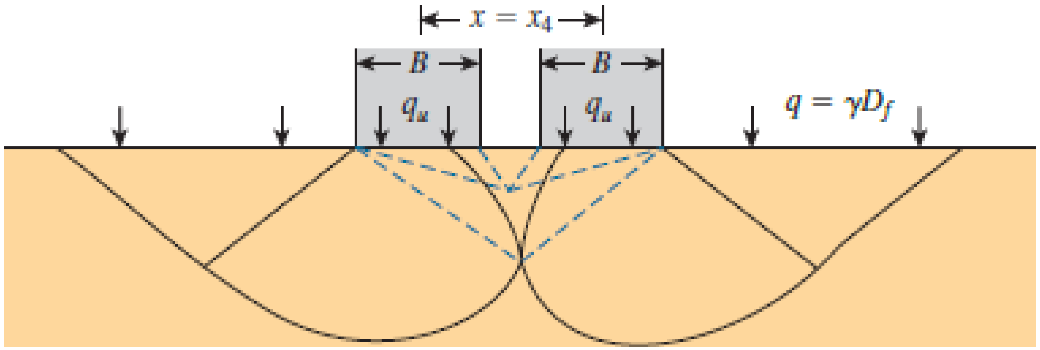
Case IV. $x = x_4 < x_3,$

- If the spacing of the foundation is **further reduced** such tha $x = x_4 < x_3,$ blocking will occur and the pair of foundations will act as a **single** foundation.
- The soil between the individual units will form an **inverted arch** which travels down with the foundation as the load is applied.
- When the two foundations **touch**, the zone of arching disappears, and the system behaves as a single foundation with a width equal to **2*B***.
- The ultimate bearing capacity for this case can be given by Eq. of **Case I**, with ***B*** being replaced by **2*B*** in the second term.

$$q_u = qN_q + \frac{1}{2}\gamma BN_\gamma$$

$$q_u = qN_q + \frac{1}{2}\gamma BN_\gamma$$

2*B*



Bearing Capacity of Foundations on Top of a Slope

Meyerhof (1957) developed the following theoretical relation for the ultimate bearing capacity for *continuous foundations*:

$$q_u = c' N_{cq} + \frac{1}{2} \gamma B N_{\gamma q}$$

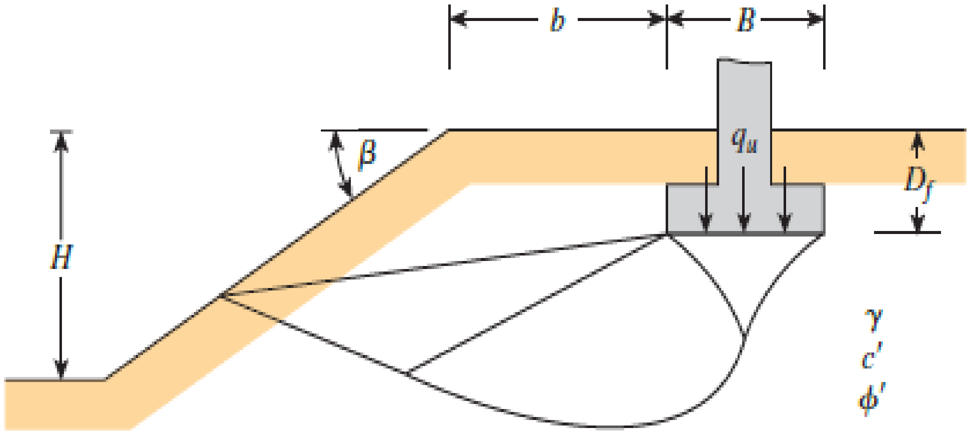
For purely granular soil, $c' = 0$; thus,

$$q_u = \frac{1}{2} \gamma B N_{\gamma q}$$

Again, for purely cohesive soil, $\phi = 0$ (the undrained condition); hence,

$$q_u = c_u N_{cq}$$

where c_u is the undrained cohesion.



