Chapter 3

Solving Problems by Search: Uninformed Search

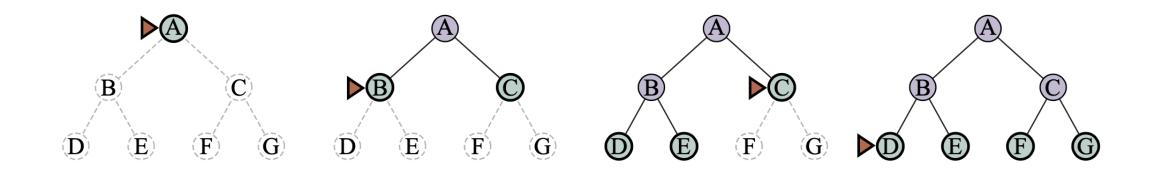
Uninformed search algorithm

- Uninformed search (blind search) strategies use only the information available in the problem definition
- Strategies that know whether one non-goal state is better than another are called informed search or heuristic search

Uninformed strategies: use only the information available in the problem definition:

- 1. Breadth First Search (BFS)
- 2. Uniform Cost Search (UCS)
- 3. Depth First Search (DFS)
- 4. Depth Limited Search (DLS)
- 5. Iterative Deepening Search (IDS)
- 6. Bidirectional search

1. Breadth First Search (BFS)



- Main idea: Expand all nodes at depth *i* before expanding nodes at depth *i* + 1.
 (Shallow nodes are expanded before deeper nodes)
- Implementation:
 - The frontier list is a First-In-First-Out queue (FIFO).
 - Test for **goal** before putting in FIFO.

1. Breadth First Search (BFS)

function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure

 $node \leftarrow a node with STATE = problem.INITIAL-STATE, PATH-COST = 0$ if problem.GOAL-TEST(node.STATE) then return SOLUTION(node) frontier $\leftarrow a$ FIFO queue with node as the only element explored \leftarrow an empty set

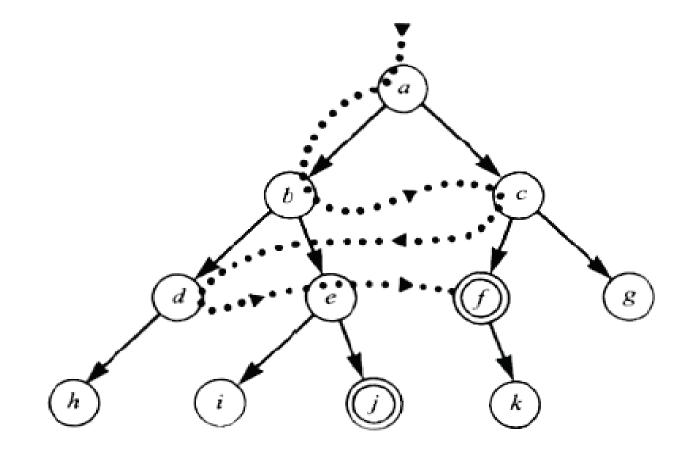
loop do

if EMPTY?(frontier) then return failure node ← POP(frontier) /* chooses the shallowest node in frontier */ add node.STATE to explored for each action in problem.ACTIONS(node.STATE) do child ← CHILD-NODE(problem, node, action)

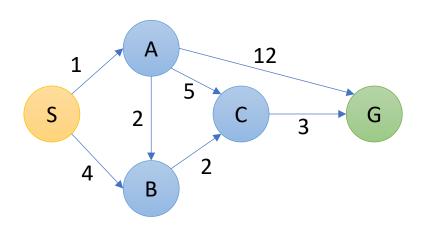
if *child*.STATE is not in *explored* or *frontier* then

if problem.GOAL-TEST(child.STATE) **then return** SOLUTION(child) frontier ← INSERT(child, frontier)

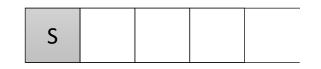
Breadth First Search (BFS)



1. BFS: Example



- Initial State = S, Path-Cost = 0
- Frontier:



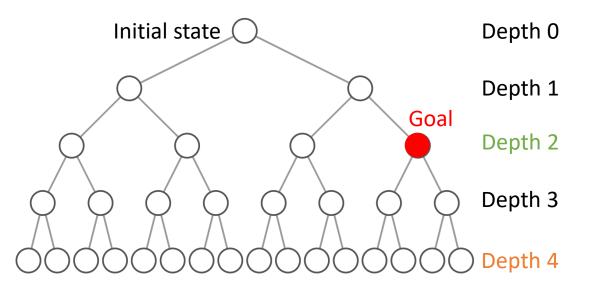
• Explored:

Recall: Evaluating search algorithms

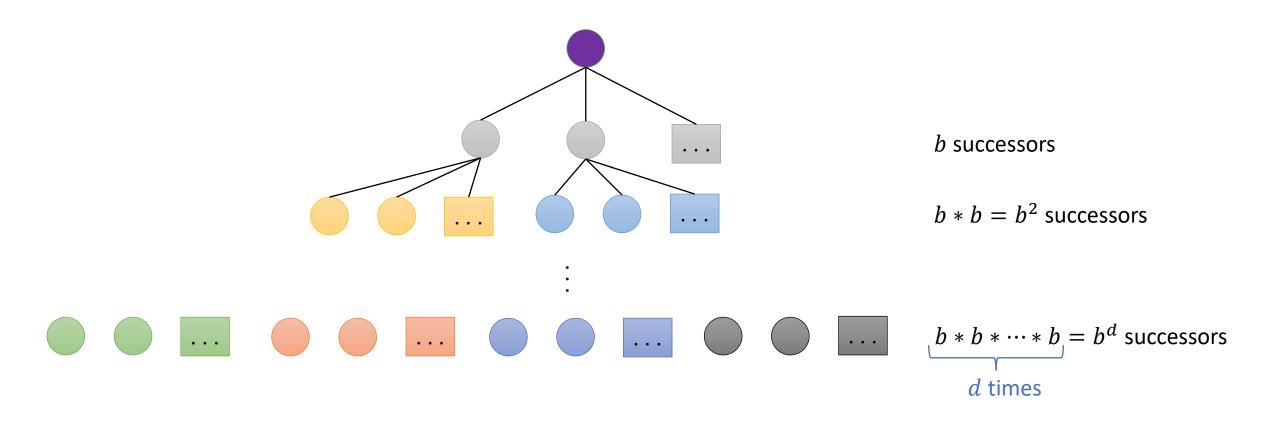
- 1. Completeness: is the algorithm guaranteed to find a solution if one exists?
- 2. Time complexity: number of operations (time) necessary to find a solution.
- 3. Space complexity: memory requirement.
- **4. Optimality**: if there are multiple solutions, does the algorithm find the best one (minimum cost)?
- Time and space complexity are measured in terms of
 - **b**: maximum branching factor of the search tree
 - *d*: depth of the best solution
 - *m*: maximum depth of the state space (may be infinite)

How to find complexity ?

- Take the worst possible example and compute its running time
- Difficult to do with graphs: use trees
- We use a complete tree with branching factor *b* and depth *m*. The goal is at depth *d*.
- Example: b = 2, m = 4, d = 2



1. BFS Performance



1. BFS Performance ()

- Completeness: Guaranteed for finite space. Guaranteed when a solution exists.
- Optimality: Yes, if step-costs are equal, otherwise no.
- Time Complexity: Total number of nodes generated: $O(b^d)$
 - What if the goal test was done when the node was selected for expansion instead of added to the frontier? $O(b^{d+1})$ (This is not BFS)
- Space Complexity: $O(b^{d-1})$ nodes in the explored set and $O(b^d)$ nodes in the frontier

1. BFS Complexity

Bigger Problem

Depth	Nodes	Time		Ν	Memory	
2	110	.11	milliseconds	107	kilobytes	
4	11,110	11	milliseconds	10.6	megabytes	
6	10^{6}	1.1	seconds	1	gigabyte	
8	10^{8}	2	minutes	103	gigabytes	
10	10^{10}	3	hours	10	terabytes	
12	10^{12}	13	days	1	petabyte	
14	10^{14}	3.5	years	99	petabytes	
16	10^{16}	350	years	10	exabytes	
Figure 3 13	Time and memor	ry requirem	ents for breadth-first	search The	numbers shown	

Figure 3.13 Time and memory requirements for breadth-first search. The numbers shown assume branching factor b = 10; 1 million nodes/second; 1000 bytes/node.

- When all step costs are equal, breadth-first search is optimal
 - it always expands the *shallowest* unexpanded node
- With any step cost function, an optimal algorithm expands the *lowest path* cost g(n), instead of expanding the shallowest node

UCS:

• Main idea: Expand the cheapest node, where the cost is the path cost g(n)

• Implementation:

• The frontier list is a priority queue with g(n) as the priority

```
node \leftarrow a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
frontier \leftarrow a priority queue ordered by PATH-COST, with node as the only element
explored \leftarrow an empty set
loop do
   if EMPTY?(frontier) then return failure
   node \leftarrow POP(frontier) /* chooses the lowest-cost node in frontier */
   if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
   add node.STATE to explored
   for each action in problem.ACTIONS(node.STATE) do
       child \leftarrow CHILD-NODE(problem, node, action)
       if child.STATE is not in explored or frontier then
           frontier \leftarrow \text{INSERT}(child, frontier)
       else if child.STATE is in frontier with higher PATH-COST then
           replace that frontier node with child
```

function UNIFORM-COST-SEARCH(problem) returns a solution, or failure

 $node \leftarrow a node with STATE = problem.INITIAL-STATE, PATH-COST = 0$ frontier \leftarrow a priority queue ordered by PATH-COST, with node as the only element explored \leftarrow an empty set

loop do

if EMPTY?(*frontier*) then return failure

 $node \leftarrow POP(frontier)$ /* chooses the lowest-cost node in frontier */

if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)

add node.STATE to explored

for each action in problem.ACTIONS(node.STATE) do

 $child \leftarrow CHILD-NODE(problem, node, action)$

if child.STATE is not in explored or frontier then

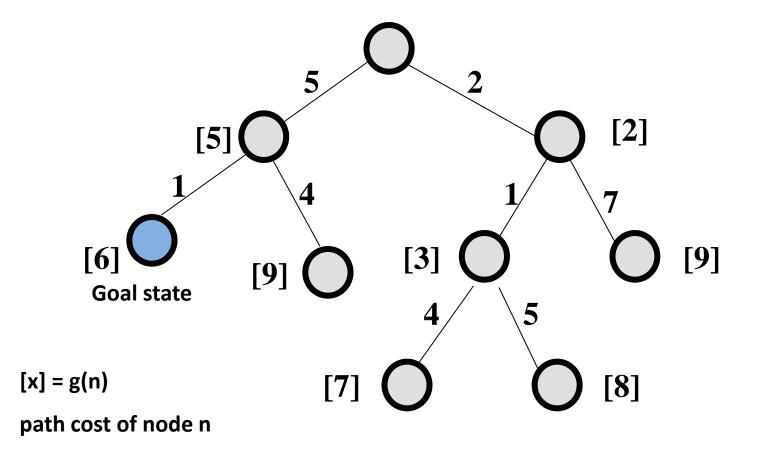
 $frontier \leftarrow \text{INSERT}(child, frontier)$

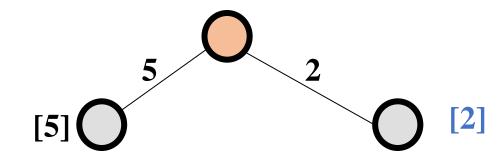
else if *child*.STATE is in *frontier* with higher PATH-COST then replace that *frontier* node with *child*

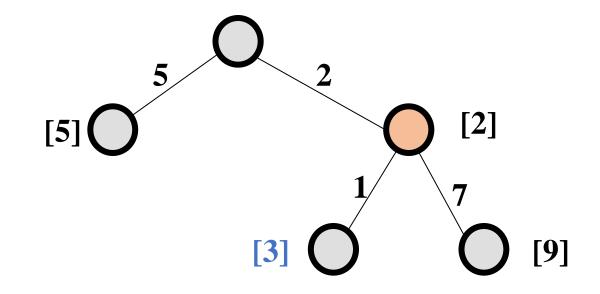
```
node \leftarrow a \text{ node with } STATE = problem.INITIAL-STATE, PATH-COST = 0
frontier \leftarrow a priority queue ordered by PATH-COST, with node as the only element
explored \leftarrow an empty set
                                                                       Expand node with smallest cost
loop do
   if EMPTY?(frontier) then return failure
    node \leftarrow POP(frontier) /* chooses the lowest-cost node in frontier */
   if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    add node.STATE to explored
   for each action in problem.ACTIONS(node.STATE) do
       child \leftarrow CHILD-NODE(problem, node, action)
       if child.STATE is not in explored or frontier then
           frontier \leftarrow \text{INSERT}(child, frontier)
       else if child.STATE is in frontier with higher PATH-COST then
           replace that frontier node with child
```

