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| KSU logo tiff.tif |  **King Saud University**  |
|  **College of Sciences** |
|  **Department of Mathematics** |
|  **373 Math** |
|  **Second Midterm** |
|  **First semester 1433-1434** |

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**Question 1:**

1. Give a definition of a base for a topological space.
2. Prove that $B=\left\{\left(a,b\right)⊆R:a<b and a,b\in Q\right\}$ is a base for $R$ with usual topology.

**Question 2:** Let $\left(X\_{1},τ\_{1}\right),…,\left(X\_{n},τ\_{n}\right) $be topological spaces, and let $B=\left\{U\_{1}×…×U\_{n}: U\_{i}\in τ\_{i}, 1\leq i\leq n\right\}$. Prove that $B$ is a base for a topology on the set $X=\prod\_{i=1}^{n}X\_{i}$. What is call the topology generated by $B$.

**Question 3:** Prove that the function $ρ:R^{n}×R^{n}\rightarrow R$, $n\geq 1$, defined by $ρ\left(x,y\right)=\max\_{1\leq i\leq n}\left\{\left|x\_{i}-y\_{i}\right|\right\}$ is a metric on $R^{n}$, where $x=\left(x\_{1},…,x\_{n}\right)$ and $y=\left(y\_{1},…,y\_{n}\right)$.

**Question 4:**

1. Prove that if $\left(X,d\right)$ is metric space and $B(x,ϵ)$ is open ball center at $x$ with radius $ϵ$ and $z\in B(x,ϵ)$, then there is $δ>0$ such that $B(z,δ)⊆B(x,ϵ)$.
2. What does we mean by the metrizability problem? Is every topological space metrizable?

**Question 5:** Prove that any infinite set with Co-finite topology is not Hausdorff.

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