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| KSU logo tiff.tif |  **King Saud University**  |  |
|  **College of Sciences** |  |
|  **Department of Mathematics** |  |
|  **373 Math** |  |
|  **First Midterm** |  |
|  **Second Semester 1433-1434** |  |

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**Question 1:** Let $X=N$. Let $τ$ be the collection of subsets of $X$ consisting of $∅$ and all subsets of $X$ with the form $U\_{n}=\left\{n,n+1,n+2,…\right\}$, $n\in N$.

1. Prove that $τ$ is a topology on $X$.
2. List the closed subsets of $X$.
3. Let $A=\left\{5,7,50\right\}$, find $\overline{A}$, $Int(A)$, and $Bd(A)$. **JUSTEFY YOUR ANSWER**

**Question 2:**

1. Give a definition of a base for a topological space.
2. Prove that $B=\left\{\left\{x\right\}:x\in R\right\}$ is a base for a topology on $R$. Describe the topology.

**Question 3:** Let $(X,τ)$ be a topological space and let $\left\{U\_{i}:1\leq i\leq n\right\}$ be a finite collection of closed subsets of $X$. Prove that $\bigcup\_{i=1}^{n}U\_{i}$ is closed subset of $X$. Is the union of an infinite number of closed sets closed set. **JUSTEFY YOUR ANSWER**

**Question 4:** Let $A$ and $B$ be subsets of a topological space. Prove the following:

1. $\left(A∪B\right)^{'}=A'∪B'$
2. If $Bd\left(A\right)=∅$, then $A$ is open and closed set.