373 Math Problems

From Text book:

2.1: 1, 4, 5, 6, 7.

2.2: 1,2 , 4, 5, 6, 7, 8, 9.

Additional Problems:

1. List all topologies for a set containing three distinct elements.
2. Prove that for a non empty set X, the collection $τ=\left\{X,Φ\right\}∪\{U:X-U is countable\}$ is a topology on X, this topology is called co-countable topology.
3. Is there a set in which discrete and indiscrete topologies coincide on it?
4. Give an example of a nontrivial topology on an infinite set X which has only a finite number of elements.
5. If $τ\_{1} and τ\_{2} $are two topologies on X, is $τ\_{1}∩τ\_{2}$ a topology on X? Is $τ\_{1}∪τ\_{2}$ a topology on X?
6. Prove that $τ$ is the discrete topology on X iff every point in X is an open set.
7. Let $X=N$. For each $n\in N$ define $U\_{n}=\{n,n+1,n+2,…….\}$ .Let $τ=\left\{X,ϕ\right\}∪\{U\_{n}:n\in N\}$. Prove that $τ$ is a topology on X.