Bearing Capacity of Foundations on Top of a Slope

$N_{\gamma q}$

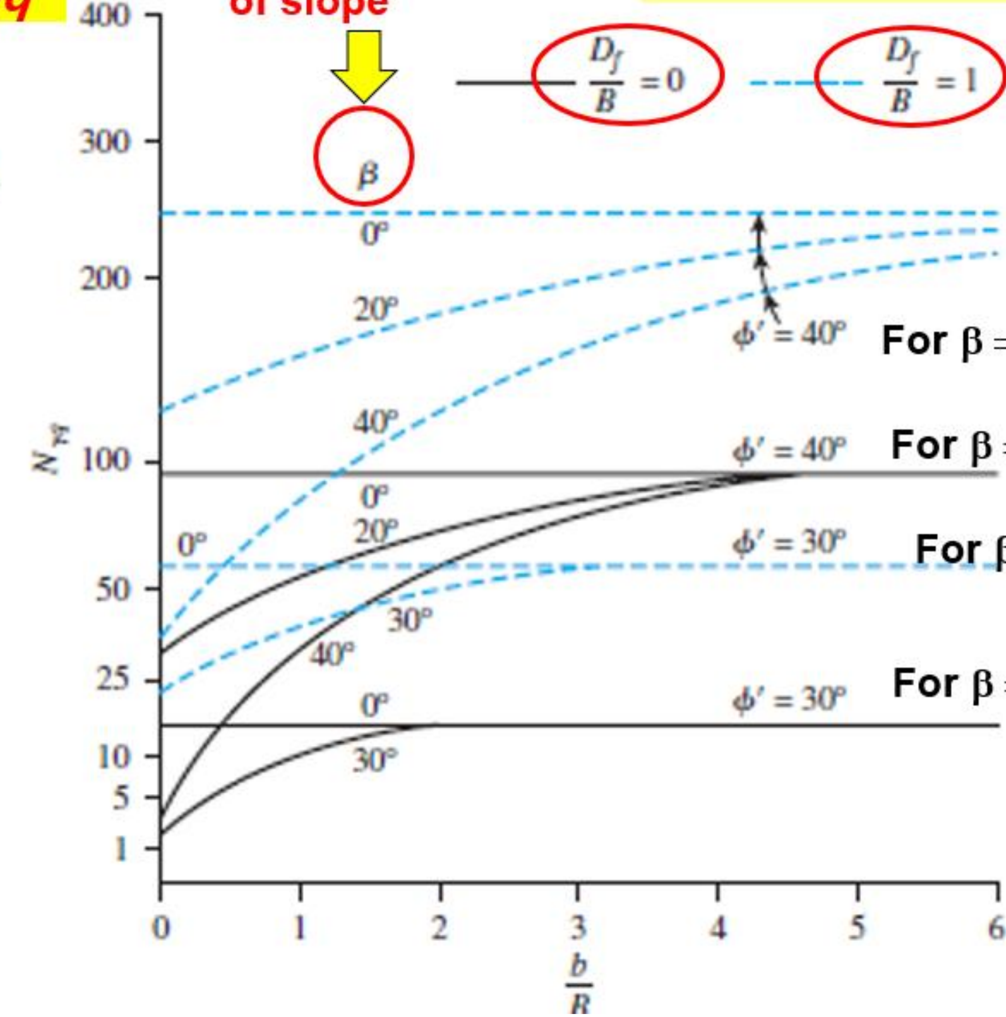
For purely granular soil, $c' = 0$,

$$q_u = \frac{1}{2}\gamma B N_{\gamma q}$$

b, B, D_f, β , ϕ

Inclination of slope

Linear interpolation for intermediate depths



For β = 0, 20, 40

For β = 0, 20, 40

For β = 0, 30

For β = 0, 30

Bearing Capacity of Foundations on Top of a Slope

for purely cohesive soil, $\phi = 0$

$$q_u = c_u N_{cq}$$

The following points need to be kept in mind in determining N_{cq} :

1. The term

$$N_s = \frac{\gamma H}{c_u}$$

is defined as the stability number.

2. If B<H, use the curves for $N_s = 0$.

3. If B>=H, use the curves for the calculated stability number N_s .

b, B, D_f, H, β , N_s

N_{cq}

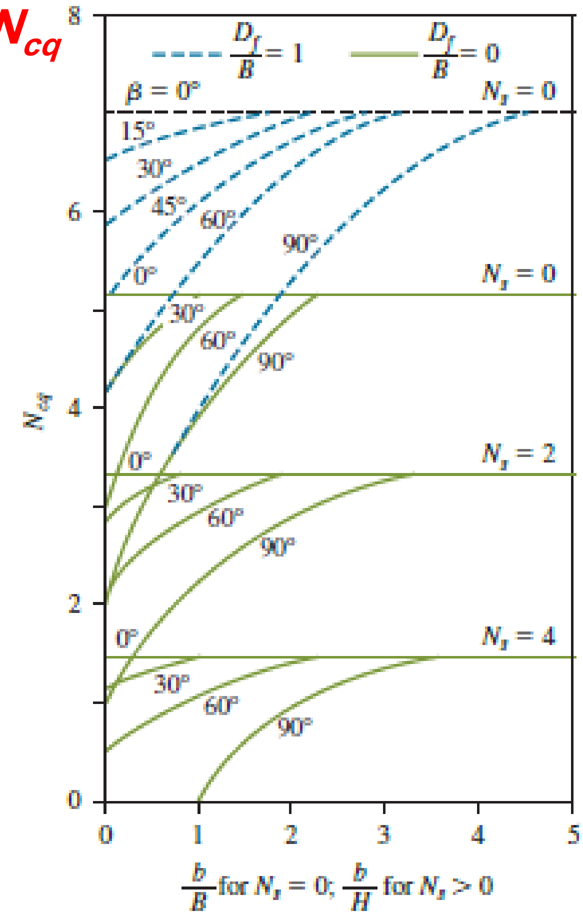


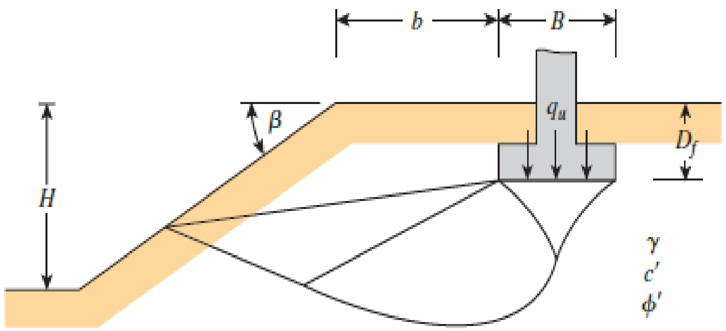
FIGURE 7.21 Meyerhof's bearing capacity factor N_{cq} for purely cohesive soil

EXAMPLE 7.8

EXAMPLE 7.8

In Figure 7.19, for a shallow continuous foundation in a clay, the following data are given: $B = 1.2$ m; $D_f = 1.2$ m; $b = 0.8$ m; $H = 6.2$ m; $\beta = 30°$; unit weight of soil $= 17.5$ kN/m^3; $\phi = 0$; and $c_u = 50$ kN/m^2. Determine the gross allowable bearing capacity with a factor of safety FS = 4.

COHESIVE



SOLUTION

Since $B < H$, we will assume the stability number $N_s = 0$. From Eq. (7.43),

$$q_u = c_u N_{cq}$$

We are given that

$$\frac{D_f}{B} = \frac{1.2}{1.2} = 1$$

and

$$\frac{b}{B} = \frac{0.8}{1.2} = 0.67$$

For $\beta = 30°$, $D_f/B = 1$, and $b/B = 0.67$, Figure 7.21 gives $N_{cq} = 6.3$. Hence,

$$q_u = (50)(6.3) = 315 \text{ kN/m}^2$$

and

$$q_{all} = \frac{q_u}{FS} = \frac{315}{4} = \mathbf{78.8\ kN/m^2}$$

EXAMPLE 7.8

$$q_u = c_u N_{cq}$$

The following points need to be kept in mind in determining N_{cq} :

1. The term

$$N_s = \frac{\gamma H}{c_u}$$

is defined as the stability number.

