373 Math Chapter 8

From Text book:

8.1: 1, 3, 5, 6, 7, 10, 11.
8.2: 5, 9, 10, 11, 12.
8.3: 2, 3, 4, 5, 8, 9, 14, 15, 16.
8.4: 2(b, c, e, h), 4, 5, 6, 8.

Additional Exercises:

Definition: Let $A \neq \phi$, $B \neq \phi$ be two subset of the metric space (X, d). Then the distance between A and B is given by

$$d(A, B) = inf\{d(x, y) : x \in A, y \in B\}.$$

If $A = a$, we write $d(a, B)$ for $d(A, B)$

Q1. In $\mathbb{R}^2$, take $A = \{(x, y) : 0 \leq x \leq 1, 0 < x < 1\}, B = \{(x, y) : 2 < x < 3, 0 < y < 1\}$,$C = \{(0, 0)\}$, $D = \left\{(\frac{1}{2}, \frac{1}{2})\right\}$. Find the following:

$$d(A, B), d(A, C), d(C, B), d(D, A), d(D, B).$$

Q2. Let (X, d) be a metric space and A a nonempty subset of X. If $x, y \in X$, prove that $d(x, A) \leq d(x, y) + d(y, A)$.

Q3. Give an example to show that for an open ball $B_r(y)$in a metric space (X,d), it is not true that $Bd(B_r(y)) = \{x \in X : d(x, y) = r\}$.

Q4. Let (X,d) be a metric space and define $e(x, y) = {}^{d(x,y)}/_{(1+d(x,y))}$. Prove that e is a bounded metric for X.