**Math 382 2nd Semester 2016-2017**

**Mid-Exam 1 29/3/2017**

**Duration: 90 Minutes**

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**Question 1:**

1. (2 points) Use the principal of mathematical inductions to prove that Prove by induction that

$$ 1 + 3 + 5 + 7 + ... +\left(2n-1\right)= n^{2},∀ n\in N.$$

1. (1.5 point) Determine the solution set of $\left|x\right|+\left|x-1\right|<3.$
2. (2 points) Let S and T be nonempty subsets of the R with the property that s < t for all s $\in $ S and t $\in $ T. Prove that $sup⁡( S) \leq inf⁡( T)$. Is it possible for $S ∩ T$ to be nonempty?

**Question 2:**

1. (3 point) State the following:
2. Definition of natural numbers.
3. The Bolzano-Weierstrass theorem.
4. Definition of a convergent sequence.
5. (0.5 points each) Give an example of :
6. A monotone sequence that is not convergent.
7. A convergent sequence that is not monotone.
8. A set that has no cluster points.
9. An isolated point.
10. A sequence that has no a convergent subsequence.

**Question 3:**

1. (1.5 points) Prove that every Cauchy sequence is bounded.
2. (1.5 points) If $A⊆B$ whate is the relation between $\hat{A}$ and $\hat{B}. $( prove!)
3. (2 points) Prove that for any $x\in R$ there exest a sequence $\left(x\_{n}\right)$ of irrational numbers that converge to $x.$

**Question 4:**

(4 points) Let $\left(x\_{n}\right)$ be a Cauchy sequence. Show directly using the definition that the

sequence $\left(x\_{n}^{2}\right)$ is also a Cauchy sequence. Carefully justify all of the steps. You may use

the result that a Cauchy sequence is bounded.