2. If B<H, use the curves for $N_s = 0$.

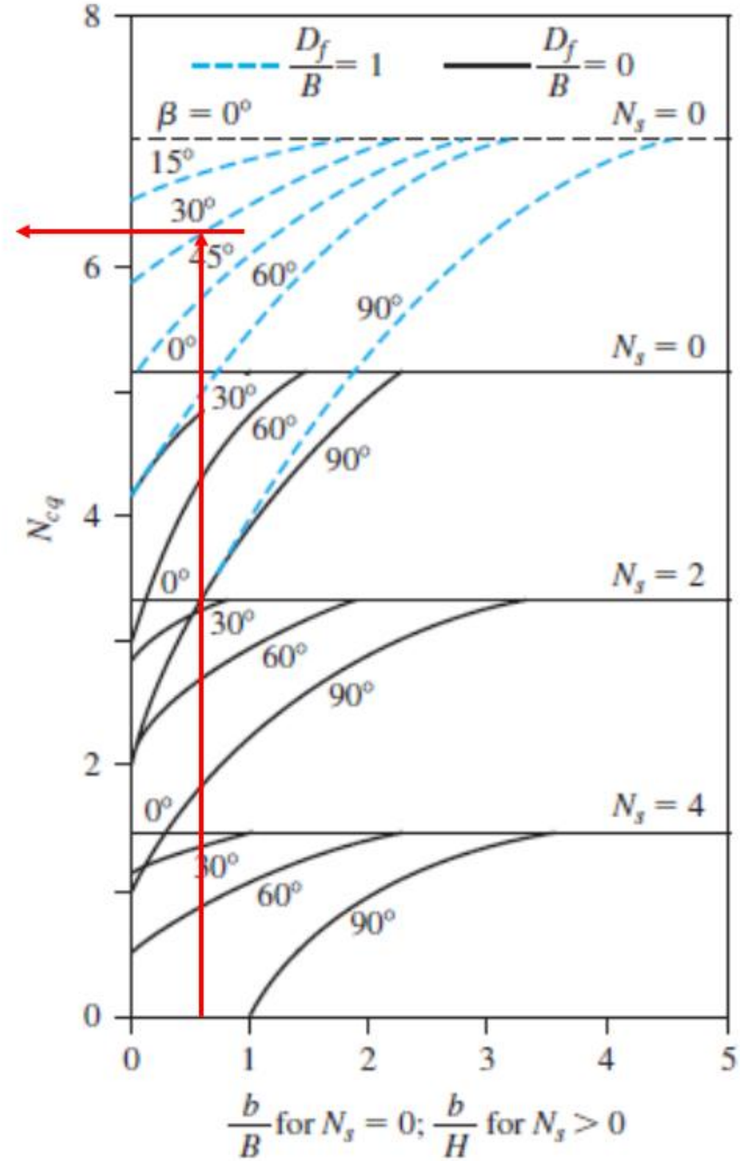
3. If B>=H, use the curves for the calculated stability number N_s .

$\beta = 30°$ $\quad \frac{D_f}{B} = \frac{1.2}{1.2} = 1$ $\quad \frac{b}{B} = \frac{0.8}{1.2} = 0.67$

B = 1.2 m

H = 6.2 m → **B < H** → $N_s = 0$

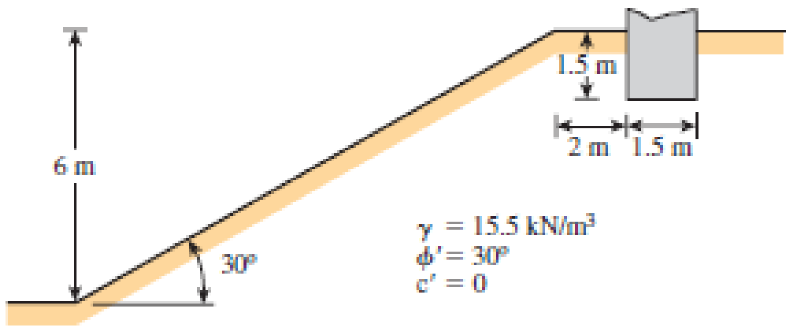
$N_{cq} = 6.3$



EXAMPLE 7.9

EXAMPLE 7.9

Figure 7.22 shows a continuous foundation on a slope of a granular soil. Estimate the ultimate bearing capacity.



GRANULAR

FIGURE 7.22 Foundation on a granular slope

SOLUTION

For granular soil ($c' = 0$), from Eq. (7.42),

$$q_u = \frac{1}{2}\gamma B N_{\gamma q}$$

We are given that $b/B = 2/1.5 = 1.33$, $D_f/B = 1.5/1.5 = 1$, $\phi' = 30°$, and $\beta = 30°$.

