**Math 382 2nd Semester 2016-2017**

**Mid-Exam 2 4/5/2017**

**Duration: 90 Minutes**

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**Question 1:**

1. State the definition of continuous function at a point c. [Definition 6.1]
2. State the definition of a right hand limit of a function f at a point c. [Definition 5.2]
3. Give an example of two functions one of which is not continuous at c but the composition is continuous at c.

**Question 2:**

1. Let $f:D\rightarrow R$ and $c\in \hat{D}$. Prove that if for any sequence $(x\_{n})$ in $D$, $x\_{n}\ne c$ and $x\_{n}\rightarrow c$ satisfy $f(x\_{n)}\rightarrow l$, **then** $\lim\_{x\to c}f(x)=l.$ [Theorem 5.1].
2. Let $f:R\rightarrow R$ satisfy the relation $f\left(x+y\right)=f\left(x\right)+f(y)$ for all $x,y. $If $\lim\_{x\to c}f(x)=l$for some $c\in R.$ **Prove that f has a limit at every point in** $R.$[ Exercise 9 in section 5.2]

**Question 3:**

1. Prove that$ \lim\_{x\to 0}sin(x)=0. $[Example 5.8]
2. Let $f:D\rightarrow R, c$ an isolated point in $D.$ **Show that** $f$ **is continuous at** $c.$ [Note after Definition 6.1]
3. Prove that if a function $f$ is continuous at a point $c,$ then so is the function $\sqrt{f(x)}$. [Example 6.11].

**Question 4:**

1. Find the limit if it exists. Write all the details
2. $\lim\_{x\to \infty }cos(πx).$
3. $\lim\_{x\to 0}f\left(x\right),$ where $f\left(x\right)=\left\{\begin{array}{c}x, x\in Q\\-x, x\notin Q \end{array}\right.$. [exercise 5 in section 5.1]
4. Explain why we cannot find two functions $f,g:D\rightarrow R$ for which $f $is continuous at $c$ but $g$ is not continuous at $c.$ **But the sum** $f+g $**is continuous at**$ c$**.**
5. Explain why we cannot find two functions both are continuous at c, but the **composition is not continuous at c.**