3. EXPECTATIONS INVOLVING INDEPENDENT R. V. 'S AND MOMENT GENERATING FUNCTIONS

Q1) Let X_1, X_2 and X_3 be independent r.v.'s with means 4, 9, 3 and variances 3, 7, 5 respectively. For $Y=2X_1-3X_2+4X_3$ and $Z=X_1+2X_2-X_3$, find:

a. E(Y) and E(Z).

b. V(Y) and V(Z).

Q2) If X and Y are independent r.v.'s with E(X)=3, E(Y)=5, V(X)=2, and V(Y)=5, find:

a. E(XY)

b. $E(X^2Y)$

Q3) Let X and Y are independent r.v's with p.d.f $f(x) = e^{-x}; x > 0$,

$f(y) = e^{-y};\ y > 0$, find :

a. E(Y) and V(X).

b. E(Y) and V(Y).

c. E(XY).

d. $E(X^2 Y^3)$.

Q4) A r.v. has $f(x) = \frac{1}{2}e^{-|x|}\ ; for\ -\infty < x < \infty$, find E(X) and V(X).

Q5) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed having mean μ and variance σ^2. Let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that $E\left[\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2\right] = (n-1)\sigma^2$.

Q6) If we have

a. $f(x) = \frac{1}{b-a}\ ; a \le x \le b$

b. $f(x) = \lambda e^{-\lambda x}\ ; x > 0$

c. $f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right]; -\infty < x < \infty$

Find E(X) and V(X).

Q7) If $X \sim Exp(2)$ independent of $Y \sim Gamma(3,4)$, find:

a. E(XY).

b. $E(X^2 Y^3)$.

c. V(X-Y)

d. V(3X+2Y)

where

	pdf	E(X)	V(X)
$Exp(\lambda)$	$f(x) = \lambda e^{-\lambda x}; x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$Gamma(\alpha, \beta)$	$f(x) = \frac{\beta^\alpha}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x}; x > 0$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$

Q8) Find the moment generating function of X If you know that $f(x)=2e^{-2x}$, x>0

Q9) Suppose independent r.v.'s X and Y are such that $M_{X+Y}(t) = \frac{e^{2t}-1}{2t-t^2}$. If $(x) = \lambda e^{-\lambda x}; x > 0$, what is the mgf of Y.

Q10) A r.v. has $f(x) = \frac{1}{2} e^{-|x|}; for\ -\infty < x < \infty$.

a. Show that its mgf is given by $M_X(t) = \frac{1}{1-t^2}$ for -1<t<1.

b. Using the mgf, find E(X) and V(X).

Q11) If X has $f(x) = \frac{3}{2}x^2, -1 < x < 1$

a. Find mgf of X.

b. Given the mgf in expanded form.

c. Use the expanded form to determine a general formula for $E(X^n)$.

Q12) X and Y are independent and identically distributed with $M(t) = e^{3t+t^2}$. Find the mgf of Z=2X-3Y+4.

Q13) Suppose X has $M_X(t) = e^{3t+t^2}$. Find the mgf of $Z = \frac{1}{4}(X-3)$ and use it to find the mean and variance of Z.

Q14) Suppose X is a r.v. for which the mgf is $M_X(t) = \frac{1}{4}(3e^t + e^{-t}), -\infty < t < \infty$.

a. Find the mean and variance of X.

b. Find the expanded form of the mgf.

Q15) Let $f(x) = 1 \;\; ; 0 \leq x \leq 1$. Use the moment generating function technique to find the moment generating function of Y=aX+b where a and b are constant.

Q16) Let $f(x) = e^{-x}\,; x > 0$, find the mgf of Z=3-2X.

Q17) X, Y and Z are independent r.v.'s with $X \sim Normal(1,3)$, $Y \sim Normal(5,2)$ and the mgf of their sum being $M_{X+Y+Z}(t) = e^{13t+3t^2}$. Determine the distribution of Z.