From Figure 7.20, $N_{\gamma q} \approx 41$. So,

$$q_u = \frac{1}{2}(15.5)(1.5)(41) = \mathbf{476.6\ kN/m^2}$$

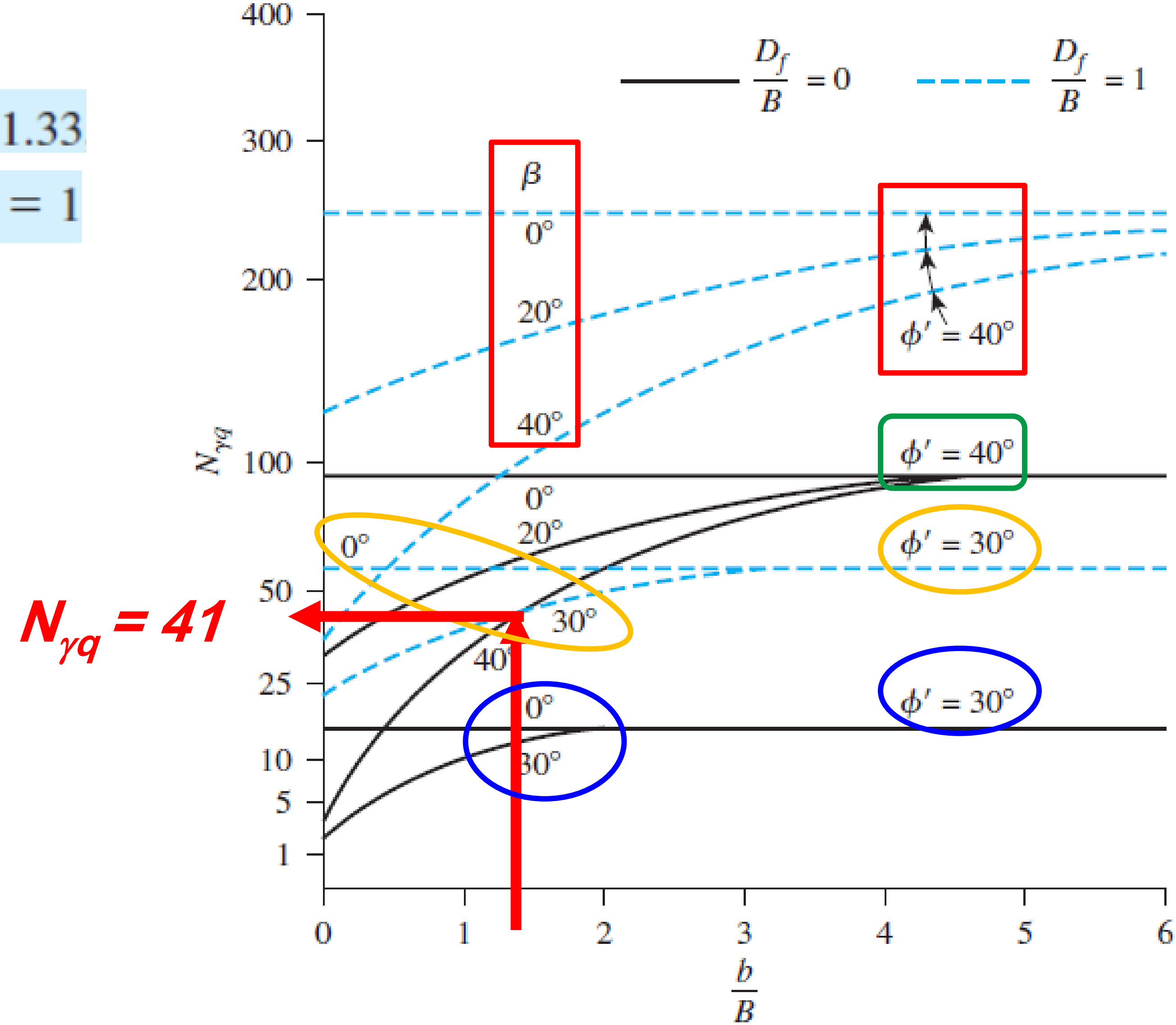
EXAMPLE 7.9

$b/B = 2/1.5 = 1.33$

$D_f/B = 1.5/1.5 = 1$

$\phi' = 30°$

$\beta = 30°$

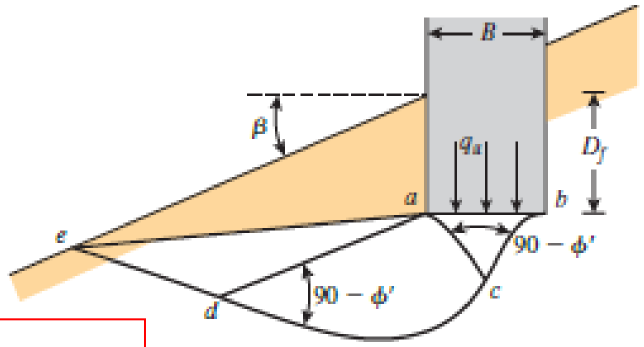


Bearing Capacity of Foundations on a Slope

A rough continuous foundation

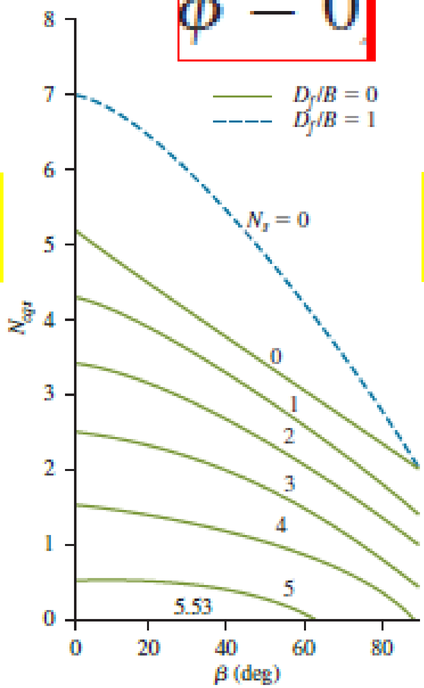
$q_u = c_u N_{cqs}$ (for purely cohesive soil, that is, $\phi = 0$)

$q_u = \frac{1}{2} \gamma B N_{\gamma qs}$ (for granular soil, that is $c' = 0$)



