(ii) $f(x) = |\sin x|$; $x \in \mathbb{R}$ | تمارين 13-2 / حل

الدالة متصلة على IR فهي متصلة قطعياً على IR .

$$f'(x) = \begin{cases} \cos x : & x \geq 0 \\ -\cos x : & x < 0 \end{cases}$$

اتصال المشتقة عند $x = 0$ ؟

$\lim_{x \to 0^+} f'(x) = 1$, $\lim_{x \to 0^-} f'(x) = -1$

$\lim_{x \to 0^+} f'(x) \neq \lim_{x \to 0^-} f'(x)$.

∴ المشتقة f' غير متصلة عند $x = 0$

⇐ f' متصلة قطعياً على IR .

∴ f ملساء قطعياً على IR .

(iii) $f(x) = \sqrt{x}$ $\quad 0 \leq x < 1$, $f(x+1) = f(x)$

الدالة متصلة على الفترة $0 \leq x < 1$

$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-\frac{1}{2}}$

الدالة f' غير متصلة عند 0

∴ الدالة متصلة قطعياً على $0 \leq x < 1$

∴ ملساء قطعياً على $0 \leq x < 1$

$f(x) = e^x$, $f(x+2) = f(x)$. $-1 \leq x < 1$

f متصلة على $[-1,1]$ فهي متصلة قطعياً
على $(-1,1)$ وكذلك f' متصلة قطعياً بالتالي
هي ملساء قطعياً .

$$S_n(x) = a_0 + \sum a_n \cos n\pi x + b_n \sin n\pi x .$$

$$a_0 = \frac{1}{2}\int_{-1}^{1} e^x \, dx = \frac{e^1 - e^{-1}}{2} = \sinh(1) .$$

$$a_n = \frac{1}{1}\int_{-1}^{1} e^x \cdot \cos n\pi x \, dx .$$

①

③ ✓ هل المتسلسلة $\sum_{k=0}^{\infty} \frac{1}{2k+1} \sin(2k+1)x$ متقاربة بانتظام، ولماذا؟

③ نضع

$$f(x) = \begin{cases} -1 & -\pi \le x < 0 \\ 1 & 0 \le x < \pi \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi}\sin nx\,dx = \frac{2}{n\pi}(1-\cos n\pi) = \begin{cases} 0 & \text{زوجي } n \\ \frac{4}{n\pi} & \text{فردي } n \end{cases}$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -1\,dx + \int_0^{\pi} 1\,dx\right] = \frac{1}{2\pi}\left\{[-x]_{-\pi}^{0} + [x]_0^{\pi}\right\} = \frac{1}{2\pi}\left\{[-(0-(-\pi))] + [\pi - 0]\right\}$$

$$= \frac{1}{2\pi}[-\pi + \pi] = 0$$

$$a_n = \frac{2}{\pi}\left[\int_0^{\pi}\cos nx\,dx\right] = \frac{2}{n\pi}[\sin nx]_0^{\pi} = \frac{2}{n\pi}[\sin n\pi - \sin 0]$$

$$= \frac{1}{n\pi}[0] = 0$$

$$\Rightarrow f(x) = \sum_{1}^{\infty} b_n \sin nx$$

$$= b_1 \sin x + b_3 \sin 3x + b_5 \sin 5x + \cdots$$

$$= \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$$

$$S_n(x) = \frac{4}{\pi}\sum_{0}^{\infty} \frac{1}{2k+1}\sin(2k+1)x$$

ولكن الدالة غير متصلة عند 0 في $[-\pi, \pi]$ لهذا فالمتسلسلة غير متقاربة بانتظام.

④ [illegible]

5) أوجد مفكوك فورير للدالة المعرفة على $[-1,1]$ بالـ...

$$f(x)=\begin{cases}0 & -1\le x<0\\ 1 & 0\le x\le 1\end{cases}$$

الحل

$$a_0=\frac{1}{2l}\int_{-l}^{l}f(x)\,dx$$

$$=\frac{1}{2}\int_{-1}^{1}f(x)\,dx=\frac{1}{2}\left[\int_{-1}^{0}0\,dx+\int_{0}^{1}dx\right]=\frac{1}{2}\int_{0}^{1}dx=\frac{1}{2}[x]_0^1=\frac{1}{2}$$

$$a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi}{l}x\,dx$$

$$=\int_{-1}^{1}f(x)\cos n\pi\,dx=\cancel{\int_{-1}^{0}0\cos n\pi x\,dx}+\int_{0}^{1}\cos n\pi x\,dx$$

$$=\frac{1}{n\pi}[\sin n\pi x]_0^1=0$$

$$b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi}{l}x\,dx$$

$$=\int_{-1}^{1}f(x)\sin n\pi x\,dx=\cancel{\int_{-1}^{0}0\sin n\pi x\,dx}+\int_{0}^{1}\sin n\pi x\,dx$$

$$=\frac{1}{n\pi}[-\cos n\pi x]_0^1=\frac{1}{n\pi}[1-\cos n\pi]=\begin{cases}0 & \text{زوجي } n\\ \frac{2}{n\pi} & \text{فردي } n\end{cases}$$