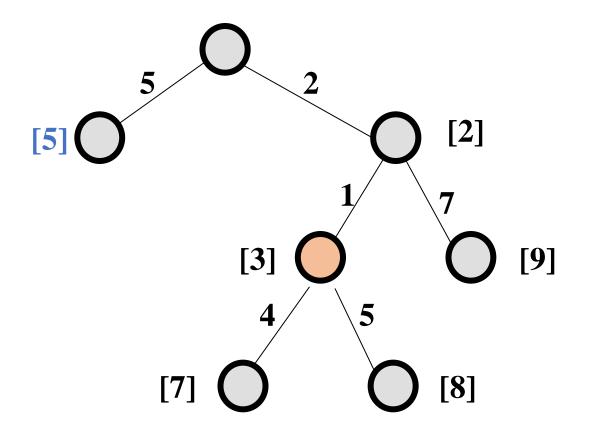
```
node \leftarrow a \text{ node with } STATE = problem.INITIAL-STATE, PATH-COST = 0
frontier \leftarrow a priority queue ordered by PATH-COST, with node as the only element
explored \leftarrow an empty set
loop do
   if EMPTY?(frontier) then return failure
    node \leftarrow POP(frontier) /* chooses the lowest-cost node in frontier */
   if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    add node.STATE to explored
   for each action in problem.ACTIONS(node.STATE) do
                                                                    Check when expanded rather than
       child \leftarrow CHILD-NODE(problem, node, action)
                                                                    when generated, because what if it is
                                                                     on a suboptimal path?
       if child.STATE is not in explored or frontier then
           frontier \leftarrow \text{INSERT}(child, frontier)
       else if child.STATE is in frontier with higher PATH-COST then
           replace that frontier node with child
```

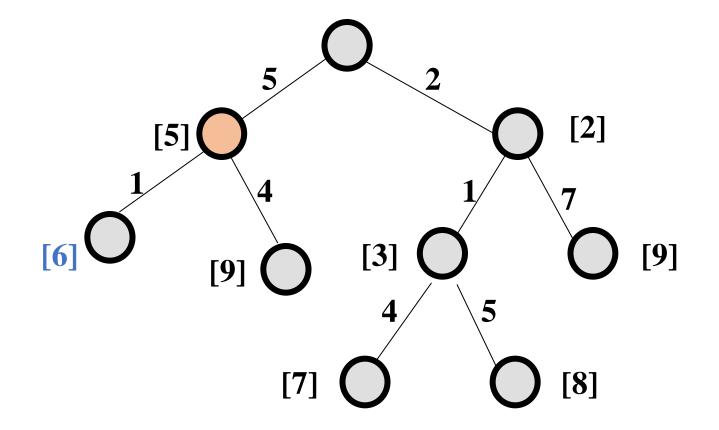
$node \leftarrow a \text{ node with STATE} = problem.INITIAL-STATE, PATH-CO$	OST = 0
<i>frontier</i> \leftarrow a priority queue ordered by PATH-COST, with node as	s the only element
$explored \leftarrow an empty set$	
loop do	
if EMPTY?(frontier) then return failure	
$node \leftarrow POP(frontier)$ /* chooses the lowest-cost node in free	ontier */
if problem.GOAL-TEST(node.STATE) then return SOLUTION	N(node)
add node.STATE to explored	
for each action in problem.ACTIONS(node.STATE) do	
$child \leftarrow CHILD-NODE(problem, node, action)$	
if child.STATE is not in explored or frontier then	What if a better path is found?
$frontier \leftarrow \text{INSERT}(child, frontier)$	
else if child.STATE is in frontier with higher PATH-COST	then
replace that <i>frontier</i> node with <i>child</i>	
	17

