

Student's Name	Student's ID	Group No.

Question No.	I	II	III	IV	Total
Mark					

[I] Determine whether the following is **True** or **False**. **Justify** your answer. [4 Points]

1. If  $p^*$  approximates  $\pi$  to 3 significant digits, then  $p^* \in [0.995\pi, 1.005\pi]$ . (            )

2. If  $x = \frac{6}{7}$  and  $y = \frac{2}{3}$  then the 4-digit rounding value of  $x \ominus y$  is 0.1905. (            )

3. If the growth of error for an algorithm is linear, then this algorithm is stable. (            )

4.  $g(x) = x^3 - 4x^2$  has two fixed-points in  $[-1, 5]$ . (            )

**[II]** Let  $\alpha_n = \frac{n+4}{(2n+1)^3}$  and  $F(h) = \frac{1-e^h}{h}$ . [4 Points]

- (a) **Determine** the rate of convergence for  $\alpha_n$ .
- (b) **Determine** the rate of convergence for  $F(h)$ .

**[III]** Let  $f(x) = x^5 - 2x - 1$ . [5 Points]

- (a) **Show** that  $f$  has a root in  $[1, 1.5]$ .
- (b) Use the Bisection method to **find the root** of  $f$  on  $[1, 1.5]$  accurate to within  $10^{-2}$ .
- (c) **Estimate** the relative error in (b).

**[IV]** [7 Points]

A. Let  $f(x) = x^3 - x^2 - 3$ .

- (i) If  $f(p) = 0$ , **show** that each of the functions  $g_1(x) = \sqrt[3]{3 + x^2}$  and  $g_2(x) = \frac{3}{x^2} + 1$  has a fixed point at  $p$ .
- (ii) **Which** of the functions  $g_1$  and  $g_2$  in (i) is better for the fixed-point iterations on  $[1, 2]$ ? **Justify** your answer.

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B. Use an appropriate fixed-point iteration to **find the root** of  $\pi + \frac{1}{2} \sin\left(\frac{x}{2}\right) - x$  on  $[0, 2\pi]$  with accuracy  $10^{-3}$ .

Good Luck