**<u>Question 1</u>.** [4+3]

**a.** Let $(X, \mathfrak{J})$ be a topological space with $A \subseteq X$ and let $\mathcal{B}$ be a base for $\mathfrak{J}$ . Show that the collection $\mathcal{B}^* = \{A \cap B : B \in \mathcal{B}\}$ is a base for $\mathfrak{J}_A$.

**b.** Let $X = \{a, b, c, d\}$ with the topology $\mathfrak{J} = \{X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$. If $A = \{a, b, c\}$ and $B = \{b, c\}$

**i.** Find $\mathfrak{J}_A$.

**ii.** Find $Int_A(B)$ and $Cl_A(B)$.

**<u>Question 2.</u>** [2+2]

**a.** Show that every constant function between topological spaces is continuous.

**b.** Let $f: \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1, & x \geq 0 \\ x, & x < 0 \end{cases}.$$

Is $f$ $\mathcal{U} - \mathcal{H}$ continuous? Is $f$ $\mathcal{U} - \mathcal{H}$ open? Justify your answer.

**Question 3.** [3+1]

**a.** Show that being Hausdorff is hereditary.

**b.** Let $X = \{a, b, c, d\}$ with topology $\mathcal{T} = \{X, \emptyset, \{a, c\}, \{b, d\}\}$ and $Y = \{x, y, z, w\}$ with topology $\mathcal{T}' = \{Y, \emptyset, \{x\}, \{x, y\}\}$. Is $(X, \mathcal{T}) \cong (Y, \mathcal{T}')$? Justify your answer.

**<u>Question 4.</u>** [3+2]

**a**. Let $X = \mathbb{R}$ with the usual topology $\mathcal{U}$ and $Y = \mathbb{R}$ with the left ray topology $\mathcal{L}$.

1. Find a base for the product topology on $X \times Y$.

**2.** Let $A = (0,1)$ and $B = (0, \infty)$. Find $\overline{A \times B}$ and $(A \times B)'$

**Question 5.** [2+3]

**a**. Show that $(\mathbb{R}, \mathcal{T}_{cof})$ is not Hausdorrf.

**b.** 1. Show that $(0,1)$ is open in $([0,3], \mathcal{U}_{[0,3]})$.

2. Show that $[1,2]$ is closed in $((0,3), \mathcal{H}_{(0,3)})$.