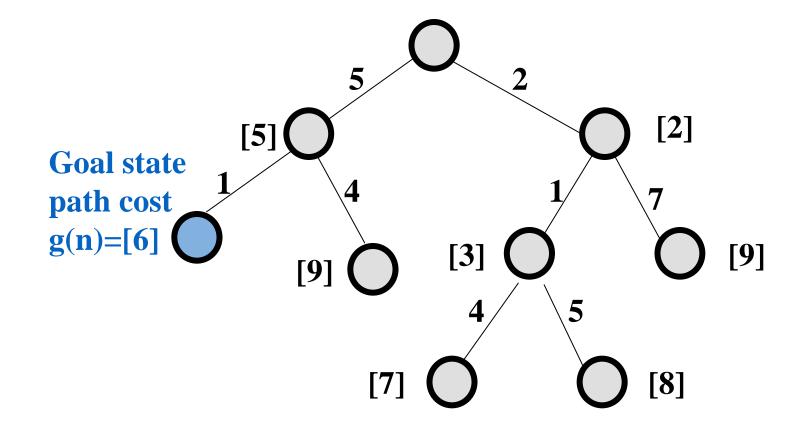


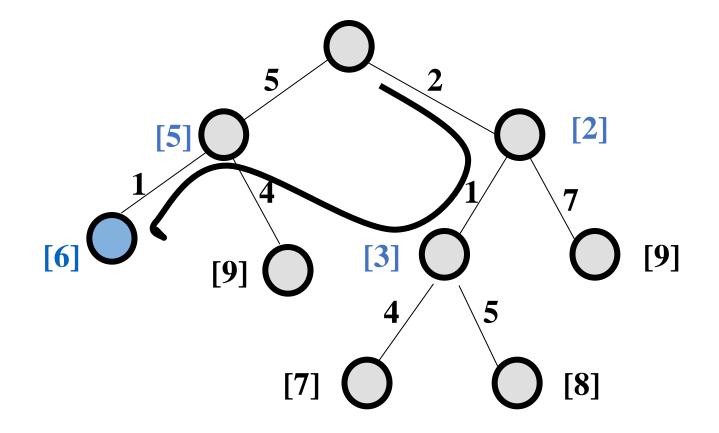




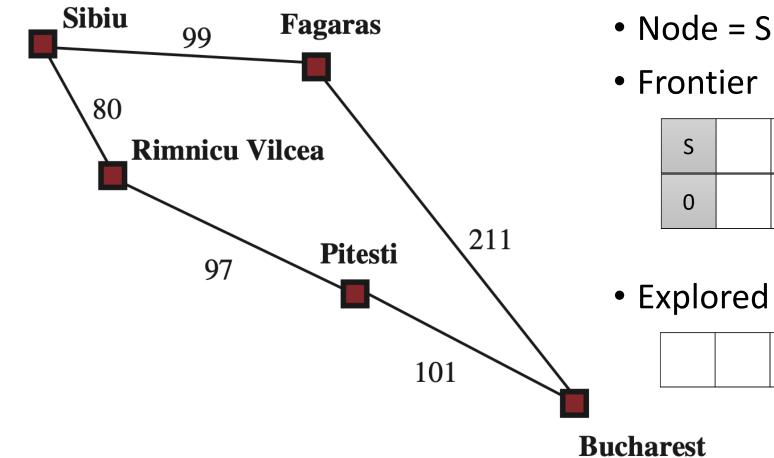




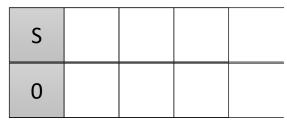




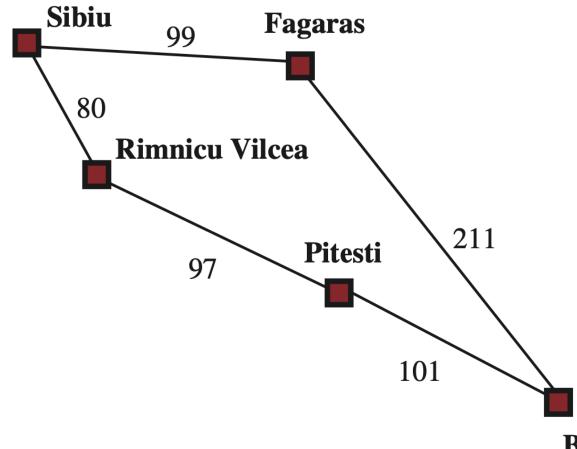
2. UCS: Example



- Node = Sibiu
- Frontier



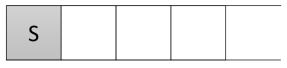
2. UCS: Example



- Pop Node = Sibiu, Goal? No
- Frontier

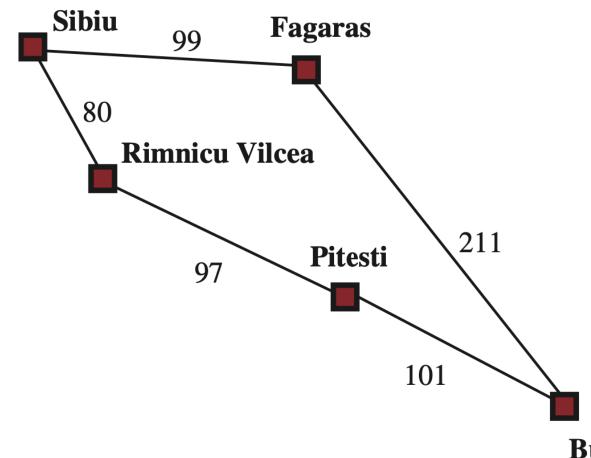
R	F		
80	99		

• Explored



Bucharest

2. UCS: Example



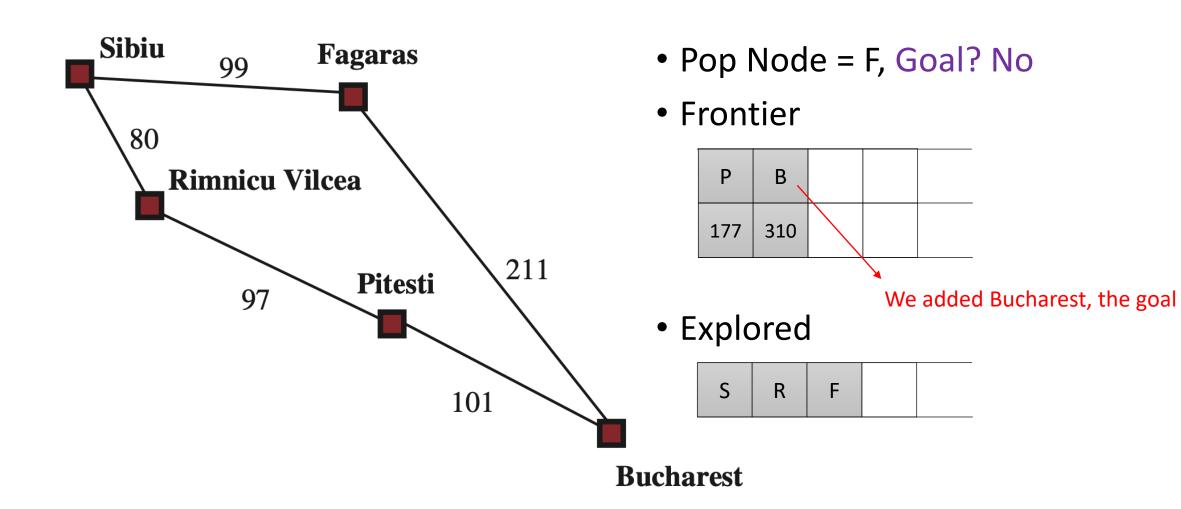
- Pop Node = R, Goal? No
- Frontier

F	Р		
99	177		

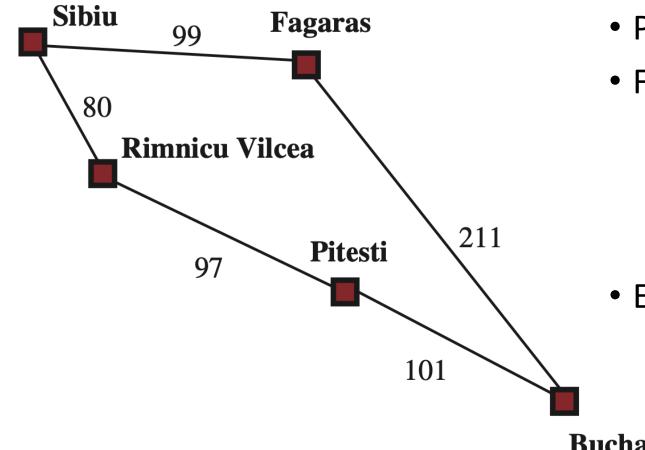
• Explored



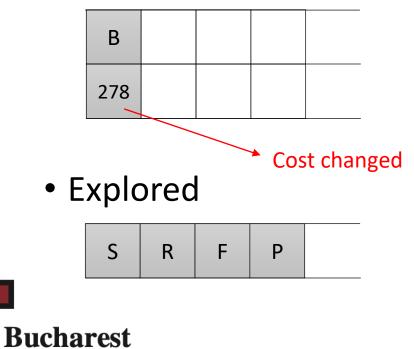
2. UCS: Example



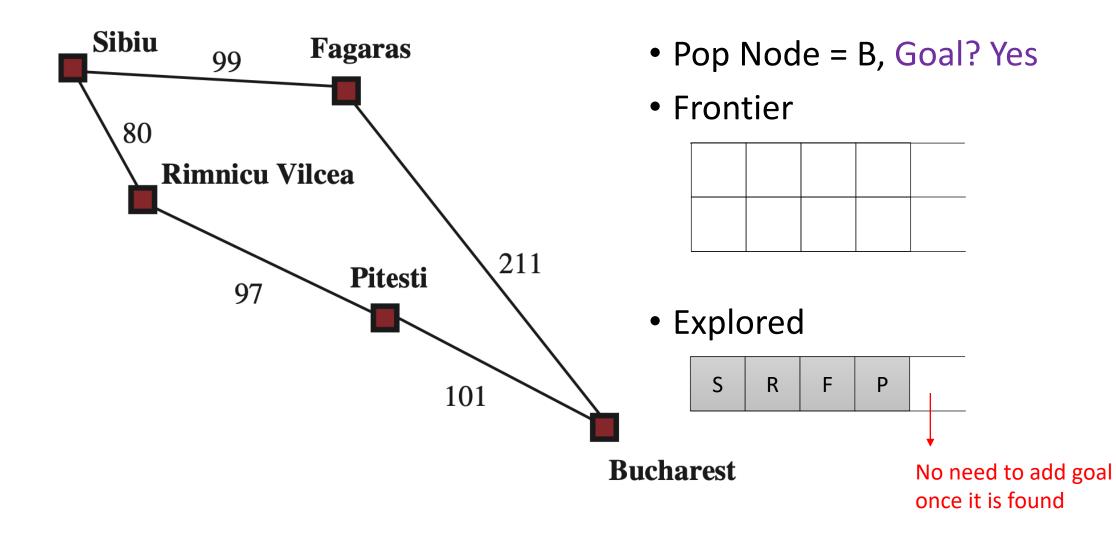
2. UCS: Example



- Pop Node = P, Goal? No
- Frontier



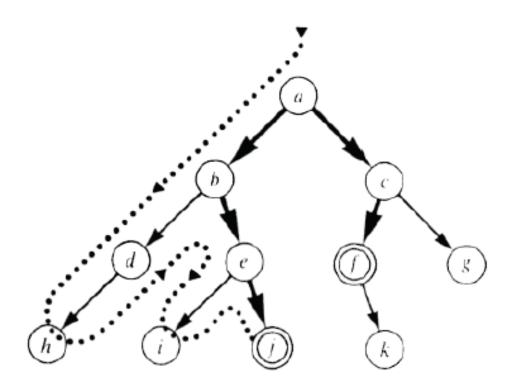
2. UCS: Example



2. UCS Performance ()

- Optimality: Yes, given that step costs are nonnegative
- Completeness: Yes, if the cost of every step exceeds some small positive constant $\varepsilon > 0$
 - UCS does not care about the number of steps
 - Gets stuck in an infinite loop if there is a path with an infinite sequence of zero-cost actions
- Time and Space Complexity:
 - *C*^{*} be the cost of the optimal solution
 - Every action costs at least ε
 - $O(b^{1+\lfloor C^*/\varepsilon \rfloor})$, if steps are equal: $O(b^{d+1})$

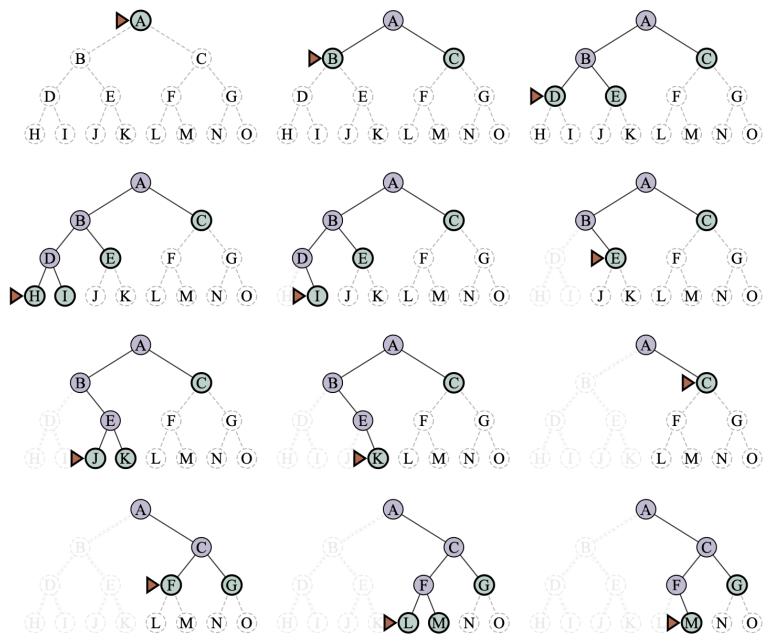
3. Depth First Search (DFS)



3. Depth First Search (DFS)

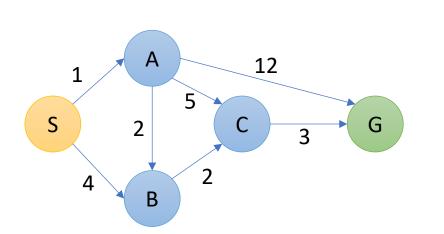
- Main idea: Expand node at the deepest level (breaking ties left to right).
- Implementation: Same as UCS, but the frontier list is a stack, Last-In-First-Out (LIFO).

 Grayed out means removed from memory



3. DFS: Example

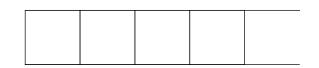
• Initial State = S, Path-Cost = 0



• Frontier:

• Explored:

S

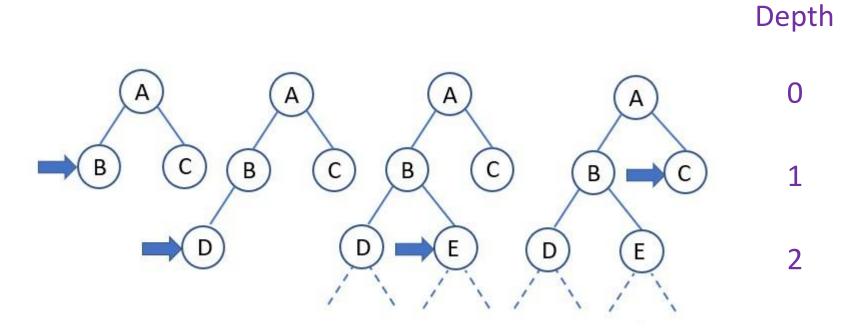


3. DFS Performance ()

- Optimality: No
- Completeness: Guaranteed for finite space. (Tree search not complete!)
- Time Complexity: generate $O(b^m)$ nodes in the search tree, where m is the maximum depth of any node
 - m itself can be much larger than d
- Space Complexity: O(b * m)
 - *b* branching factor
 - *m* maximum depth

4. Depth-Limited Search (DLS)

• Assume Depth limit = 2



4. DLS

- It is simply DFS with a depth bound (limit)
 - Searching is not permitted beyond the depth bound
- Works well if we know what the depth of the solution is
- Termination is guaranteed
- If the solution is beneath the depth bound, the search cannot find the goal

Students

4. DLS

function DEPTH-LIMITED-SEARCH(*problem*, ℓ) returns a node or *failure* or *cutoff frontier* \leftarrow a LIFO queue (stack) with NODE(*problem*.INITIAL) as an element $result \leftarrow failure$ while not IS-EMPTY(frontier) do $node \leftarrow POP(frontier)$ **if** *problem*.IS-GOAL(*node*.STATE) **then return** *node* if DEPTH(*node*) > ℓ then $result \leftarrow cutoff$ else if not IS-CYCLE(node) do for each *child* in EXPAND(*problem*, *node*) do add child to frontier return result

4. DLS Performance ()

- Completeness: No
- Optimality: No
- Time Complexity: $O(b^L)$, where L is the depth limit.
- Space Complexity: O(b * L), where L is the depth limit.

5. Iterative Deepening Search (IDS)

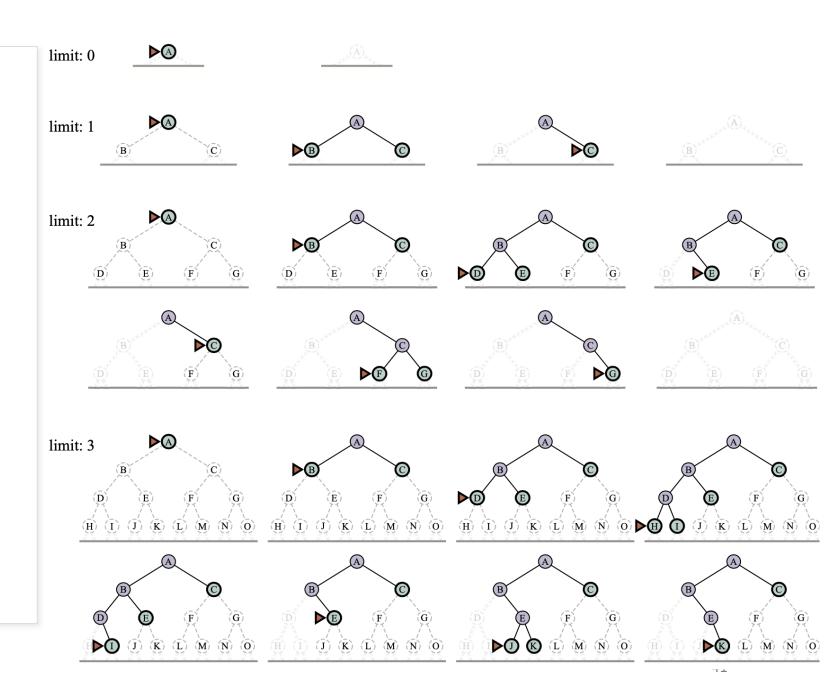
• If the depth is unknown, use Iterative deepening search (IDS)

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution node or *failure* **for** *depth* = 0 **to** ∞ **do** *result* \leftarrow DEPTH-LIMITED-SEARCH(*problem*, *depth*) **if** *result* \neq *cutoff* **then return** *result*

Students

5. IDS

- Main idea: Expand node at depth zero, if not goal, increase level
- Grayed out means removed from memory



5. IDS Performance ()

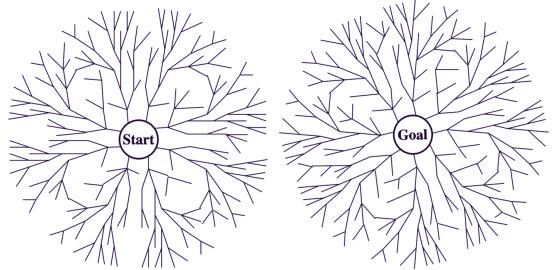
- IDS combines the benefits of BFS and DFS:
 - Like DFS the memory requirements are very modest O(b * d)
 - Like BFS, it is complete when the branching factor is finite
- The total number of generated nodes:

 $N(IDS) = (d)b + (d - 1)b^2 + \dots + (1)b^d$

- In general, IDS is the preferred uninformed search method, when search space is large, and depth of solution is unknown
- Completeness: Yes (if b is finite).
- Optimality: Yes when the path cost is a non-decreasing function of depth.
- Time Complexity: $O(b^d)$
- Space Complexity: O(b * d)

6. Bidirectional Search (BDS)

- Main idea: Start searching from both the initial state and the goal state (if applicable), meet in the middle (the frontiers intersect)
- Why? Because time (and space) complexity is $b^{\frac{n}{2}} + b^{\frac{n}{2}}$ is much less than b^d
 - Area of the two small circles is less than the area of one big circle



6. Bidirectional Search (BDS)

- Use BFS or UCS for search
- Can not be used in implicit goals
- Difficult when the actions are not reversible
- Requires a method for computing predecessors
 - Difficult when e.g. goal is no queen attacks another queen
 - Easy in finding a route from a map

6. BDS Performance ()

- Completeness: Yes, if b is finite and use BFS or UCS in both directions
- Optimality: Yes, if UCS is used in both directions or the step costs are all identical and BFS is used in both directions
- Time Complexity: $O(b^{\frac{d}{2}})$
- Space Complexity: $O(b^{\frac{d}{2}})$

Summary

Criterion	Breadth-	Uniform-	Depth-	Depth-	Iterative	Bidirectional
	First	Cost	First	Limited	Deepening	(if applicable)
Complete? Time	$egin{array}{l} \operatorname{Yes}^a \ O(b^d) \ O(b^d) \end{array}$	$egin{aligned} & ext{Yes}^{a,b} \ O(b^{1+\lfloor C^*/\epsilon floor}) \ O(b^{1+\lfloor C^*/\epsilon floor}) \end{aligned}$	No $O(b^m)$	No $O(b^{\ell})$ $O(b^{\ell})$	Yes ^a $O(b^d)$ O(bd)	$egin{array}{l} \operatorname{Yes}^{a,d} \ O(b^{d/2}) \ O(b^{d/2}) \end{array}$
Space	$O(0^{a})$	$O(b^{-1} [s^{-1}, s^{-1}])$	O(bm)	$O(b\ell)$ No	O(bd)	$O(b^{c,d})$
Optimal?	Yes ^c	Yes	No		Yes ^c	Yes ^{c,d}

Figure 3.21 Evaluation of tree-search strategies. *b* is the branching factor; *d* is the depth of the shallowest solution; *m* is the maximum depth of the search tree; *l* is the depth limit. Superscript caveats are as follows: ^{*a*} complete if *b* is finite; ^{*b*} complete if step costs $\geq \epsilon$ for positive ϵ ; ^{*c*} optimal if step costs are all identical; ^{*d*} if both directions use breadth-first search.

