Chapter 6

Ultimate Bearing Capacity of Shallow Foundations

Omitted parts:

Section 6.7, 6.8

Ultimate Bearing Capacity of Shallow Foundations

To perform satisfactorily, shallow foundations must have two main characteristics:

1. **They have to be safe against overall shear failure in the soil that supports them.**
2. **They cannot undergo excessive displacement, or excessive settlement.**

The term excessive is relative, because the degree of settlement allowed for a structure depends on several considerations.

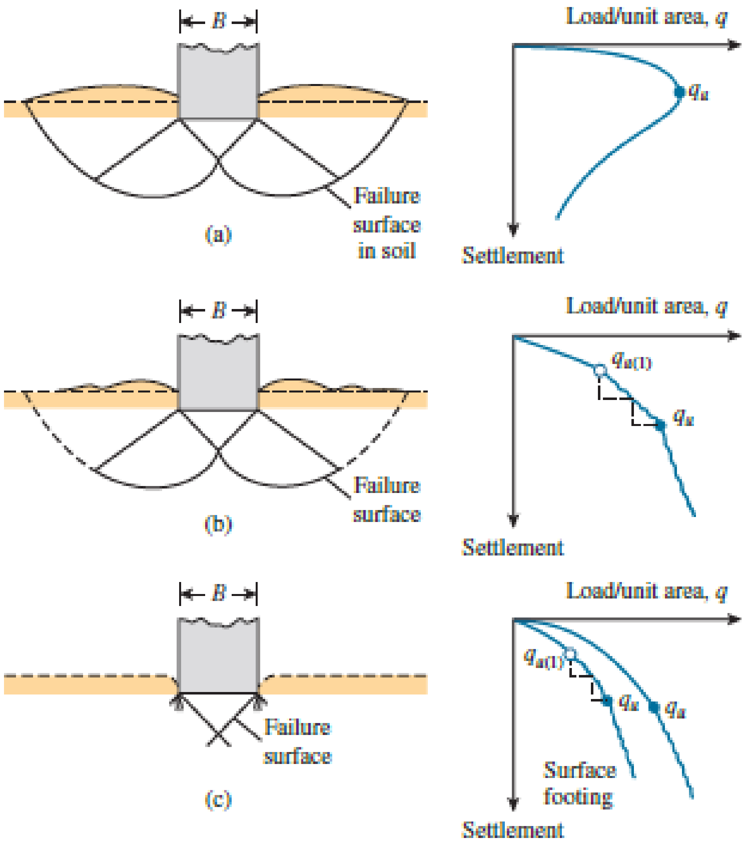
TYPES OF SHEAR FAILURE

Types of Shear Failure

Shear Failure: Also called "Bearing capacity failure" and it's occur when the shear stresses in the soil exceed the shear strength of the soil.

There are three types of shear failure in the soil:

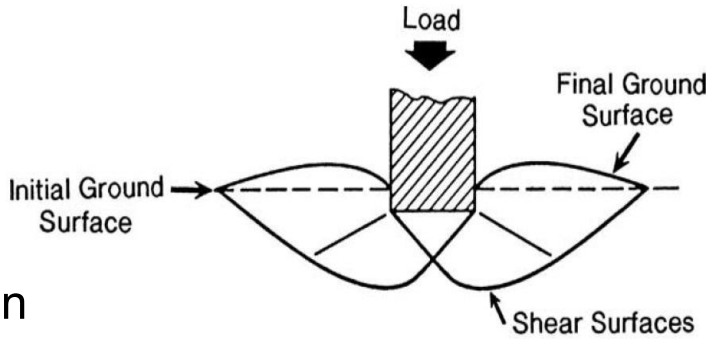
a) **General Shear Failure**
b) **Local Shear Failure**
c) **Punching Shear Failure**

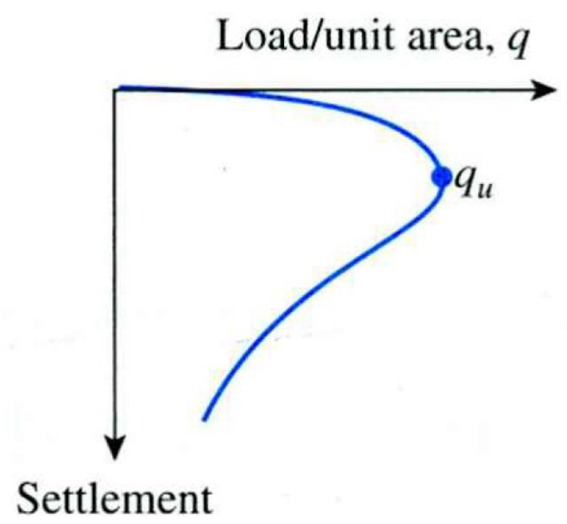


GENERAL SHEAR FAILURE

The following are some characteristics of general shear failure:

