PHYSICS 501
3rd HOMEWORK-FALL 2019
Dr. V. Lempesis

1. In the field of atom cooling when we consider the interaction of atoms with a laser beam we find the concept of *scattering* or *dissipative* force. This force is given by:

$$\mathbf{F} = \frac{\hbar p}{\left(1+p\right)} \vec{\nabla}\Theta$$

where Θ is the phase of the laser electric field, $\hbar$ is the reduced Planck's constant ($h/2\pi$), while p is the so called *saturation parameter.* In the case where the beam is a plane wave $\Theta = kz$ (where k is the wave number equal to $2\pi/\lambda$, where λ is the laser wave length) and p is a constant. Calculate the quantity $\vec{\nabla} \cdot \mathbf{F}$.

(5 marks)

Solution

$$\vec{\nabla} \cdot \mathbf{F} = \vec{\nabla} \cdot \left[\frac{\hbar p}{\left(1+p\right)} \vec{\nabla}\Theta \right] = \frac{\hbar p}{\left(1+p\right)} \vec{\nabla} \cdot \vec{\nabla}\Theta = \frac{\hbar p}{\left(1+p\right)} \vec{\nabla} \cdot k = 0$$

2. Find an equation in spherical coordinates of the sphere: $x^2 + y^2 + (z-1)^2 = 1$.

(5 marks)

Solution

We know that in spherical coordinates we have:

$$x = r\sin\theta\cos\varphi,\ \ y = r\sin\theta\sin\varphi,\ \ z = r\cos\theta$$

Thus we have

$$x^2 + y^2 + (z-1)^2 = 1 \Rightarrow \left(r\sin\theta\cos\varphi\right)^2 + \left(r\sin\theta\sin\varphi\right)^2 + (r\cos\theta - 1)^2 = 1$$

$$r^2\sin^2\theta\cos^2\varphi + r^2\sin^2\theta\sin^2\varphi + r^2\cos^2\theta + 1 - 2r\cos\theta = 1$$

$$r^2\sin^2\theta\left(\cos^2\varphi + \sin^2\varphi\right) + r^2\cos^2\theta - 2r\cos\theta = 0$$

$$r\sin^2\theta\left(\cos^2\varphi + \sin^2\varphi\right) + r\cos^2\theta - 2\cos\theta = 0$$

$$r\sin^2\theta + r\cos^2\theta - 2\cos\theta = 0$$

$$r - 2\cos\theta = 0$$

$$r = 2\cos\theta$$

3. Find the work done going around a unit circle in the xy-plane, going clockwise from 0 to $-\pi$, *against* a force field given by:

$$\mathbf{F} = -\mathbf{i}\frac{y}{x^2+y^2} + \mathbf{j}\frac{x}{x^2+y^2}.$$

Use cylindrical coordinates in your calculations. Solution based on Cartesians is not going to be accepted.

(5 marks)

Solution:

The answer can be given using cylindrical coordinates

$$\mathbf{F}\cdot d\mathbf{l} = -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = -ydx + xdy$$

But (with ρ=1)

$$x = \rho\cos\varphi = \cos\varphi, \quad dx = -\sin\varphi d\varphi, \quad y = \rho\sin\varphi = \sin\varphi, \quad dy = \cos\varphi d\varphi$$

$$\mathbf{F}\cdot d\mathbf{l} = -ydx + xdy = \sin^2\varphi d\varphi + \cos^2\varphi d\varphi = d\varphi$$

$$W = -\int_0^{-\pi}\mathbf{F}\cdot d\mathbf{l} = -\int_0^{-\pi} d\phi = \pi$$

Vasileios Lembessis 2/11/2019 10:03
Comment [1]: You need the minus sign here because you are asked to find the work done against the force!

4. Calculate the work you do in going **along a straight line** from point (1, 1) to point (3, 3). The force you exert is given by

$$\mathbf{F} = \mathbf{i}(x-y) + \mathbf{j}(x+y).$$

(5 marks)

Solution

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Comment [2]: A lot of you calculated the integral taking other paths as well. But I asked you "**along a straight line**"!!!! Be careful when you read a question.

The line joining the two points has the equation $y = x$ (so dy=dx along it). Thus

$$\int_{(1,1)}^{(3,3)}\mathbf{F}\cdot d\mathbf{l} = \int_{(1,1)}^{(3,3)} F_x dx + F_y dy = \int_{(1,1)}^{(3,3)} (x-y)dx + (x+y)dy \underset{x=y}{=} \int_{(1,1)}^{(3,3)} (x+x)dx = 2\left[\frac{x^2}{2}\right]_1^3 = (3^2 - 1^2) = 8$$

For the girls: Please send your answers in pdf form (typed or in clearly handwritten form) in my email address (vlempesis@ksu.edu.sa). Please use ONE

file for your entire homework NOT one file per page. Please do not forget to put your name and your ID number on it AND on your file name.
Your deadline is on Wednesday 7th November 2018 at 23:59.

For the boys: You will hand in your homework in hard copy in my office on Thursday 8th November up to 11:59.