Tutorial

Blind Search Algorithms

Tree Search:

BFS, DFS, DLS, IDS

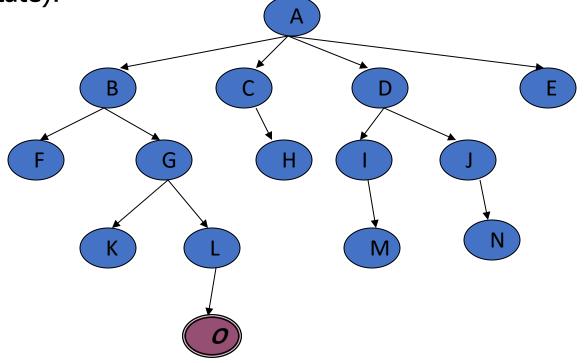
Basic Search Algorithms

Breadth First Search

BFS

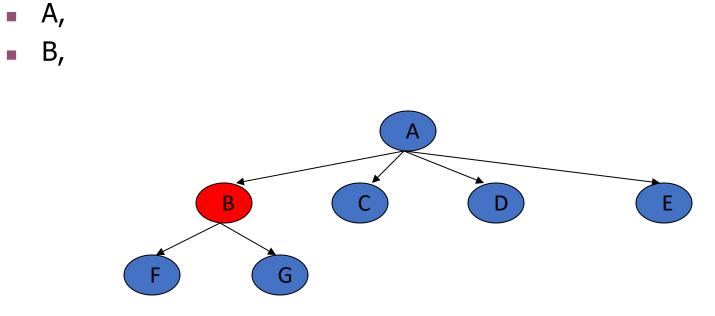
• Application1:

Given the following state space (tree search), give the sequence of visited nodes when using BFS (assume that the node *O* is the goal state):

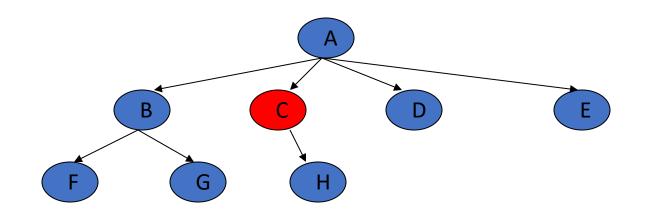


A,

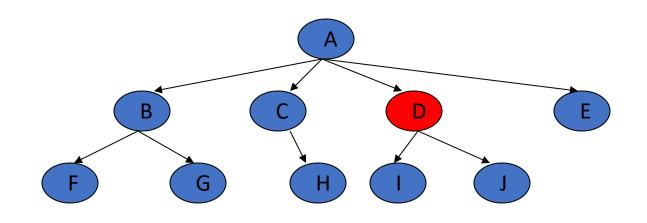
B C D E



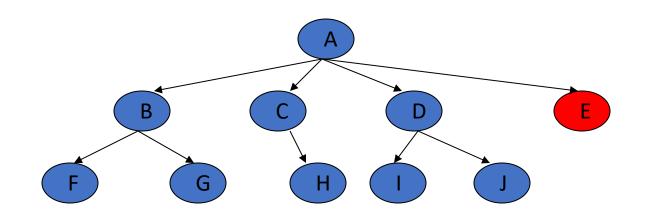
A,B,C

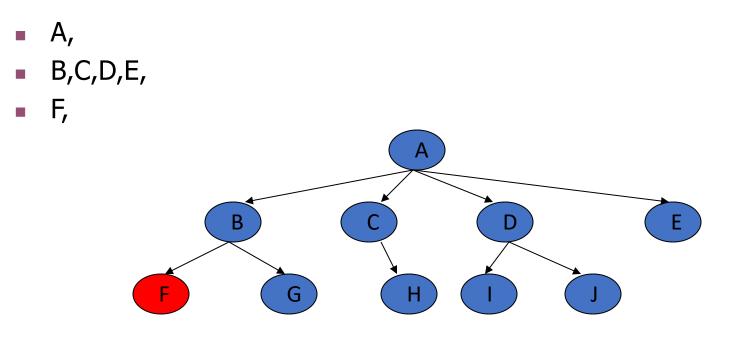


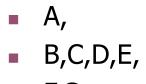
A,B,C,D



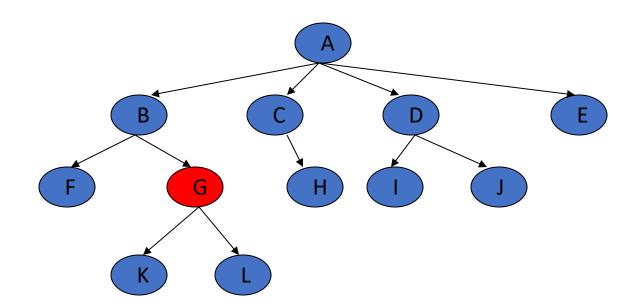
A,B,C,D,E

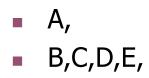




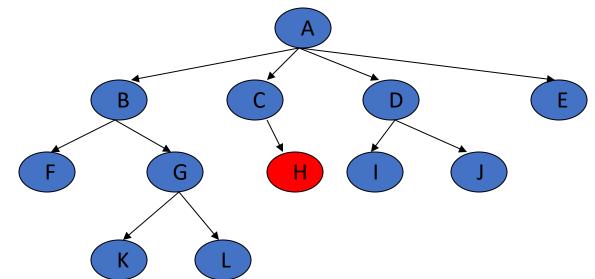


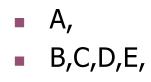
■ F,G



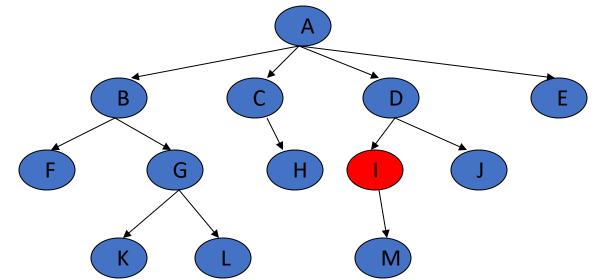


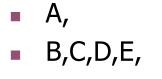
■ F,G,H



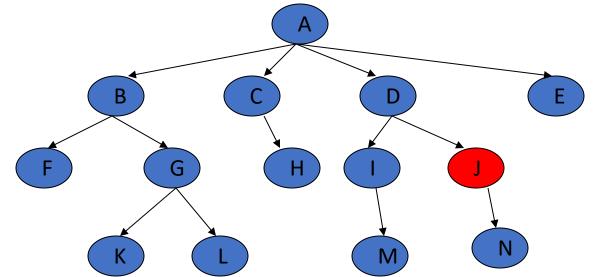


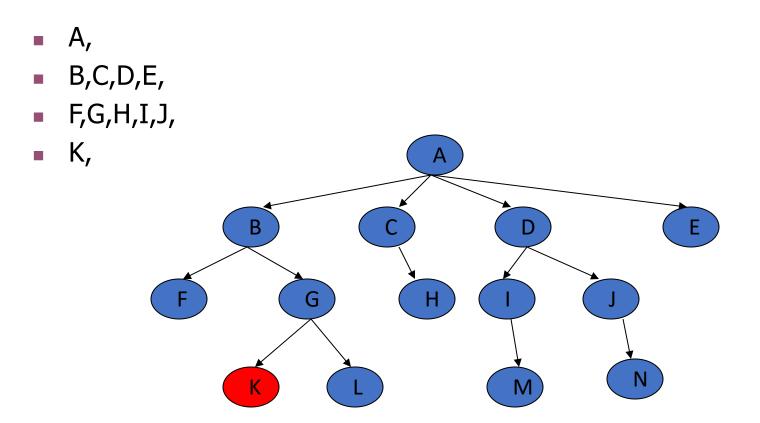
■ F,G,H,I

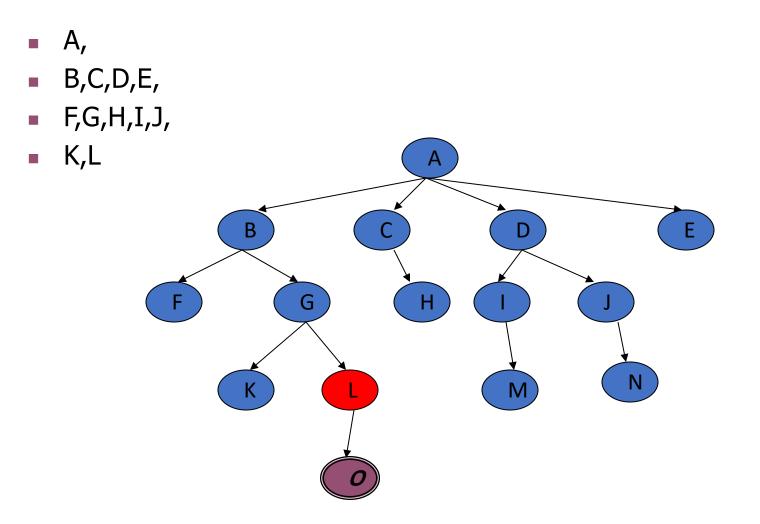


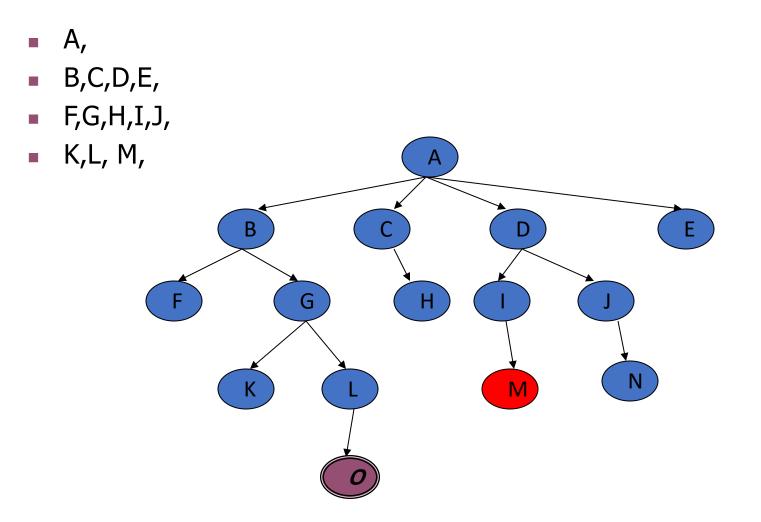


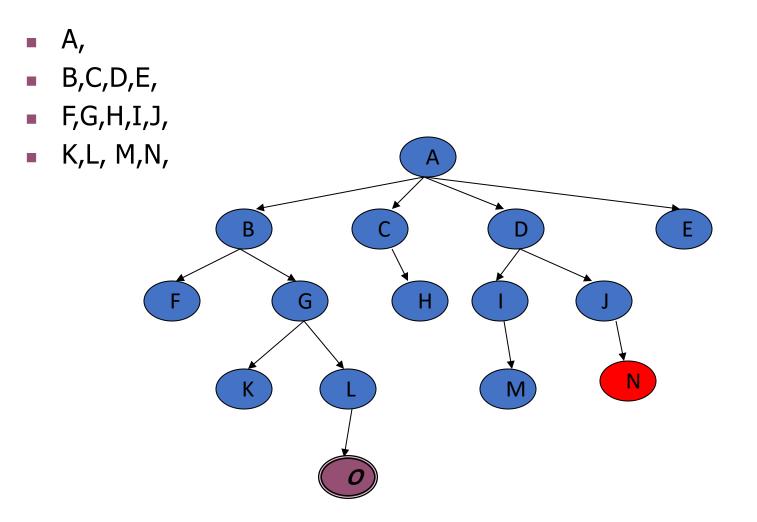


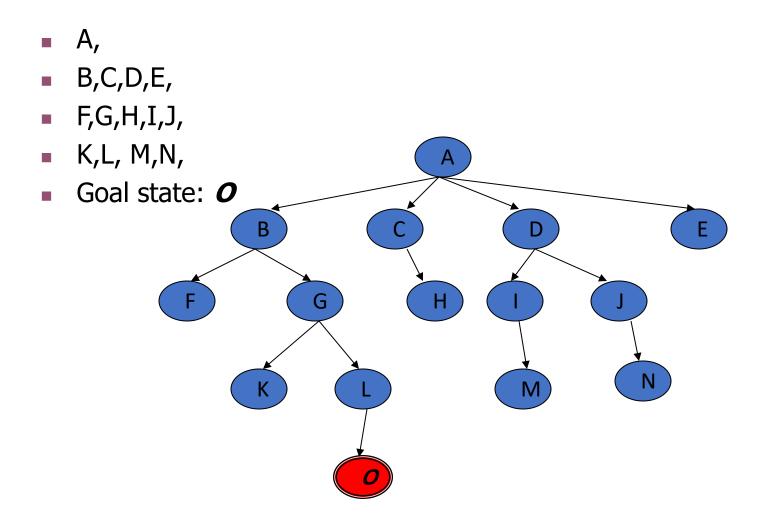






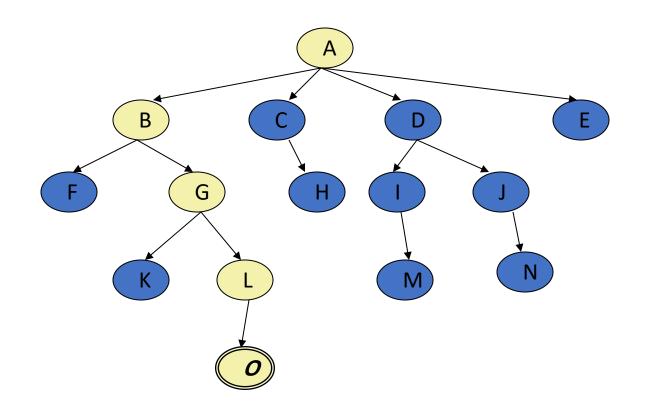






• The returned solution is the sequence of operators in the path:

A, B, G, L, O



Basic Search Algorithms

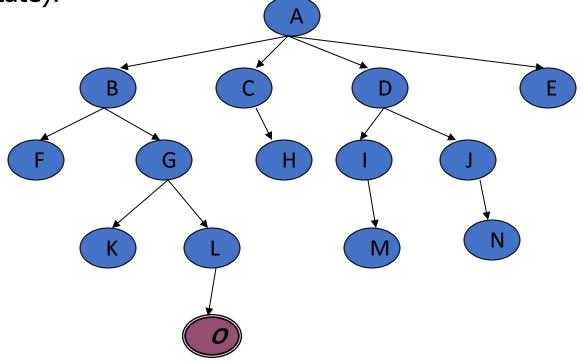
Depth First Search

DFS

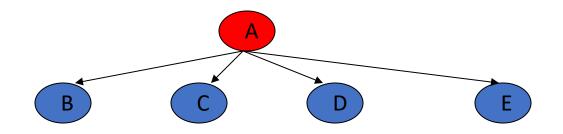
Depth First Search (DFS)

• Application2:

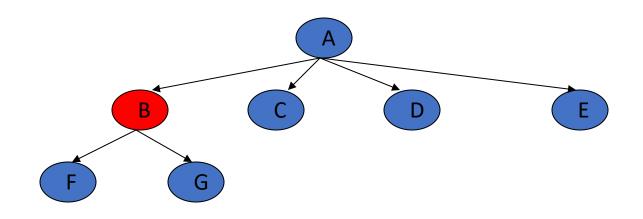
Given the following state space (tree search), give the sequence of visited nodes when using DFS (assume that the node *O* is the goal state):



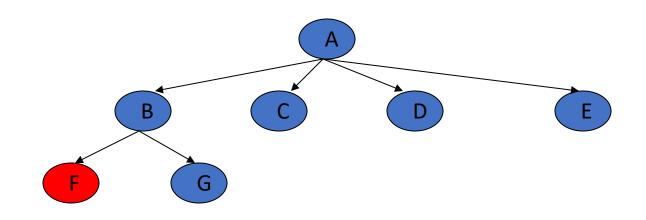
A,



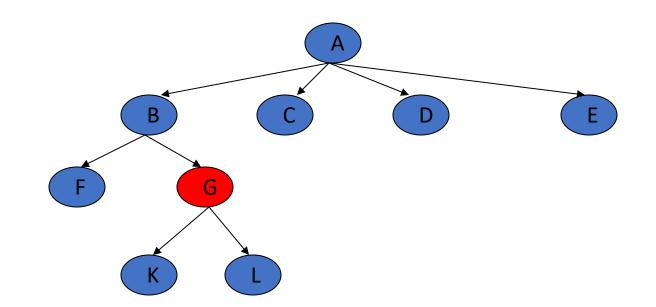
A,B,



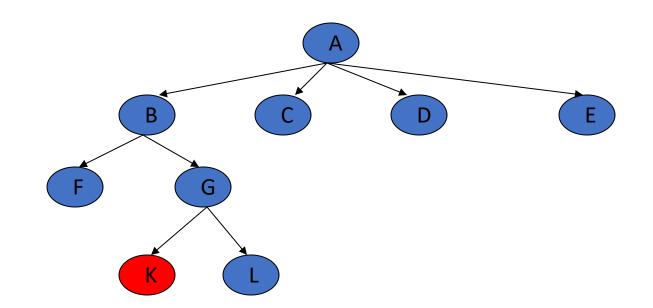
A,B,F,

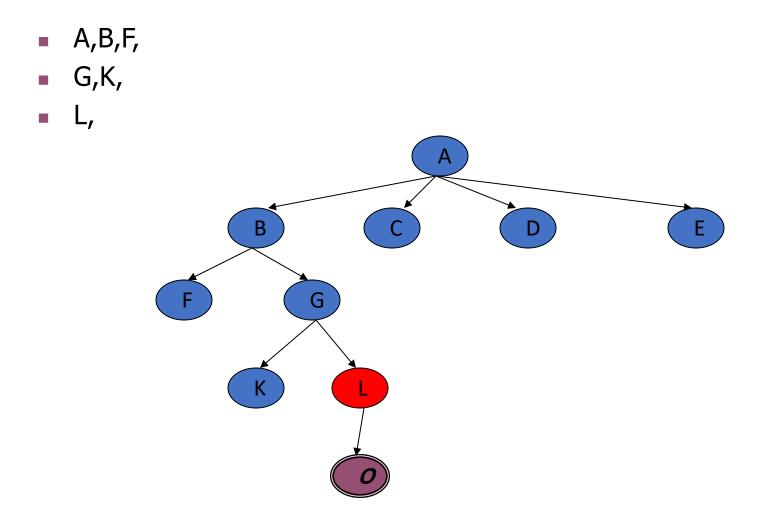


- A,B,F,
- G,

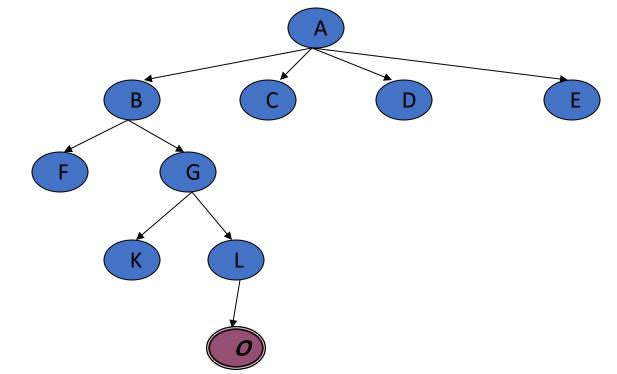


- A,B,F,
- G,K,

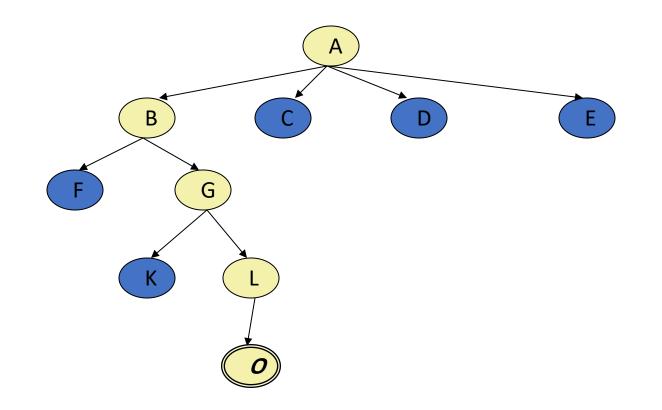




- A,B,F,
- G,K,
- L, O: Goal State



The returned solution is the sequence of operators in the path: *A*, *B*, *G*, *L*, *O*



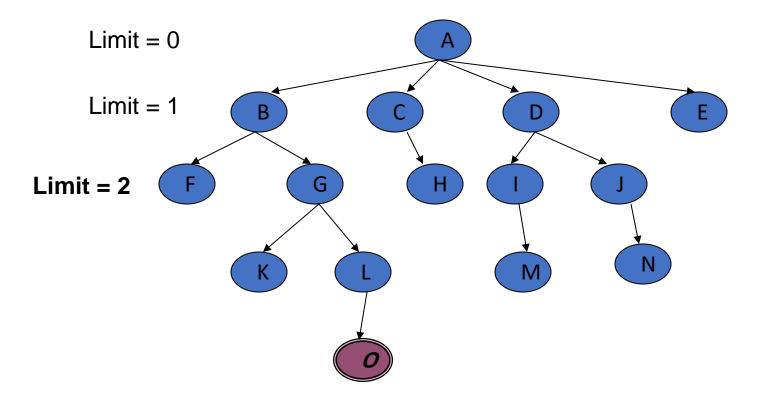
Basic Search Algorithms

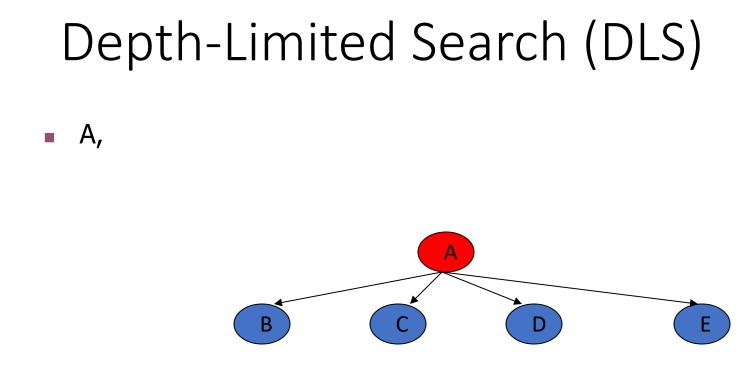
Depth-Limited Search

DLS

• Application3:

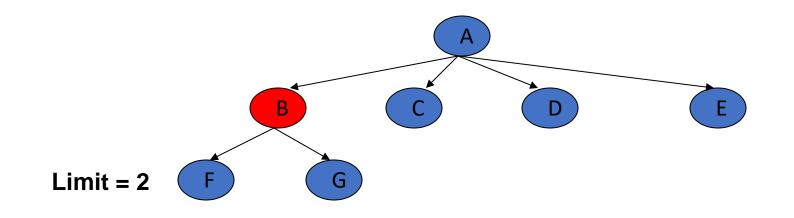
Given the following state space (tree search), give the sequence of visited nodes when using DLS (Limit = 2):