(ص) ع

5 ص ع

$$\Longrightarrow f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} b_n \sin n\pi x$$

$$= \frac{1}{2} + [b_1 \sin \pi x + b_2 \sin 2\pi x + b_3 \sin 3\pi x + \cdots$$

$$= \frac{1}{2} + \frac{2}{n\pi}[\sin \pi x + \sin 3\pi x + \sin 5\pi x + \cdots]$$

$$= \frac{n\pi + 4}{2n\pi}[\sin \pi x + \sin 3\pi x + \sin 5\pi x + \cdots]$$

$$v(t) = (2\cos 100\pi t) \qquad (2.2)$$

$$v(t) = \frac{2}{\pi} + \cos 100\pi t - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2-1} \cos 200n\pi t$$

$$V(0) = 2\cos 100\pi(0) = 2 = \frac{2}{\pi} + 1 - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$$

$$\Rightarrow 2 = \frac{2+\pi}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$$

$$\Rightarrow \pi = 1 + \frac{\pi}{2} - 2\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$$

$f(x) = x^2 \;, -2 < x < 2 \quad f(x+4) = f(x)$ (13)

$$C_0 = \frac{1}{4}\int_{-2}^{2} x^2\,dx = \frac{1}{4}\left.\frac{x^3}{3}\right|_{-2}^{2} = \frac{1}{4}\left(\frac{8}{3} - \frac{(-8)}{3}\right) = \frac{1}{4}\left(\frac{16}{3}\right) = \frac{4}{3}$$

$$C_n = \frac{1}{4}\int_{-2}^{2} x^2 e^{-in\pi x/2}\,dx = \frac{1}{4}\left[\left.\frac{-2x^2}{in\pi}\cancel{e^{-in\pi x/2}}\right|_{-2}^{2} + \frac{4}{in\pi}\int_{-2}^{2} x\, e^{-in\pi x/2}\,dx\right]$$

$$= \frac{1}{4}\left[\frac{4}{in\pi}\left(\left.\frac{-2x}{in\pi}e^{-in\pi x/2}\right|_{-2}^{2} + \frac{2}{in\pi}\int_{-2}^{2} e^{-in\pi x/2}\,dx\right)\right] = \frac{1}{in\pi}\left[\frac{-4}{in\pi}e^{-in\pi} - \frac{4}{in\pi}e^{+in\pi}\right] + \frac{1}{2in\pi}\left(\left.\frac{-2}{in\pi}\cancel{e^{-in\pi x/2}}\right|_{-2}^{2}\right)$$

$$= \frac{1}{in\pi}\left[\frac{-4}{in\pi}(\cos n\pi - \cancel{i\sin n\pi}) - \frac{4}{in\pi}(\cos n\pi + \cancel{i\sin n\pi})\right] = \frac{4}{n^2\pi^2}(2\cos n\pi)$$

$$= \frac{8}{n^2\pi^2}(-1)^n$$

$$C_{-n} = \frac{1}{4}\int_{-2}^{2} x^2 e^{in\pi x/2}\,dx = \frac{1}{4}\left[\left.\frac{2x^2}{in\pi}\cancel{e^{in\pi x/2}}\right|_{-2}^{2} - \frac{4}{in\pi}\int_{-2}^{2} x\, e^{in\pi x/2}\,dx\right]$$

$$= \frac{1}{4}\left[-\frac{4}{in\pi}\left(\left.\frac{2x}{in\pi}e^{in\pi x/2}\right|_{-2}^{2} - \frac{2}{in\pi}\int_{-2}^{2} e^{in\pi x/2}\,dx\right)\right] = \frac{1}{4}\left[\frac{-4}{in\pi}\left(\left(\frac{4}{in\pi}e^{in\pi} + \frac{4}{in\pi}e^{-in\pi}\right) + \left.\frac{4}{n^2\pi^2}\cancel{e^{in\pi x/2}}\right|_{-2}^{2}\right)\right]$$

$$= \frac{-1}{in\pi}\left[\frac{4}{in\pi}(\cos n\pi + \cancel{i\sin n\pi}) + \frac{4}{in\pi}(\cos n\pi - i\sin n\pi)\right] = \frac{4}{n^2\pi^2}(2\cos n\pi)$$

$$= \frac{8}{n^2\pi^2}(-1)^n$$

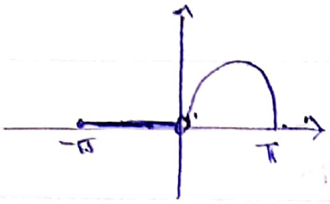
$$\cancel{S_n(x) = \frac{4}{3} + \sum_{n=1}^{\infty}\left[\frac{8}{n^2\pi^2}(-1)^n \cos\frac{n\pi}{2}x + i\,\frac{8}{n^2\pi^2}(-1)^n \sin\frac{n\pi}{2}x\right]}$$

