**Math 382 2nd Semester 2016-2017**

**Mid-Exam 1 29/3/2017**

**Duration: 90 Minutes**

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**Question 1:**

1. (2 points) Use the principal of mathematical inductions to prove that Prove by induction that
2. (1.5 point) Determine the solution set of
3. (2 points) Let S and T be nonempty subsets of the R with the property that s < t for all s S and t T. Prove that . Is it possible for to be nonempty?

**Question 2:**

1. (3 point) State the following:
2. Definition of natural numbers.
3. The Bolzano-Weierstrass theorem.
4. Definition of a convergent sequence.
5. (0.5 points each) Give an example of :
6. A monotone sequence that is not convergent.
7. A convergent sequence that is not monotone.
8. A set that has no cluster points.
9. An isolated point.
10. A sequence that has no a convergent subsequence.

**Question 3:**

1. (1.5 points) Prove that every Cauchy sequence is bounded.
2. (1.5 points) If whate is the relation between and ( prove!)
3. (2 points) Prove that for any there exest a sequence of irrational numbers that converge to

**Question 4:**

(4 points) Let be a Cauchy sequence. Show directly using the definition that the

sequence is also a Cauchy sequence. Carefully justify all of the steps. You may use

the result that a Cauchy sequence is bounded.