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| KSU logo tiff.tif | **King Saud University** |
| **College of Sciences** |
| **Department of Mathematics** |
| **373 Math** |
| **Final Exam** |
| **Second Semester 1433-1434** |

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**Question 1:**

1. Let be a metric space. Show that the function defined by is a metric on . Show that and induce the same topology.
2. Let be a metric space. Prove that if is a compact subset of , then is closed and bounded. Give an example to show the converse of the statement does not hold.
3. What do we mean by the metrizability problem? Is every topological space metrizable? (Justify your answer)

**Question 2:**

1. Let be a metric space. Prove that the set is closed, where and . (This set is called the closed ball with center and radius
2. Let be the usual topology on , . Prove that is a metrizable.

**Question 3:**

1. Prove that in a Hausdorff space any convergent sequence has a unique limit. Give an example to show the converse of the statement does not hold.
2. Let and be sequences in the spaces and , respectively. Prove that the sequence converges to if and only if converges to and converges to .

**Question 4:**

1. Define a compact space.
2. Prove that with Co-finite topology is compact, but with usual topology is not compact.
3. Prove that any closed set of a compact space is compact.

**Question 5:**

1. Prove that if is a continuous function from a compact space into , then attains its maximal and its minimal.
2. Prove that if is a continuous bijection function from a compact space onto a Hausdorff space , then is a homeomorphism.

**Question 6:**

1. Let be a metrizable space. Prove that is limit point compact space if and only if is sequentially compact.
2. If is not a metrizable space, then prove that the statement in I is not true.

**Bonus:**

Let be a metric on . Prove that for any

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