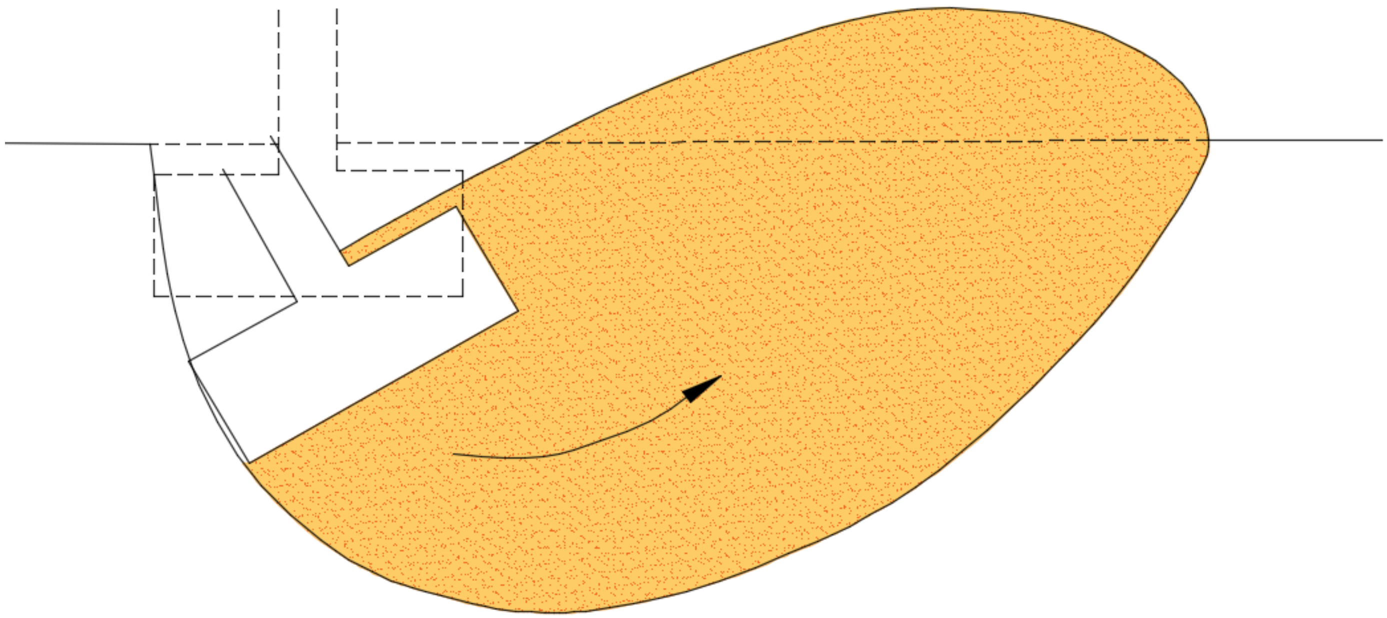
- ❑ Occurs over dense sand or stiff cohesive soil.
- ❑ Involves total rupture of the underlying soil.
- ❑ There is a continuous shear failure of the soil from below the footing to the ground surface (solid lines in the figure).
- ❑ When the (load / unit area) plotted versus settlement of the footing, there is a distinct load at which the foundation fails (Q_u)
- ❑ The value of (Q_u) divided by the area of the footing is considered to be the ultimate bearing capacity of the footing(q_u).
- ❑ For general shear failure, the ultimate bearing capacity has been defined as the bearing stress that causes **a sudden** catastrophic failure of the foundation.
- ❑ As shown in the figure, a general shear failure ruptures occur and pushed up the soil on both sides of the footing (In laboratory).