$\phi = 0$

N_{cqs}

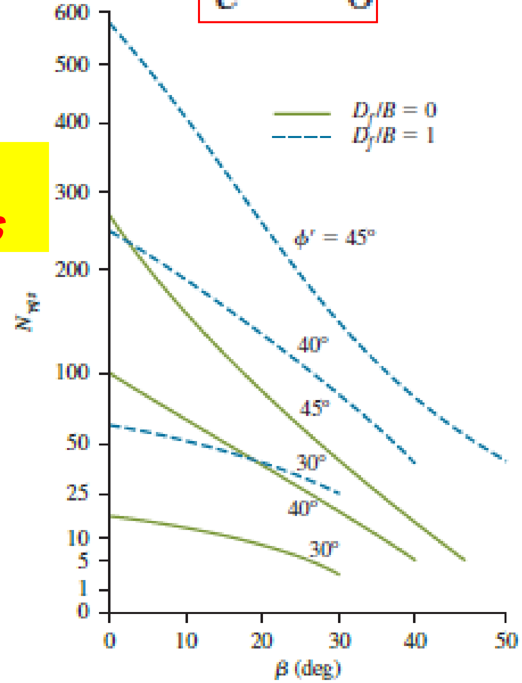


Variation of N_{cqs} with β.

(Note: $N_s = \gamma H/c_u$)

$c' = 0$

$N_{\gamma qs}$



Variation of $N_{\gamma qs}$ with β

Foundations on Rock

$$q_u = c'N_c + qN_q + 0.5\gamma B N_\gamma$$

$$N_c = 5\tan^4\left(45 + \frac{\phi'}{2}\right)$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right)$$

$$N_\gamma = N_q + 1$$

$$q_{uc} = 2c'\tan\left(45 + \frac{\phi'}{2}\right)$$

where

q_{uc} = unconfined compression strength of rock
ϕ' = angle of friction

$$q_{u(\text{modified})} = q_u(\text{RQD})^2$$

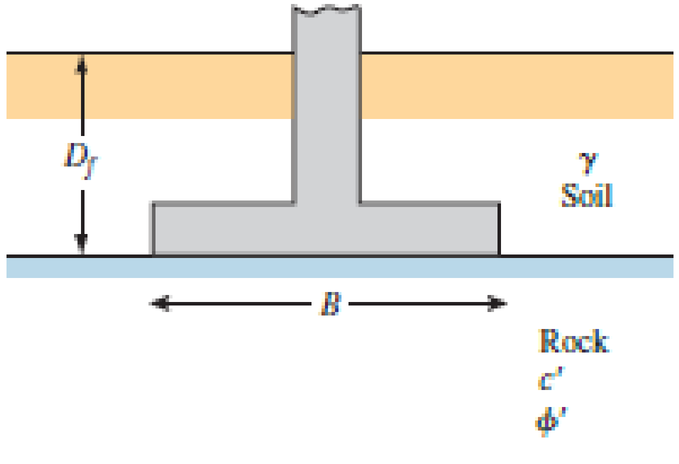


TABLE 7.4 Range of the Unconfined Compression Strength of Various Types of Rocks

Rock type	q_{uc} MN/m²	ϕ' (deg)
Granite	65–250	45–55
Limestone	30–150	35–45
Sandstone	25–130	30–45
Shale	5–40	15–30

EXAMPLE 7.12

EXAMPLE 7.12

Refer to Figure 7.32. A square column foundation is to be constructed over siltstone. Given:

Foundation: $B \times B = 2.5\text{ m} \times 2.5\text{ m}$
$D_f = 2\text{ m}$
Soil: $\gamma = 17\text{ kN/m}^3$
Siltstone: $c' = 32\text{ MN/m}^2$
$\phi' = 31°$
$\gamma = 25\text{ kN/m}^3$
$\text{RQD} = 50\%$

Estimate the allowable load-bearing capacity. Use FS = 4. Also, for concrete, use $f_c' = 30\text{ MN/m}^2$.

SOLUTION

From Eq. (6.19),

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma$$

$$N_c = 5\tan^4\left(45 + \frac{\phi'}{2}\right) = 5\tan^4\left(45 + \frac{31}{2}\right) = 48.8$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right) = \tan^6\left(45 + \frac{31}{2}\right) = 30.5$$

$$N_\gamma = N_q + 1 = 30.5 + 1 = 31.5$$

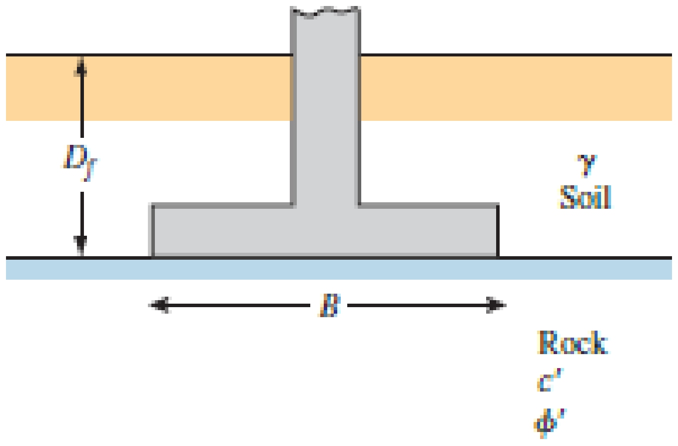
Hence,

$$q_u = (1.3)(32 \times 10^3\text{ kN/m}^2)(48.8) + (17 \times 2)(30.5) + (0.4)(25)(2.5)(31.5)$$
$$= 2030.08 \times 10^3 + 1.037 \times 10^3 + 0.788 \times 10^3$$
$$= 2031.9 \times 10^3\text{ kN/m}^2 \approx 2032\text{ MN/m}^2$$

$$q_{u(\text{modified})} = q_u(\text{RQD})^2 = (2032)(0.5)^2 = 508\text{ MN/m}^2$$ 1

$$q_{\text{all}} = \frac{508}{4} = 127\text{ MN/m}^2$$ 2

Since 127 MN/m^2 is greater than f_c', use $q_{\text{all}} = \mathbf{30\text{ MN/m}^2}$. 3

