$e^{i\beta x} = \cos\beta x + \sin\beta x$

(2.3)

$$uLv - vLu = [p(uv' - u'v)]' \quad (1)$$

$$L = \frac{d}{dx}\left(p\frac{d}{dx}\right) + r$$

الطرف الأيمن

$p'(uv' - vu') + p(u'v' + uv'' - v'u' - u''v)$

$\Rightarrow p'(uv' - vu') + p(uv'' - vu'')$ (1)

الطرف الأيسر

~~$u\left[\left[p\frac{dv}{dx}\right]' + rv\right] - v\left[p\frac{du}{dx} + ru\right]$~~

~~$= upv' + urv - vpu' + vru$~~

$uLv - vLu =$

$= u\left[\left(p\frac{dv}{dx}\right)' + rv\right] - v\left[\left(p\frac{du}{dx}\right)' + ru\right]$

$= u\left((pv'' + p'v') + rv\right) - v(pu'' + p'u' + ru)$

$= upv'' + up'v' + urv - vpu'' - vp'u' - vru$

$= p'(uv' - vu') + p(uv'' - vu'')$ وهو الطرف الايمن وبذلك تتحقق المساواة.

(5)

(2)

$$py'' + qy' + ry + \lambda y = 0 \; ; \; p(x) > 0 \quad (*)$$

$$\tilde{p}y'' + \tilde{p}'y' + \tilde{r}y + \lambda\tilde{w}y = 0$$

البرهان: نضرب المعادلة (*) بدالة عامة S

$$Spy'' + Sqy' + Sry + \lambda Sy = 0$$

$$\tilde{p} = Sp \Rightarrow \tilde{p}' = (Sp)' = Sq$$

$$\Rightarrow S'p + Sp' = Sq \Rightarrow \frac{S'}{S} + \frac{p'}{p} = \frac{q}{p}$$

$$\Rightarrow \frac{ds}{s} + \frac{dp}{p} = \frac{q}{p}dx \Rightarrow \ln sp = \int \frac{q}{p}dx$$

$$\Rightarrow Sp = e^{\int \frac{q}{p}dx} \Rightarrow S = \frac{1}{p}\exp\left(\int q/p \, dx\right)$$

$$\tilde{p} = Sp = \tilde{p}' = Sq, \; \tilde{r} = Sr, \; \lambda\tilde{w} = \lambda s$$

$$(*) \Rightarrow \tilde{p}y'' + \tilde{p}'y' + \tilde{r}y + \lambda\tilde{w}y = 0$$

(4)

(iii)

$$\underbrace{\cos x}_{p(x)}\frac{d^2}{dx^2} + \underbrace{\sin x}_{q(x)}\frac{d}{dx} - \underbrace{\cos^2 x}_{r(x)}$$

$$S\cos x\frac{d^2}{dx^2} + S\sin x\frac{d}{dx} - S\cos^2 x$$

$$\tilde{p} = S\cos x \Rightarrow \tilde{p}' = (S\cos x)' = S.\sin x$$

$$\Rightarrow S'\cos x - S\sin x = S\sin x$$

$$\Rightarrow S'\cos x = 2S\sin x$$

لا نضع ثابت لأن نفترض أي قيمة له $\ln(1)=0$

$$\Rightarrow \frac{ds}{s} = 2\frac{\sin x}{\cos x}dx = \ln S = 2\ln|\sec x| = \ln\sec^2 x$$

$$= \boxed{S = \sec^2 x}$$

(c7)

$$\sec^2 x \left[\cos x \frac{d^2}{dx^2} + \sin x \frac{d}{dx} - \cos^2 x \right]$$

$$= \frac{1}{\cos x} \frac{d^2}{dx^2} + \frac{\sin x}{\cos^2 x} \frac{d}{dx} - 1 = \tilde{P}(x) \frac{d^2}{dx^2} + \tilde{P}(x) \frac{d}{dx} + \tilde{R}(x)$$