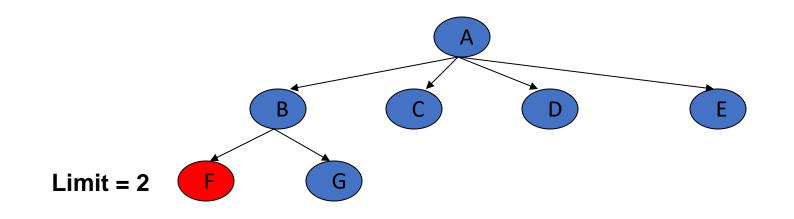


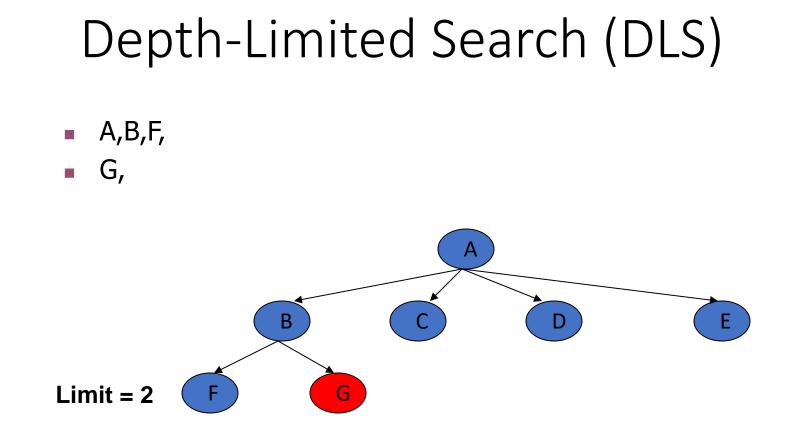
Limit = 2

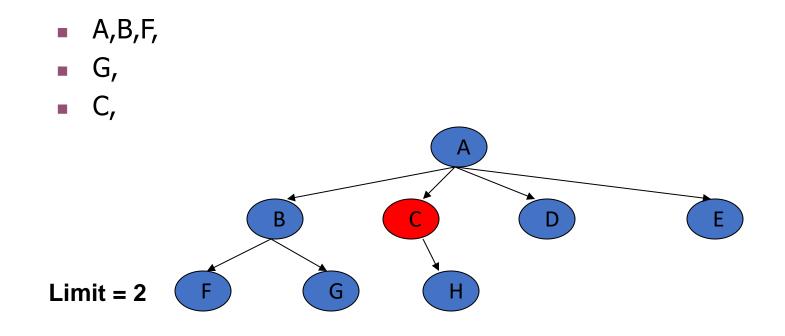
A,B,

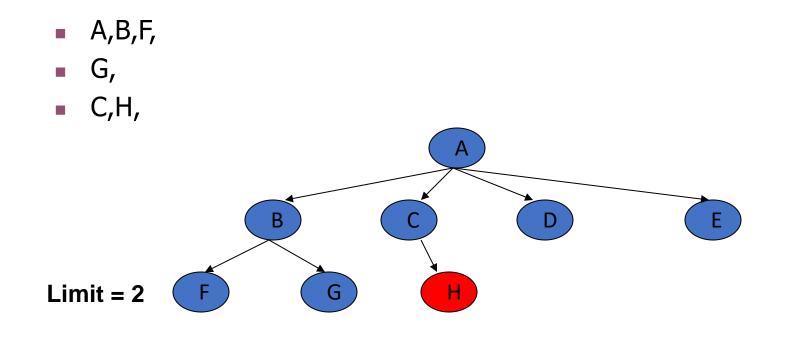


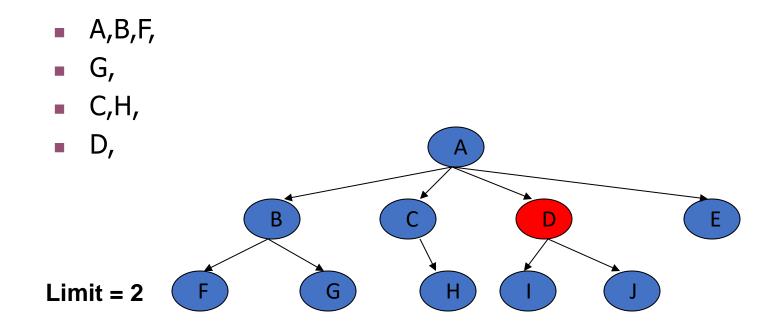
A,B,F,

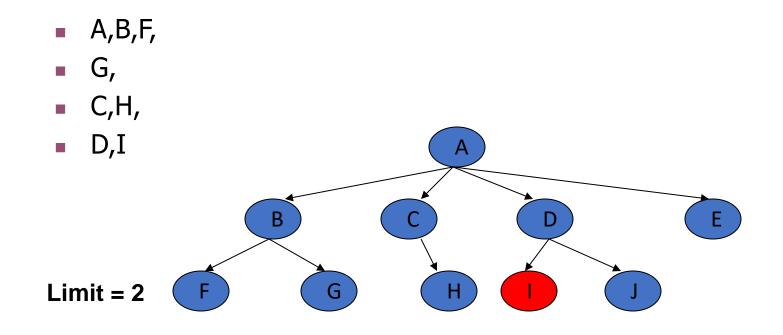


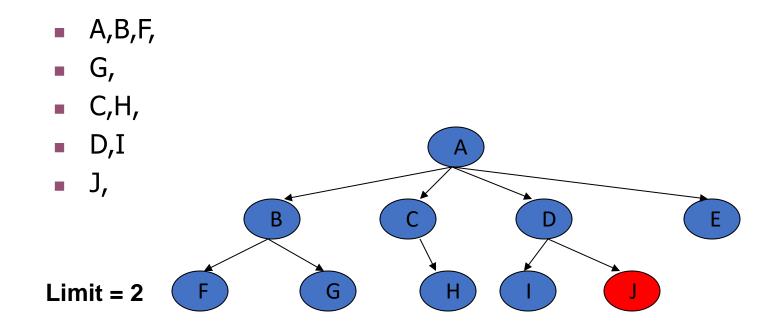


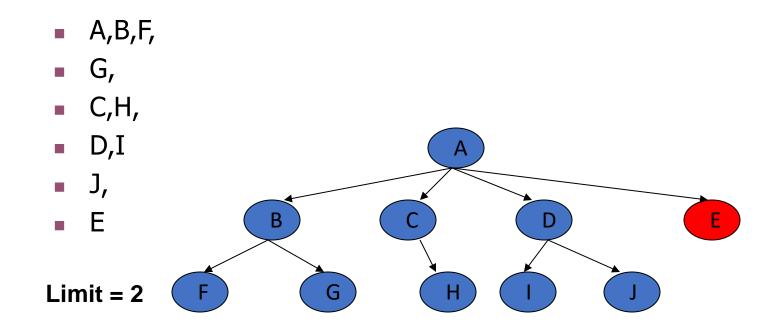


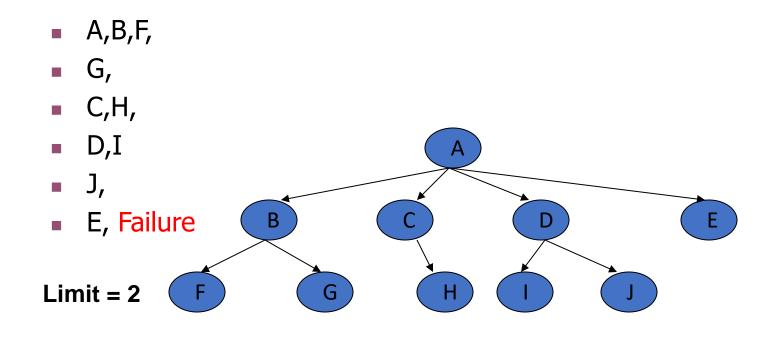




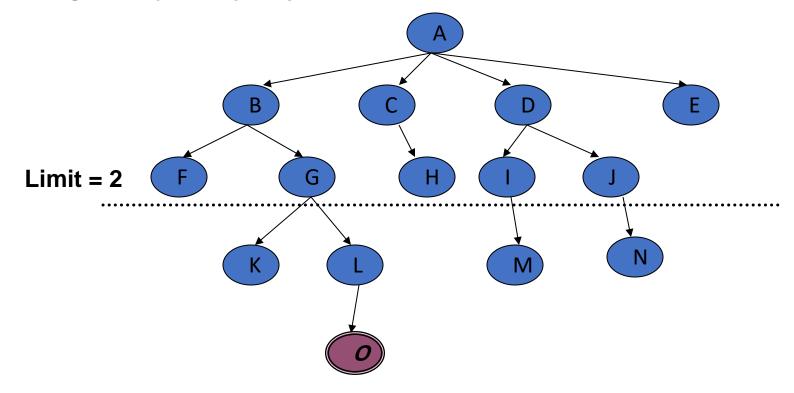








- DLS algorithm returns Failure (no solution)
- The reason is that the goal is beyond the limit (Limit =2): the goal depth is (d=4)



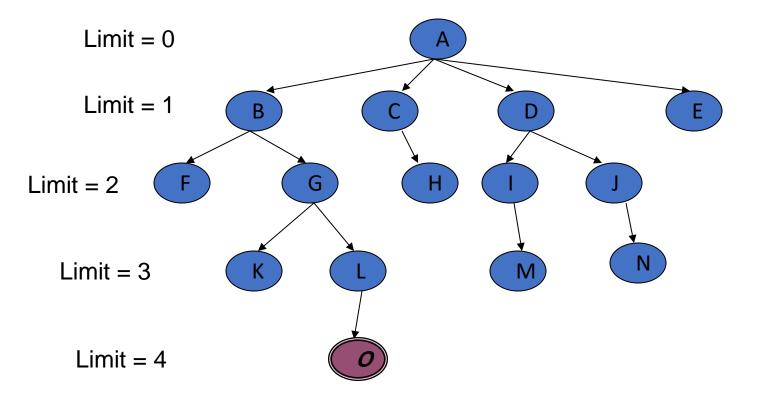
Basic Search Algorithms

Iterative Deepening Search

IDS

• Application4:

Given the following state space (tree search), give the sequence of visited nodes when using IDS:



DLS with bound = 0

Limit = 0

A,

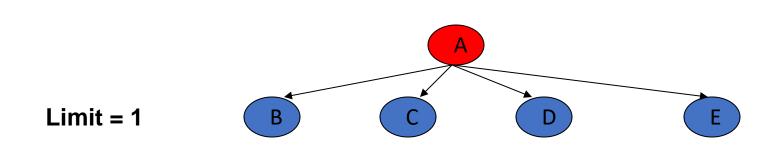


• A, Failure

Limit = 0

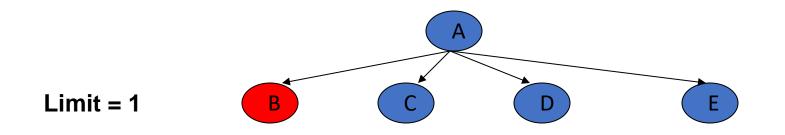


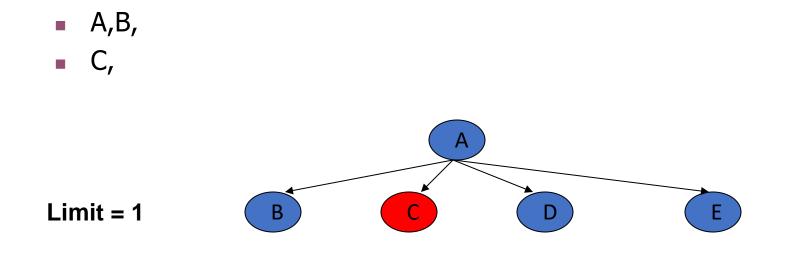
DLS with bound = 1

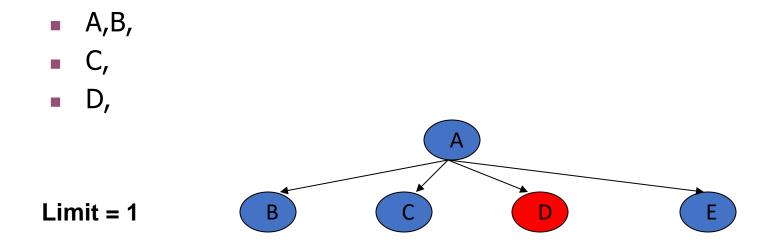


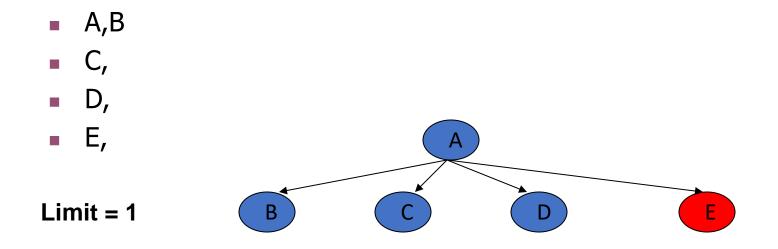
A,

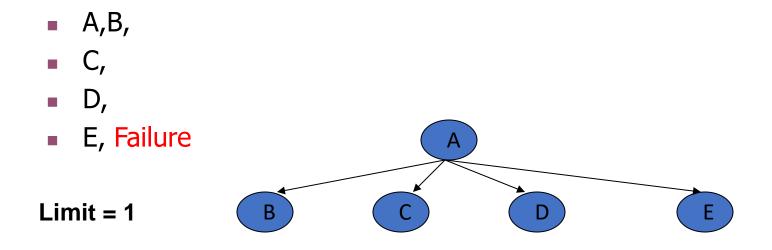
A,B,

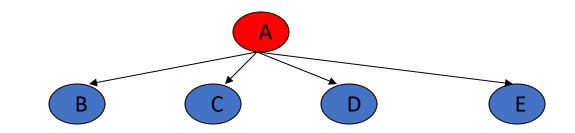








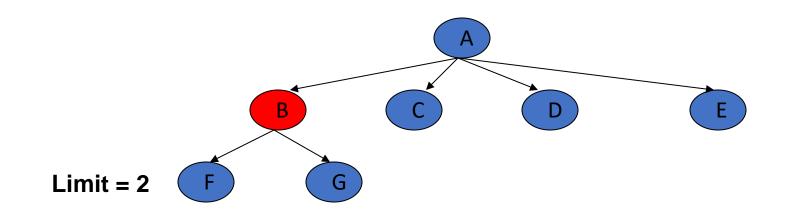




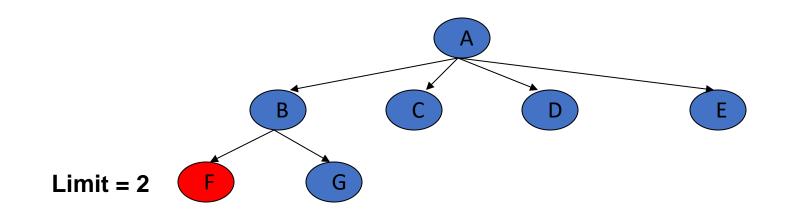
Limit = 2

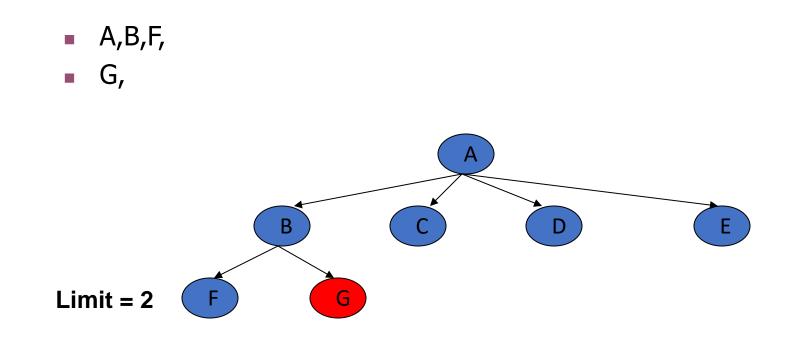
A,

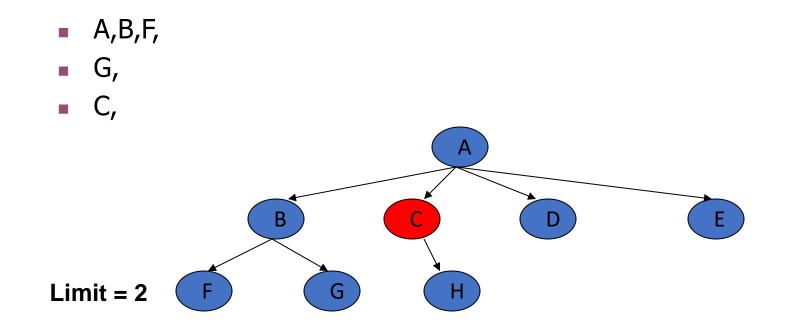
A,B,

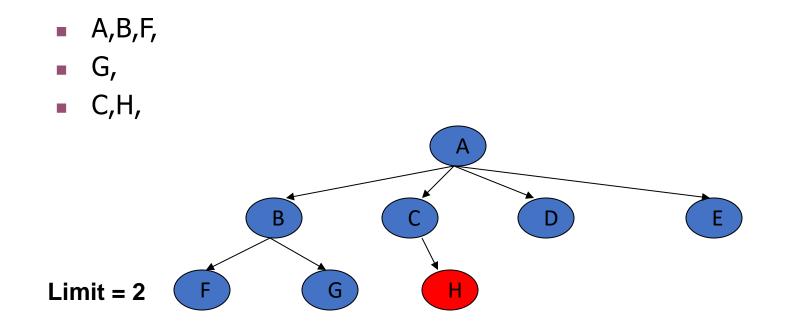


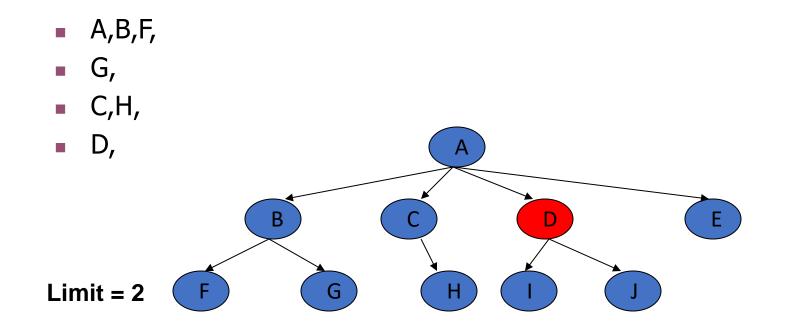
A,B,F,

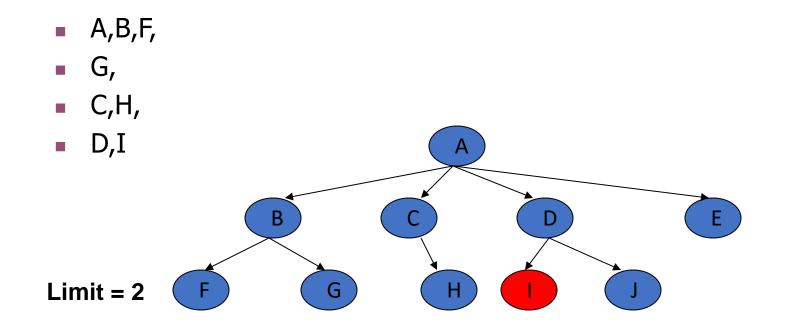


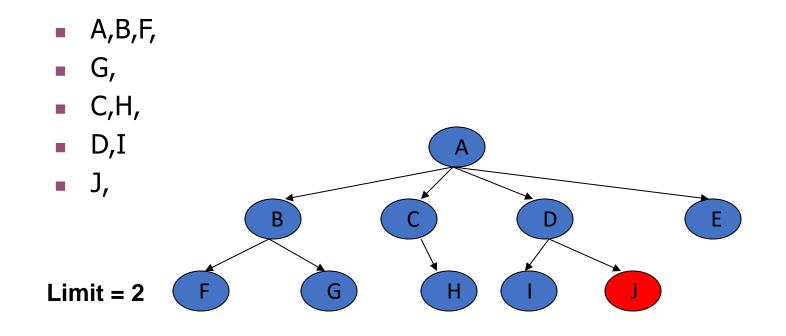


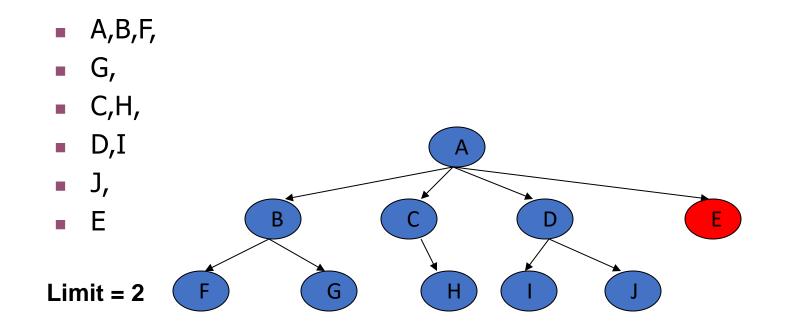


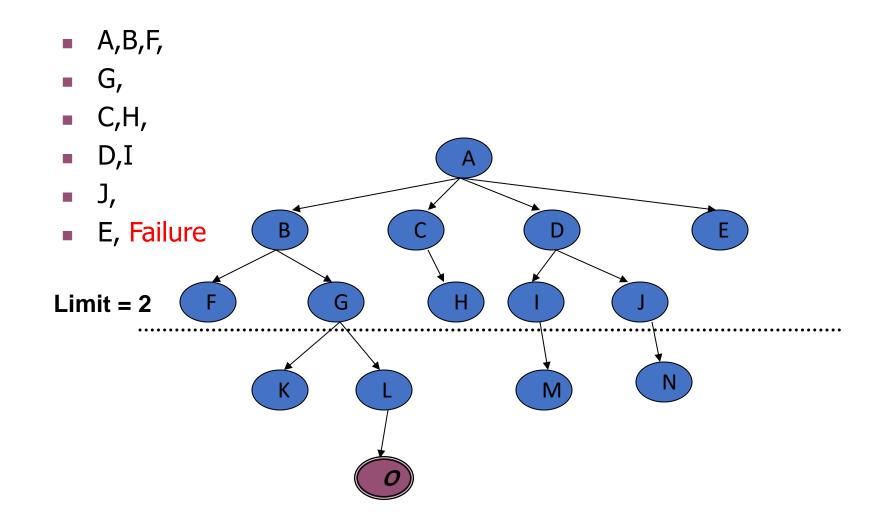




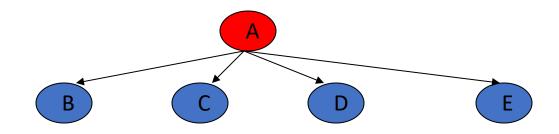








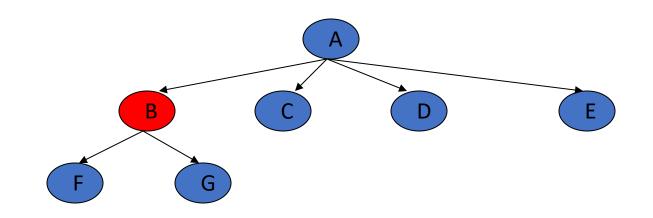
DLS with bound = 3



Limit = 3

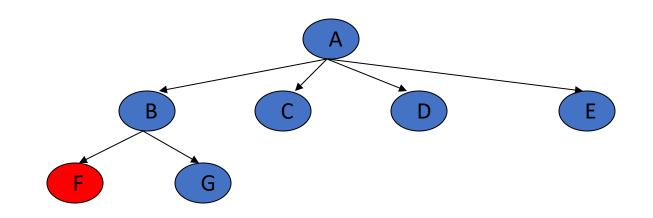
A,

A,B,

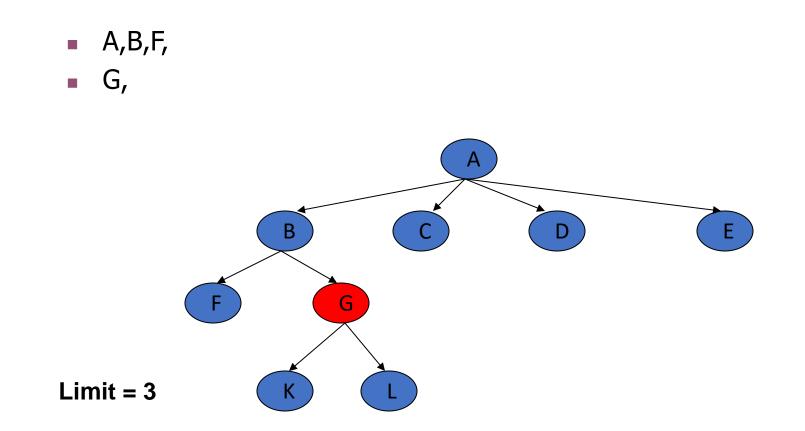


Limit = 3

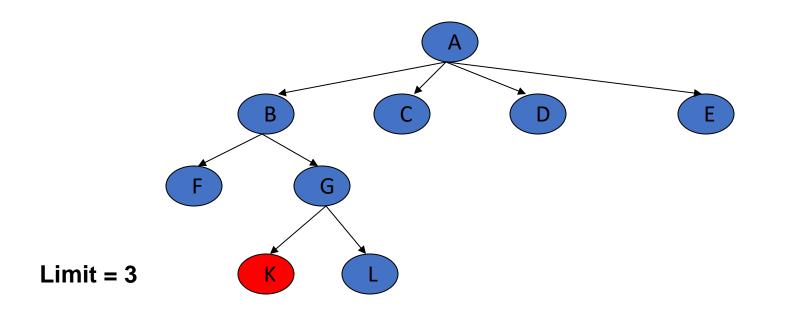
A,B,F,

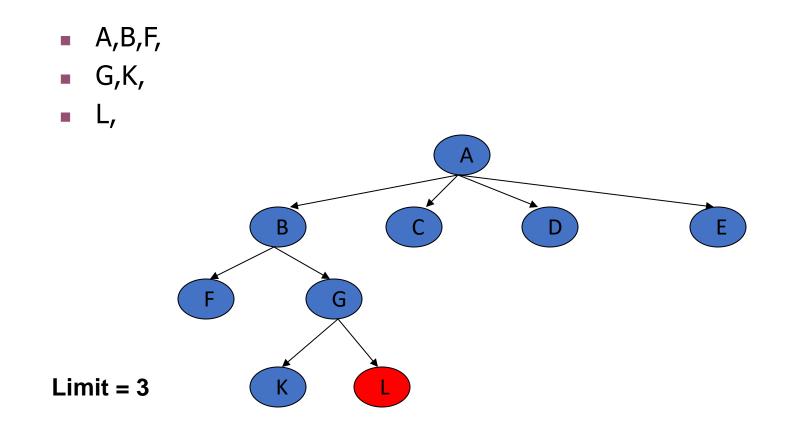


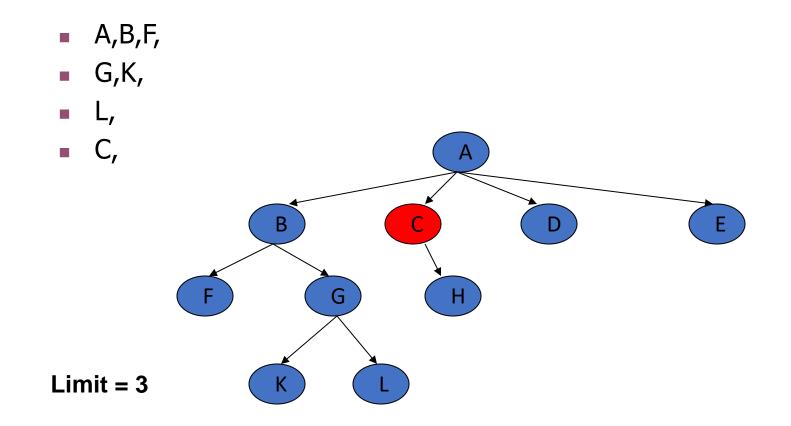
Limit = 3

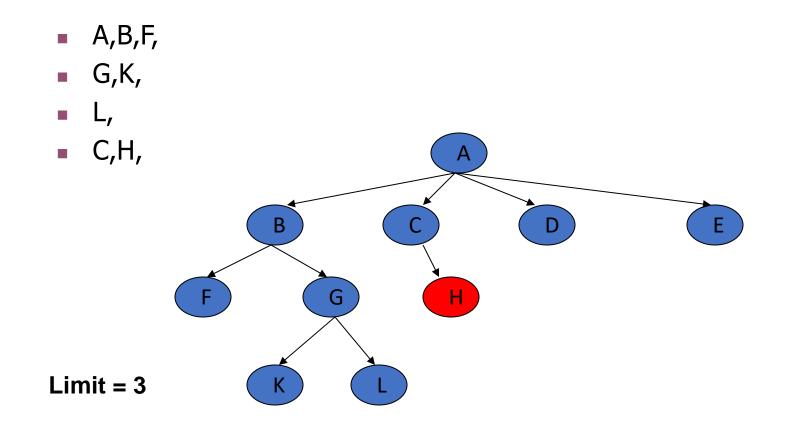


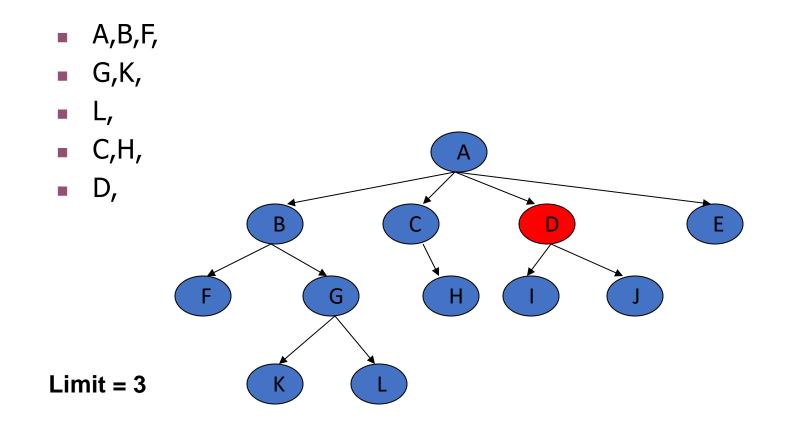
- A,B,F,
- G,K,

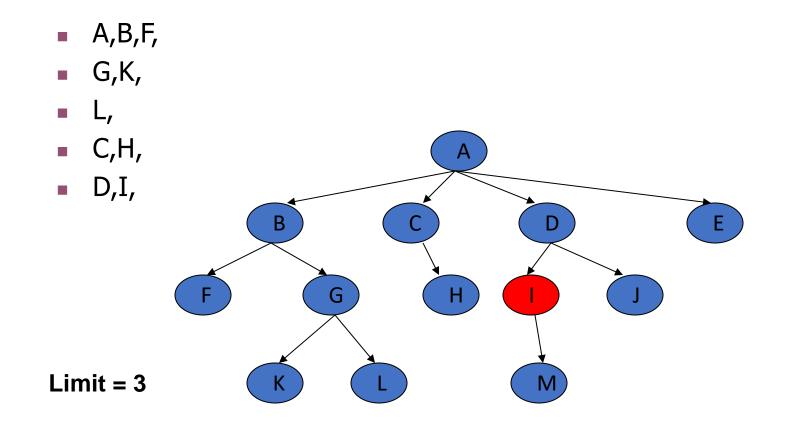


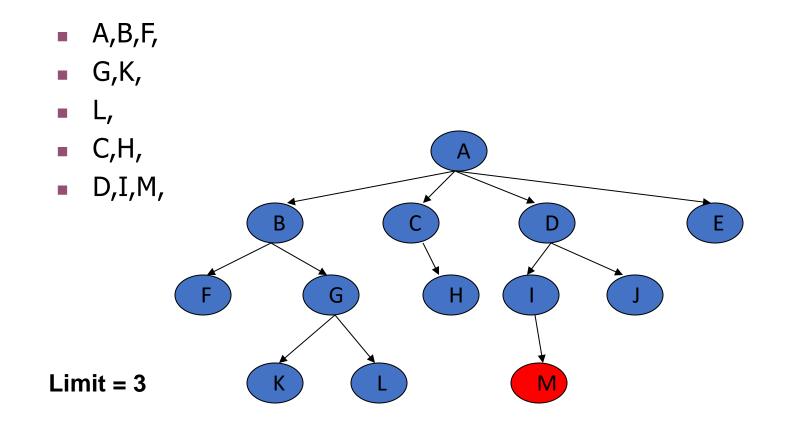


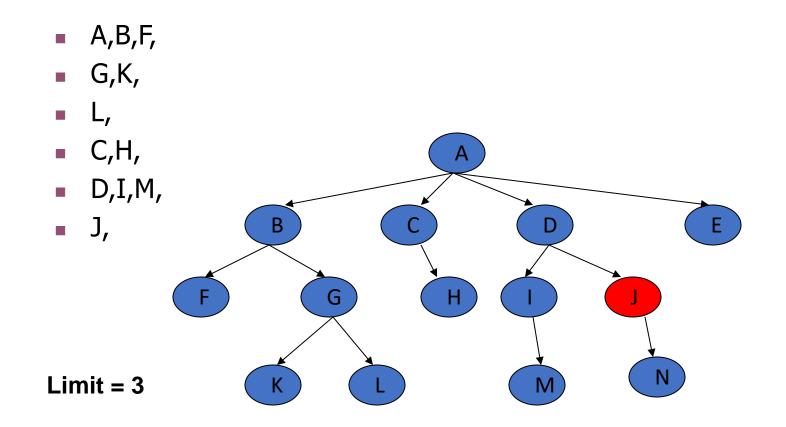