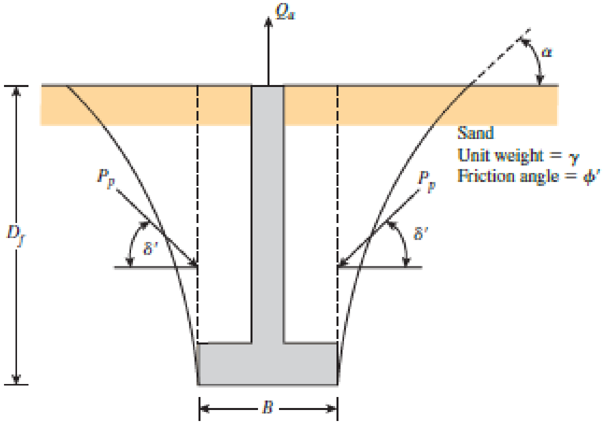


Uplift Capacity of Foundations

Foundations (such as transmission tower foundations) may be subjected to uplift forces under special circumstances.

The intersection of the failure surface at the ground level will make an angle α with the horizontal.

The magnitude of α will vary with the D_r in the case of sand and with the consistency in the case of clay soils.



Shallow continuous foundation subjected to uplift

Uplift Capacity of Foundations

Shallow and Deep Foundations Under Uplift

- When the failure surface in soil extends up to the ground surface at ultimate load, it is defined as a shallow foundation under uplift.

- For larger values of D_f/B, failure takes place around the foundation and the failure surface does not extend to the ground surface. These are called deep foundations under uplift.

Critical Embedment Ratio

- The embedment ratio, D_f/B, at which a foundation changes from shallow to deep condition is referred to as the critical embedment ratio, $(D_f/B)_{cr}$.

- In sand the magnitude of $(D_f/B)_{cr}$ can vary from 3 to about 11 and, in saturated clay, it can vary from 3 to about 7.

Uplift Capacity of Foundations

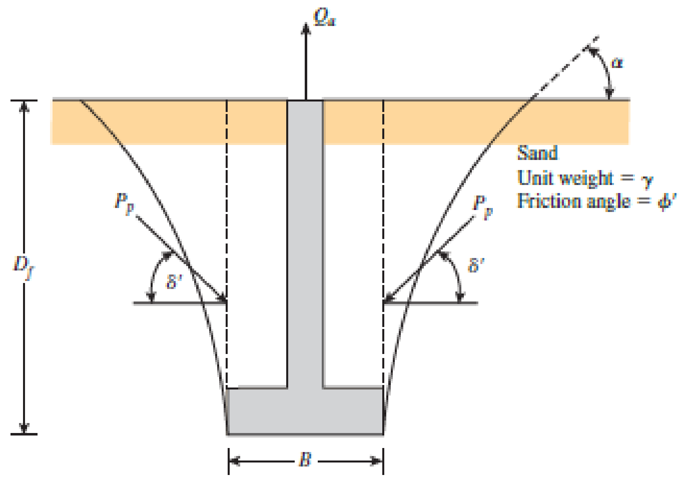
Foundations in Granular Soil ($c' = 0$)

The ultimate load can be expressed as

$$Q_u = F_q A \gamma D_f$$

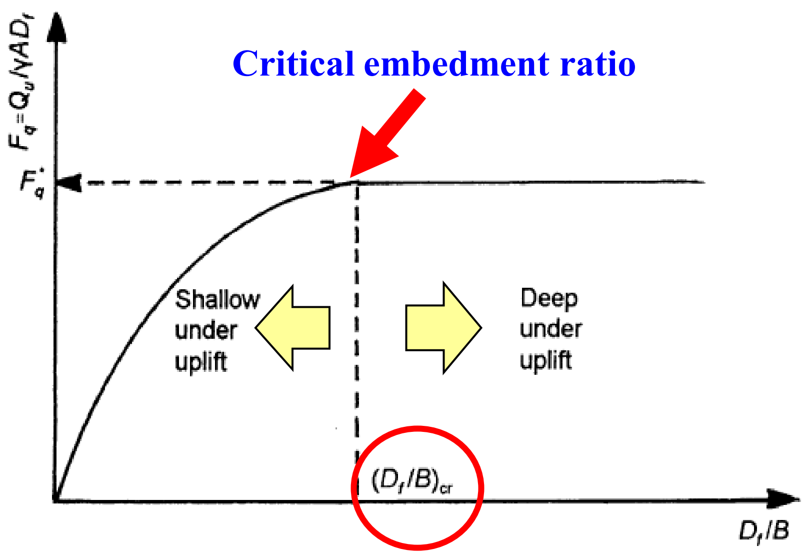
where A = area of the foundation.

F_q = breakout factor



Shallow continuous foundation subjected to uplift

The breakout factor increases with D_f/B up to a maximum value of $F_q = F_q^*$ at $D_f/B = (D_f/B)_{cr}$. For $D_f/B > (D_f/B)_{cr}$ the breakout factor remains practically constant (that is, F_q^*).



