

Student's Name	Student's ID	Group No.

Question No.	I	II	III	IV	Total
Mark					

[I] Determine whether the following is **True** or **False**. **Justify** your answer. [4 Points]

1. If  $k$  is a positive integer, then the convergence of the sequence  $p_n = \frac{1}{n^k}$  to zero is **linear**. (            )

2.  $x = 1$  is a **simple** root of  $g(x) = x^3e^x - 2x^2e^x + xe^x$ . (            )

3. If  $f[3, 5, 6] = 7$  and  $f[5, 6] = 1$  are given divided differences for a function  $f$ , then  $f[3, 5] = 22$ . (            )

**[II]** Let  $f(x) = x^5 - x^2 + x + 1$ . [4 Points]

- (a) **Use** two iterations of the Modified Newton Method to **approximate** the root of  $f$  on  $[-1, 1]$ .
- (b) **How** accurate is the approximation in (a)? **Justify** your answer.

**[III]** If  $f(0.25) = 1.648$ ,  $f(0.5) = 2.718$ , and  $f(0.75) = 4.481$ , [5 Points]

- (i) **use** an appropriate Lagrange polynomial of degree two to **approximate**  $f(0.43)$ .
- (ii) Given that  $f(x) = e^{2x}$ , **use** the error formula to **find** an upper bound for the error in (i).
- (iii) **Compare** the bound in (ii) with the actual relative error.

[V] Use the data in the following table for all parts of this question. [7 Points]

x	0	0.2	0.4	0.6
f(x)	1	1.22	1.49	1.82

- (i) **Approximate**  $f(0.5)$  by the Newton Forward-Difference formula with 3 points.
- (ii) **Approximate**  $f''(0.4)$  by a second derivative midpoint formula.
- (iii) **Approximate**  $f'(0.2)$  by a 3-point formula.
- (iv) **Estimate** the error in (iii) if you know that  $f(x) = e^x \cos x$ .