Math 382
Mid-Exam 1

1st Semester 2021
Duration: 90 Minutes

Name:

Question	1	2	3	total
Mark				
Full mark	7	7	6	20

Question 1:

i) (2 points) Prove that if $x > -1$ then $(1+x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$. (Exercise 2.3.5)

ii) (2 points) Let $A = [a, b)$ find $\sup A$, $\inf A$. Verify your answer. (Example 2.9)

iii) (1 point) Let B be bounded sets and $\emptyset \neq A \subseteq B$. Prove that A is bounded and that $\sup A \leq \sup B$. (Exercise 2.4.3)

iv) (2 points) Let $(x_n), (y_n)$ be two convergent sequences. Prove that $\lim x_n . y_n = \lim x_n . \lim y_n$. (Theorem 3.4).

Question 2:

i) (3 points) Prove the limit of the following sequences, and **use the definition** of convergence to prove:

a. $\lim\left(\frac{1}{n}+\frac{n}{n+1}\right)=1$

b. $\lim\left(\frac{2^n+1}{2^n}\right)=1$

ii) (2 points) Prove the squeezing theorem:
If $(x_n), (y_n), (z_n)$ are sequences of real numbers such that $x_n \leq y_n \leq z_n$ for all n, and that $\lim x_n = \lim z_n = l$. Then (y_n) is convergent with limit l. (Theorem 3.6)

iii) (2 points) Prove that if $x_n \geq 0$, $\forall n \quad and \lim x_n = x$. Then $\lim \sqrt{x_n} = \sqrt{x}$. (Theorem 3.9)

<u>Question 3:</u> Prove the following:

i) (1 points) The rational numbers $\mathbb{Q}$ is not a neighborhood of 0.

ii) (1 points) If $(x_n)$is a bounded sequence, then $\lim \frac{x_n}{n} = 0$.

iii) (1 points) It is not true that if the sequence $|x_n|$ is convergent, then (x_n) is convergent.

iv) (3 points) Given $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$ for all $n \geq 1$, prove that the sequence (x_n) is convergent and **calculate its limit**. (Example 3.11)