Uplift Capacity of Foundations

Failure Conditions

$D_f/B \leq (D_f/B)_{cr}$ → **Shallow foundation condition**

$D_f/B > (D_f/B)_{cr}$, → **Deep foundation condition**

TABLE 7.5 Variation of K_u, m, and $(D_f/B)_{cr}$

Soil friction angle, ϕ' (deg)	K_u	m	$(D_f/B)_{cr}$ for square and circular foundations
20	0.856	0.05	2.5
25	0.888	0.10	3
30	0.920	0.15	4
35	0.936	0.25	5
40	0.960	0.35	7
45	0.960	0.50	9

Rectangular

$$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}} = \left(\frac{D_f}{B}\right)_{\text{cr-square}}\left[0.133\left(\frac{L}{B}\right) + 0.867\right] \leq 1.4\left(\frac{D_f}{B}\right)_{\text{cr-square}}$$

Uplift Capacity of Foundations

The Breakout Factor

$D_f/B \le (D_f/B)_{cr}$ → **Shallow**

$$F_q = 1 + 2\left[1 + m\left(\frac{D_f}{B}\right)\right]\left(\frac{D_f}{B}\right)K_u \tan\phi' \qquad (7.56)$$

(for shallow circular and square foundations)

$$F_q = 1 + \left\{\left[1 + 2m\left(\frac{D_f}{B}\right)\right]\left(\frac{B}{L}\right) + 1\right\}\left(\frac{D_f}{B}\right)K_u \tan\phi' \qquad (7.57)$$

(for shallow rectangular foundations)

where

m = a coefficient which is a function of ϕ'
K_u = nominal uplift coefficient

TABLE 7.5 Variation of K_u, m, and $(D_f/B)_{cr}$

Soil friction angle, ϕ' (deg)	K_u	m	$(D_f/B)_{cr}$ for square and circular foundations
20	0.856	0.05	2.5
25	0.888	0.10	3
30	0.920	0.15	4
35	0.936	0.25	5
40	0.960	0.35	7
45	0.960	0.50	9

$D_f/B > (D_f/B)_{cr}$, → **Deep**

- Use Eqs. 7.56 and 7.57 only use $(D_f/B)_{cr}$ in place of D_f/B

Uplift Capacity of Foundations

Step-by-step procedure to estimate the uplift capacity of foundations in granular soil

Step 1. Determine D_f, B, L, and ϕ'.

Step 2. Calculate D_f/B.

Step 3. Using Table 7.5 and Eq. (7.61), calculate $(D_f/B)_{cr}$.

Step 4. If D_f/B is less than or equal to $(D_f/B)_{cr}$, it is a shallow foundation.

Step 5. If $D_f/B > (D_f/B)_{cr}$, it is a deep foundation.

Step 6. For shallow foundations, use D_f/B calculated in Step 2 in Eq. (7.59) or (7.60) to estimate F_q. Thus, $Q_u = F_q A \gamma D_f$.

Step 7. For deep foundations, substitute $(D_f/B)_{cr}$ for D_f/B in Eq. (7.59) or (7.60) to obtain F_q, from which the ultimate load Q_u may be obtained.

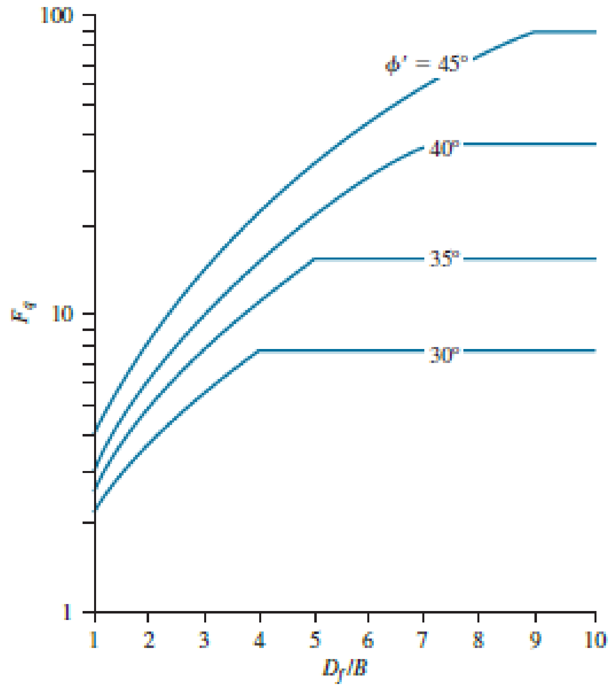


FIGURE 7.38 Variation of F_q with D_f/B and ϕ'

The variations of F_q for square and circular foundations.

Uplift Capacity of Foundations

Foundations in Cohesive Soil ($\phi = 0$, $c = cu$)

$$Q_u = A(\gamma D_f + c_u F_c)$$

A = area of the foundation

c_u = undrained shear strength of soil

F_c = breakout factor

As in the case of foundations in granular soil, the breakout factor F_c increases with embedment ratio and reaches a maximum value of $F_c = F_c^*$ at $D_f/B = (D_f/B)_{cr}$ and remains constant thereafter.

Das (1978) also reported some model test results with square and rectangular foundations. Based on these test results, it was proposed that

$$\left(\frac{D_f}{B}\right)_{\text{cr-square}} = 0.107c_u + 2.5 \le 7 \qquad (7.63)$$

where

$\left(\frac{D_f}{B}\right)_{\text{cr-square}}$ = critical embedment ratio of square (or circular) foundations

c_u = undrained cohesion, in kN/m^2

It was also observed by Das (1980) that

$$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}} = \left(\frac{D_f}{B}\right)_{\text{cr-square}}\left[0.73 + 0.27\left(\frac{L}{B}\right)\right] \le 1.55\left(\frac{D_f}{B}\right)_{\text{cr-square}} \qquad (7.64)$$

where

$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}}$ = critical embedment ratio of rectangular foundations

