

Probability and Random Variables:

$$* P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A^c) = 1 - P(A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A)P(B|A)$$

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$$

* For the partition $\{A_1, A_2, \dots, A_k\}$:

$$P(B) = \sum_{i=1}^k P(B|A_i)P(A_i)$$

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}, \quad n! = n(n-1)\dots(1)$$

$$* F(x) = P(X \leq x) = \begin{cases} \sum_{t \leq x} f(t) \\ \int_{-\infty}^x f(t) dt \end{cases}$$

$$* E(x) = \mu_X = \begin{cases} \sum_{\text{all } x} x f(x) \\ \int_{-\infty}^{\infty} x f(x) dx \end{cases}$$

$$\text{Var}(x) = \sigma_X^2 = E[(X - \mu)^2]$$

$$= \begin{cases} \sum_{\text{all } x} (X - \mu)^2 f(x) \\ \int_{-\infty}^{\infty} (X - \mu)^2 f(x) dx \end{cases}$$

$$\text{Var}(x) = \sigma_x^2 = E(X^2) - \mu^2$$

* Thebychev inequality:

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

Probability Distributions:

$$* f(x) = P(X = x) = \frac{1}{k}, \quad x = x_1, x_2, \dots, x_k$$

$$E(X) = \mu_X = \sum_{i=1}^k x_i / k,$$

$$\text{Var}(X) = \sigma_X^2 = \sum_{i=1}^k (x_i - \mu)^2 / k$$

$$* f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$E(X) = \mu_X = np,$$

$$\text{Var}(X) = \sigma_X^2 = np(1-p)$$

$$* f(x) = P(X = x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$$

$$E(X) = \mu_X = n \frac{K}{N},$$

$$\text{Var}(X) = \sigma_X^2 = n \frac{K}{N} \left(1 - \frac{K}{N}\right) \frac{N-n}{N-1}$$

$$* f(x) = P(X = x) = \frac{\mu^x e^{-\mu}}{x!}$$

$$\mu = \lambda t,$$

$$E(X) = \mu,$$

$$\text{Var}(X) = \sigma_X^2 = \mu$$

$$* f(x) = \frac{1}{B-A}, \quad A < X < B$$

$$E(X) = \mu_X = \frac{B+A}{2}$$

$$\text{Var}(X) = \sigma_X^2 = \frac{(B-A)^2}{12}$$

$$* f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}, \quad X > 0$$

$$E(X) = \mu_X = \beta$$

$$\text{Var}(X) = \sigma_X^2 = \beta^2$$

Sampling Distributions:

$$* \bar{X} = \sum_{i=1}^n X_i / n$$

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^n X_i^2 - n(\bar{X})^2}{n-1}$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

$$\frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t(n-1)$$

$$(\bar{X}_1 - \bar{X}_2) \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$* \hat{p} \sim N\left(p, \sqrt{\frac{pq}{n}}\right); \quad q = 1-p$$

$$(\hat{p}_1 - \hat{p}_2) \sim N\left(p_1 - p_2, \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}\right)$$

Confidence Intervals:

$$\bar{X} \pm Z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} \pm t_{\alpha} \frac{S}{\sqrt{n}}$$

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$\hat{p} \pm Z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{n}}; \quad \hat{q} = 1 - \hat{p}$$

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{\alpha} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$s.e.(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

$$e = Z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

$$n = \left(Z_{\alpha} \frac{\sigma}{e}\right)^2$$

Test Statistics:

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

$$T = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{X - np_0}{\sqrt{np_0 q_0}}$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$