$$S_n(x) = \frac{4}{3} + \sum_{n=1}^{\infty}\left[\left(\frac{8}{n^2\pi^2}(-1)^n + \frac{8}{n^2\pi^2}(-1)^n\right)\cos\frac{n\pi}{2}x + i\cancel{\left(\frac{8}{n^2\pi^2}(-1)^n + \frac{8}{n^2\pi^2}(-1)^n\right)}\sin\frac{n\pi}{2}x\right]$$

$$S_n(x) = \frac{4}{3} + \sum_{n=1}^{\infty} 2\left(\frac{8}{n^2\pi^2}(-1)^n\right)\cos\frac{n\pi}{2}x$$

$$= \frac{4}{3} + \frac{16}{\pi^2}\sum_{n=1}\frac{(-1)^n}{n^2}\cos\frac{n\pi}{2}x$$

وبالمقارنة مع سؤال (13) نجد انهما متساويان



$$f(x) = \begin{cases} 0 & ; -\pi \le x < 0 \\ \sin x, & 0 \le x < \pi \end{cases}, \quad f(x+2\pi) = f(x)$$

(12)

$f(-\pi^+) = 0 \qquad f(\pi^-) = \sin\pi = 0$

$f(0^-) = \lim_{x\to 0^-} f(x) = 0$ لنبحث اتصالها عند $x=0$

$f(0^+) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sin x = 0$

f دالة متصلة على $[-\pi,\pi)$ أي متصلة قطعياً على $[-\pi,\pi)$

$$f'(x) = \begin{cases} 0 & ; -\pi < x < 0 \\ \cos x; & 0 < x < \pi \end{cases}$$

لنبحث اتصال f' عند $x=0$

$\lim_{x\to 0^-} f'(x) = f'(0^-) = 0 \quad , \quad \lim_{x\to 0^+} f'(x) = f'(0^+) = \cos(0) = 1$

$\Leftarrow$ f' غير متصلة عند $x=0$ $\Leftarrow$ f' متصلة قطعياً على الفترة $(-\pi,\pi)$

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_0^{\pi}\sin x\,dx = \frac{1}{2\pi}[-\cos x]_0^{\pi} = \frac{1}{2\pi}[\cos(0) - \cos(\pi)]$$

$$= \frac{1}{2\pi}[1+1] = \frac{1}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi}\sin x\cos nx\,dx = \frac{1}{\pi}\left(-\frac{1}{2}\right)\left[\frac{\cos(1-n)x}{1-n} + \frac{\cos(1+n)x}{1+n}\right]_0^{\pi}$$

$$= \frac{-1}{2\pi}\left\{\left[\frac{\cos(1-n)\pi}{1-n} + \frac{\cos(1+n)\pi}{1+n}\right] - \left[\frac{1}{1-n} + \frac{1}{1+n}\right]\right\} = \frac{-1}{2\pi}\left\{\left[\frac{(-1)^{1-n}}{1-n} + \frac{(-1)^{1+n}}{1+n}\right] - \left[\frac{1}{1-n} + \frac{1}{1+n}\right]\right\}$$

$$= \frac{-1}{2\pi}\left[\frac{(-1)^{n+1} - 1}{1-n} + \frac{(-1)^{n+1} - 1}{1+n}\right] = \frac{1-(-1)^{n+1}}{2\pi}\left[\frac{1}{1-n} + \frac{1}{1+n}\right]$$

$$= \frac{1-(-1)^{n+1}}{2\pi}\left(\frac{2}{1-n^2}\right) = \frac{1+(-1)^n}{\pi(1-n^2)}$$

Date

$$b_n = \frac{1}{\pi}\int_0^{\pi} \sin x \sin nx \, dx = \frac{1}{\pi}\cdot\frac{1}{2}\left[\frac{\sin(1-n)x}{1-n} - \frac{\sin(1+n)x}{1+n}\right]_0^{\pi} = 0$$

$$a_1 = \frac{1}{\pi}\int_0^{\pi} \sin x \cos x \, dx = \frac{1}{2\pi}\sin^2 x\Big]_0^{\pi} = 0$$

$$f(x) \doteq \frac{1}{\pi} + \sum_{n=2}^{\infty} \frac{1+(-1)^n}{\pi(1-n^2)} \cos nx$$

$$f(x) \doteq \frac{1}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{1-(2n)^2} \cos 2nx$$

$$S_n(x) = \frac{1}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{1-4n^2} \cos 2nx$$

$$|U_n(x)| = \left|\frac{1}{1-4n^2}\cos 2nx\right| \le \frac{1}{1-4n^2} \le \frac{1}{4n^2} = M_n$$

$\sum_{n=1}^{\infty} M_n = \sum_{n=1}^{\infty} \frac{1}{4n^2}$ متقاربة لأنها متسلسلة P حيث $P = 2 > 1$

$$\Rightarrow S_n(x) \xrightarrow{u} f(x)$$

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