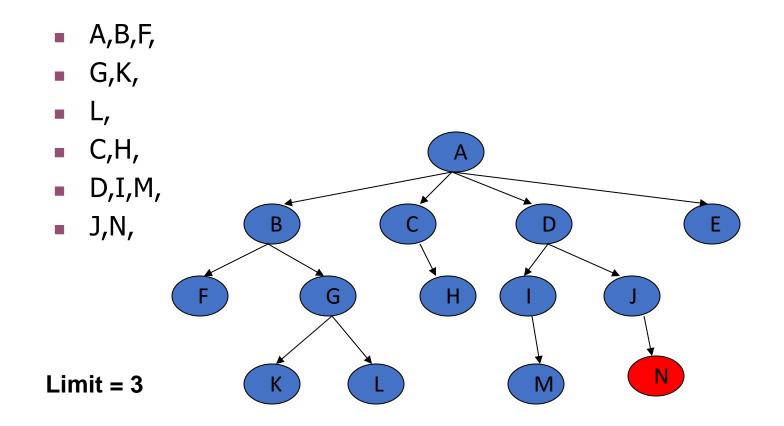


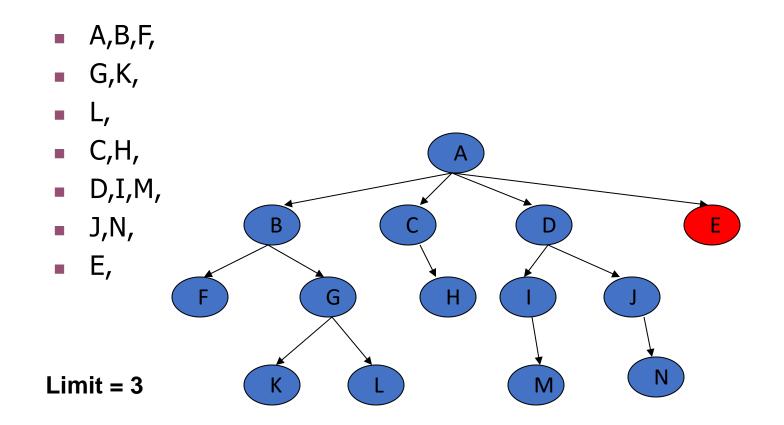


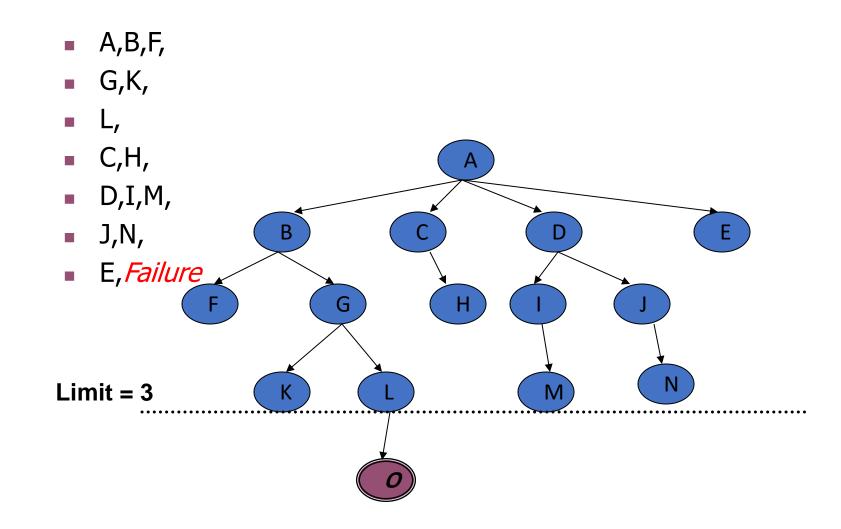










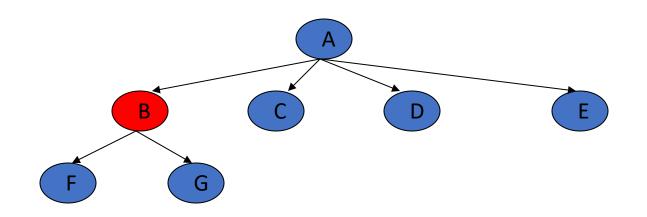


DLS with bound = 4

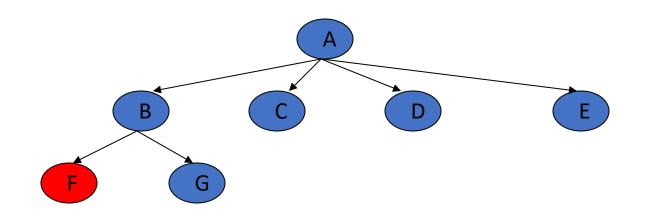
B C D E

A,

A,B,



A,B,F,

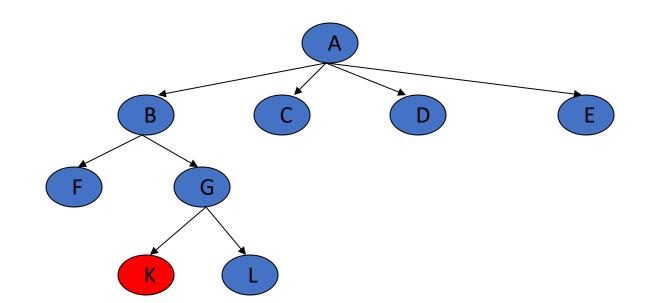


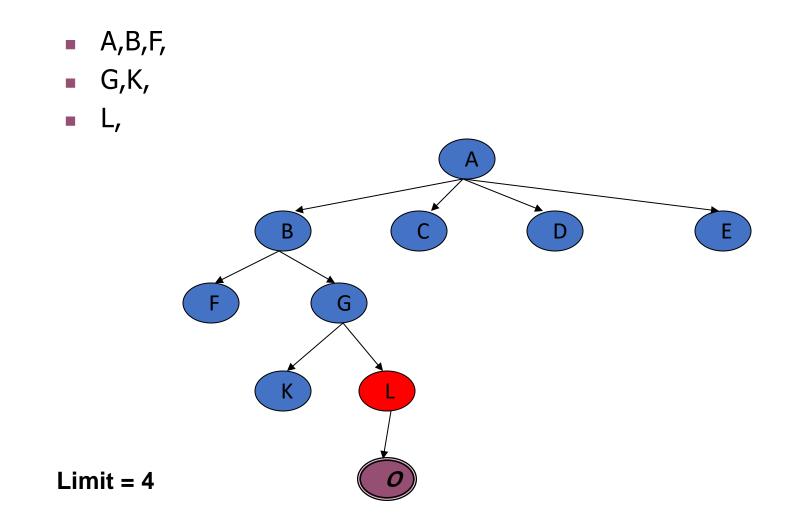
A,B,F,

• G,

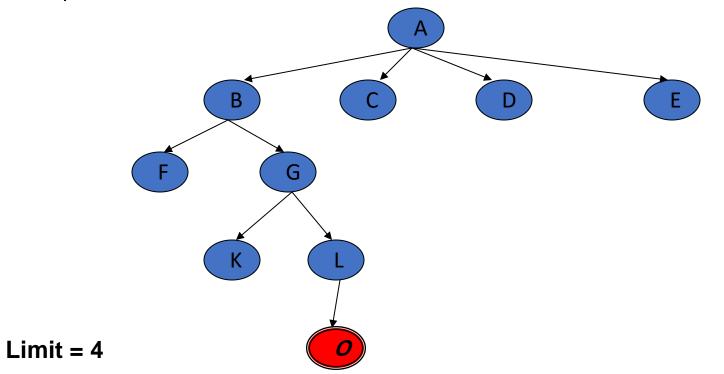
A,B,F,

• G,K,

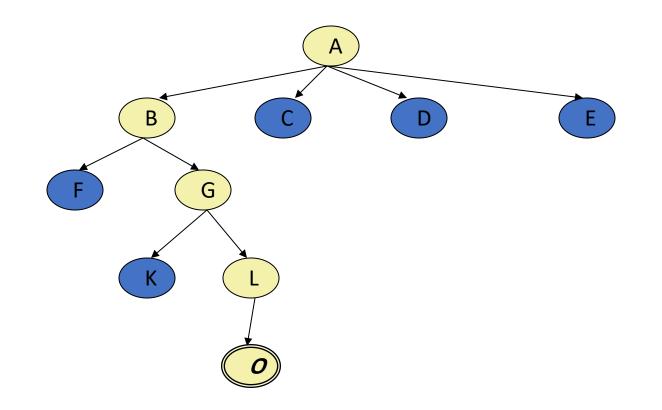




- A,B,F,
- G,K,
- L, O: Goal State



The returned solution is the sequence of operators in the path: *A*, *B*, *G*, *L*, *O*



Summary

- ✓ Search: process of constructing sequences of actions that achieve a goal given a problem.
- The studied methods assume that the environment is observable, deterministic, static and completely known.
- Goal formulation is the first step in solving problems by searching. It facilitates problem formulation.
- Formulating a problem requires specifying four components: Initial states, operators, goal test and path cost function. Environment is represented as a state space.
- ✓ A solution is a path from the initial state to a goal state.
- Search algorithms are judged on the basis of completeness, optimality, time complexity and space complexity.
- ✓ Several search strategies: BFS, DFS, DLS, IDS,...
- All uninformed searches have an exponential time complexity hopeless as a viable problem solving mechanism (unless you have a quantum computer!)