L = length of foundation

Uplift Capacity of Foundations

Cohesive Soil

Based on these findings, Das (1980) proposed an empirical procedure to obtain the breakout factors for shallow and deep foundations. According to this procedure, α' and β' are two nondimensional factors defined as

$$\alpha' = \frac{\frac{D_f}{B}}{\left(\frac{D_f}{B}\right)_{cr}} \tag{7.65}$$

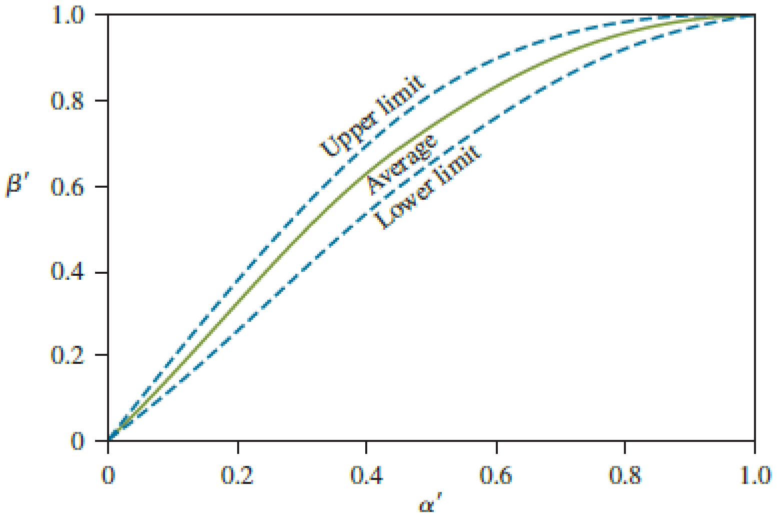
and

$$\beta' = \frac{F_c}{F_c^*} \tag{7.66}$$

For a given foundation, the critical embedment ratio can be calculated using Eqs. (7.63) and (7.64). The magnitude of F_c^* can be given by the following empirical relationship:

$$F^*_{c\text{-rectangular}} = 7.56 + 1.44\left(\frac{B}{L}\right) \tag{7.67}$$

where $F^*_{c\text{-rectangular}}$ = breakout factor for deep rectangular foundations.



Uplift Capacity of Foundations

Step-by-step procedure to estimate the uplift capacity of foundations in Cohesive Soil

Step 1. Determine the representative value of the undrained cohesion, c_u.

Step 2. Determine the critical embedment ratio using Eqs. (7.63) and (7.64).

Step 3. Determine the D_f/B ratio for the foundation.

Step 4. If $D_f/B > (D_f/B)_{cr}$, as determined in Step 2, it is a deep foundation. However, if $D_f/B \leq (D_f/B)_{cr}$, it is a shallow foundation.

Step 5. For $D_f/B > (D_f/B)_{cr}$,

$$F_c = F_c^* = 7.56 + 1.44\left(\frac{B}{L}\right)$$

Thus,

$$Q_u = A\left\{\left[7.56 + 1.44\left(\frac{B}{L}\right)\right]c_u + \gamma D_f\right\} \tag{7.68}$$

where A = area of the foundation.

Step 6. For $D_f/B \leq (D_f/B)_{cr}$,

$$Q_u = A(\beta' F_c^* c_u + \gamma D_f) = A\left\{\beta'\left[7.56 + 1.44\left(\frac{B}{L}\right)\right]c_u + \gamma D_f\right\} \tag{7.69}$$

EXAMPLE 7.13

Granular Soil

EXAMPLE 7.13

Consider a circular foundation in sand. Given for the foundation: diameter, $B = 1.5$ m and depth of embedment, $D_f = 1.5$ m. Given for the sand: unit weight, $\gamma = 17.4$ kN/m^3, and friction angle, $\phi' = 35°$. Calculate the ultimate bearing capacity.

ultimate uplift capacity

SOLUTION

$D_f/B = 1.5/1.5 = 1$ and $\phi' = 35°$. For circular foundation, $(D_f/B)_{cr} = 5$. Hence, it is a shallow foundation. From Eq. (7.59),

$$F_q = 1 + 2\left[1 + m\left(\frac{D_f}{B}\right)\right]\left(\frac{D_f}{B}\right)K_u \tan\phi'$$

For $\phi' = 35°$, $m = 0.25$, and $K_u = 0.936$ (Table 7.5). So

$$F_q = 1 + 2[1 + (0.25)(1)](1)(0.936)(\tan 35) = 2.638$$

$$Q_u = F_q \gamma A D_f = (2.638)(17.4)\left[\left(\frac{\pi}{4}\right)(1.5)^2\right](1.5) = \mathbf{121.7\ kN}$$

EXAMPLE 7.14

Cohesive Soil

EXAMPLE 7.14

A rectangular foundation in a saturated clay measures 1.5 m × 3 m. Given: $D_f = 1.8$ m, $c_u = 52$ kN/m^2, and $\gamma = 18.9$ kN/m^3. Estimate the ultimate uplift capacity.

SOLUTION

From Eq. (7.63),

$$\left(\frac{D_f}{B}\right)_{\text{cr-square}} = 0.107c_u + 2.5 = (0.107)(52) + 2.5 = 8.06$$

So use $(D_f/B)_{\text{cr-square}} = 7$. Again from Eq. (7.64),

$$\left(\frac{D_f}{B}\right)_{\text{cr-rectangular}} = \left(\frac{D_f}{B}\right)_{\text{cr-square}}\left[0.73 + 0.27\left(\frac{L}{B}\right)\right]$$

$$= 7\left[0.73 + 0.27\left(\frac{3}{1.5}\right)\right] = 8.89$$

Check: $1.55\left(\frac{D_f}{B}\right)_{\text{cr-square}} = (1.55)(7) = 10.85$

EXAMPLE 7.14

So use $(D_f/B)_{cr\text{-rectangular}} = 8.89$. The actual embedment ratio is $D_f/B = 1.8/1.5 = 1.2$. Hence, this is a shallow foundation.

$$\alpha' = \frac{\dfrac{D_f}{B}}{\left(\dfrac{D_f}{B}\right)_{cr}} = \frac{1.2}{8.89} = 0.135$$

Referring to the average curve of Figure 7.39, for $\alpha' = 0.135$, the magnitude of $\beta' = 0.2$. From Eq. (7.69),

$$Q_u = A\left\{\beta'\left[7.56 + 1.44\left(\frac{B}{L}\right)\right]c_u + \gamma D_f\right\}$$

$$= (1.5)(3)\left\{(0.2)\left[7.56 + 1.44\left(\frac{1.5}{3}\right)\right](52) + (18.9)(1.8)\right\} = \mathbf{540.6\ kN}$$

THE END