**373 Math Problems**

**Sheet #3**

**From Text book:**

2.5:1,2,3,4,5,9.10.

**Additional Problems:**

1. Find a base for the open half-line topology which is different from the topology itself.
2. Let $(X,τ)$ be a topological space and $B$ a base for $τ$. Prove that $A⊆X$ is dense in X iff each nonempty element of $B$ contains a point of *A*.

**Definition:**

Two collections $B\_{1} and B\_{2}$ of subsets of *X* are equivalent bases iff there exists a topology $τ $for *X* such that $B\_{1} and B\_{2}$ are both bases for $τ.$

1. Let $(X,τ)$ be a topological space, $B\_{1}$a base for $τ$, and $ B\_{2}$ a collection of subsets of X Prove that $B\_{1} and B\_{2}$ are equivalent bases iff
2. For each $U\_{1}\in B\_{1} and x\in U\_{1}, there is a U\_{2}\in B\_{2} such that x\in U\_{2}⊆U\_{1}$ and
3. For each $U\_{2}\in B\_{2} and x\in U\_{2}, there is a U\_{1}\in B\_{1} such that x\in U\_{1}⊆U\_{2}$.
4. Let $B\_{1} and B\_{2} $be a collections of subsets of X both satisfying (a) and (b) of Prob.(3) above. Prove that $B\_{1} and B\_{2}$ are equivalent bases iff $τ\left(B\_{1}\right)=τ\left(B\_{2}\right).$