Math 382 **1st Semester 2017-2018**
Mid-Exam 1 **22/10/2017**
Duration: 90 Minutes

Question	1	2	3	4	total
Mark					
Full mark	4	6	4	8	22

Question 1:

i) (2 points) Use the principal of mathematical inductions to prove that $2^{n-1} \leq n!, \forall\, n \in \mathbb{N}$.

ii) (1 point) Prove that $|x + y| \leq |x| + |y|$.

iii) (1 point) Show that $\max\{a, b\} = \frac{1}{2}(a + b + |a + b|)$.

Question 2:

i) (4 point) Let A, B be two bounded nonempty subsets of $\mathbb{R}$, and let

$A + B = \{a + b: a \in A \text{ and } b \in B\}$ (جمع مجموعات)

$AB = \{ab: a \in A \text{ and } b \in B\}$ ضرب مجموعات

and $kA = \{ka: a \in A\}$ for any $k \in \mathbb{R}$.ضرب مجموعه في عدد ثابت

1) Prove that $\sup(A + B) = Sup(A) + \sup(B)$.
2) Prove that $\inf(A + B) = inf(A) + \inf(B)$
3) Prove that $\sup(AB) \neq Sup(A)\sup(B)$.

ii) (2 points) Let A, B be as above, and let $A \subseteq B$. Show $Sup(A) \leq \sup(B)$.

Question 3:

i) (2 points) Let $A = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$. Find

a. $Sup(A)$ if it exists.
b. $Inf\ (A)$ if it exists.
c. $\max(A)$ if it exists.
d. $\min(A)$ if it exists.

ii) (1 point) Show that the set $A = \{x : x = \frac{p}{3}, p \in \mathbb{Z}\}$ is an inductive set.

iii) (1 points) The rational numbers $\mathbb{Q}$ is not a neighborhood of 0.

Question 4:

i) (2 points) If $(x_n)$is a bounded sequence, then $\lim \frac{x_n}{n} = 0$.

ii)

a. (2 points) If $x_n \to x$ then $|x_n| \to |x|$.

b. (1 points) It is not true that if the sequence $|x_n|$ is convergent, then (x_n) is convergent.

iii) (3 points) let $0 < a < 1$. Show that the sequence (a^n) converges to 0. (Example 3.7)