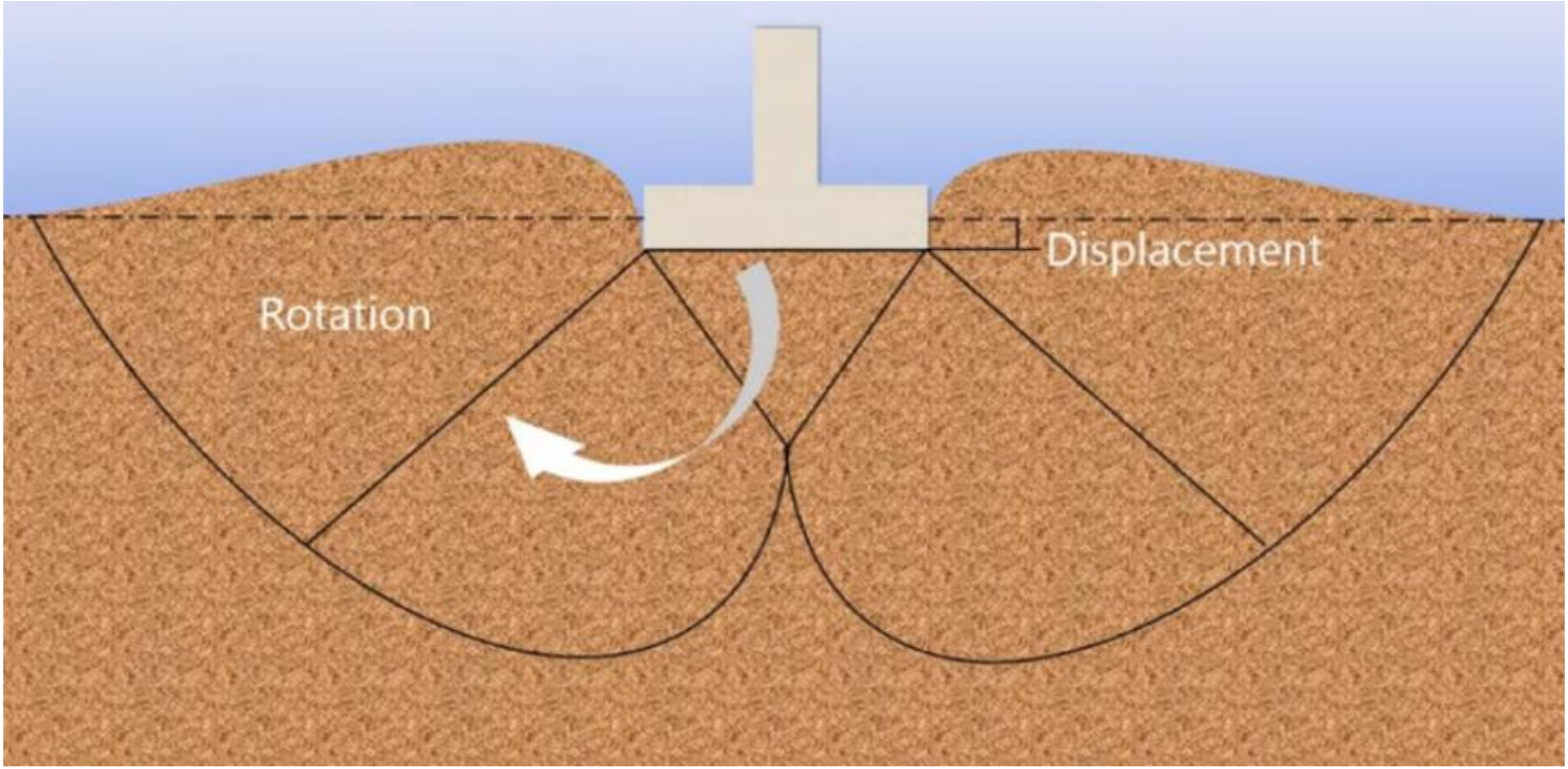


GENERAL SHEAR FAILURE

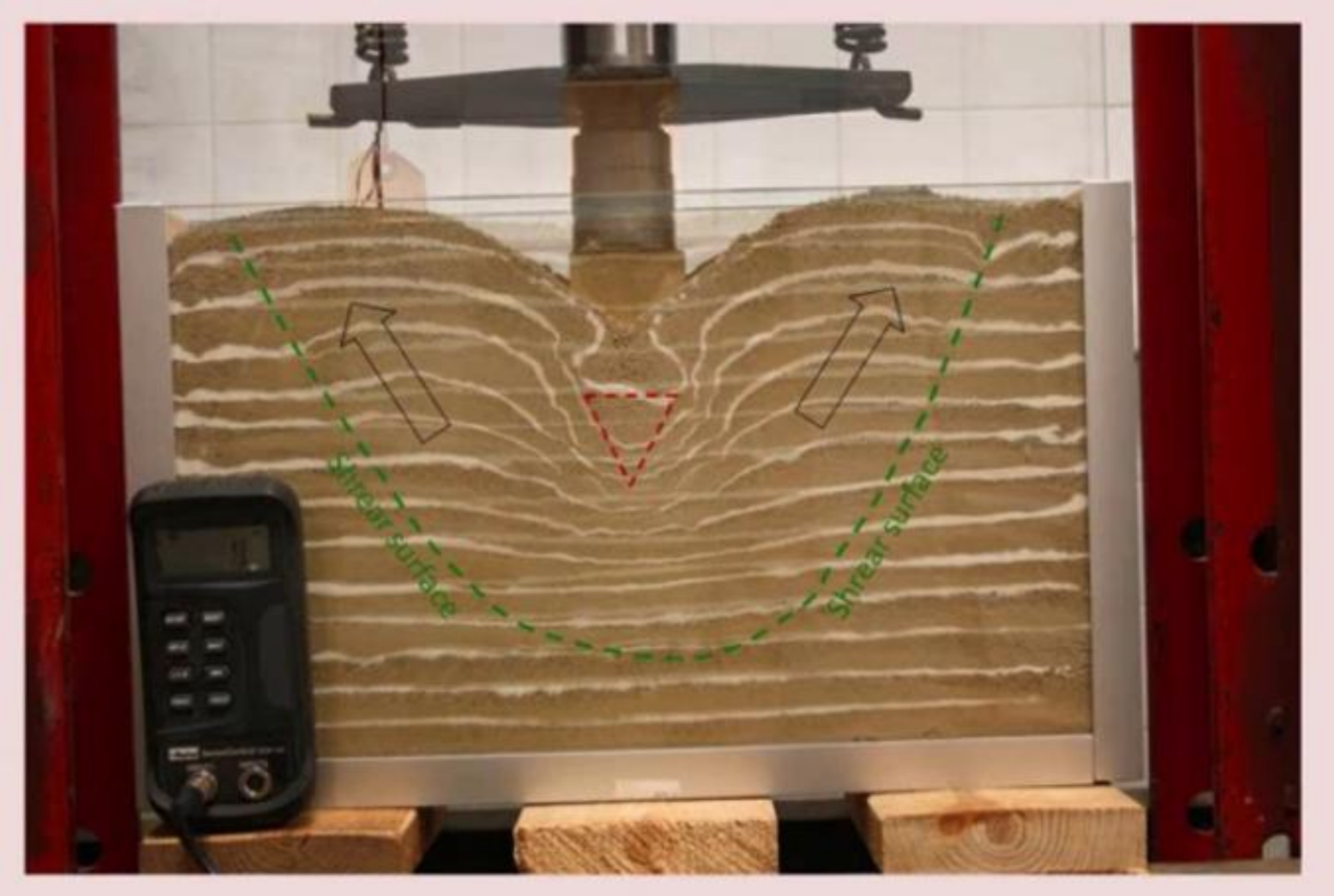
For actual failures on the field, the soil is often pushed up on **only one side** of the footing with **subsequent tilting** of the structure as shown in figure below:



GENERAL SHEAR FAILURE



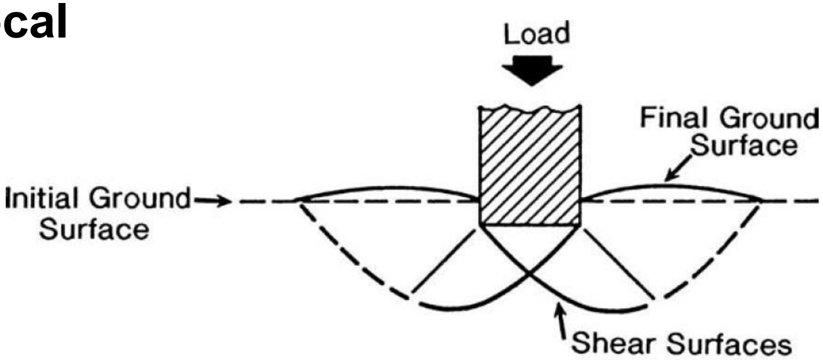
GENERAL SHEAR FAILURE

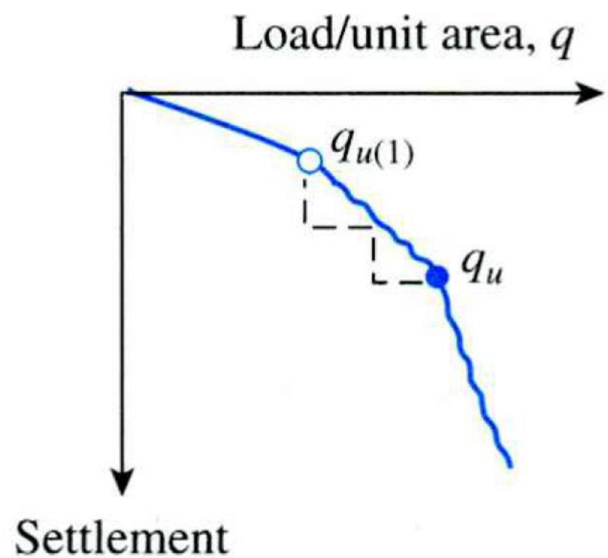


LOCAL SHEAR FAILURE

The following are some characteristics of local shear failure:

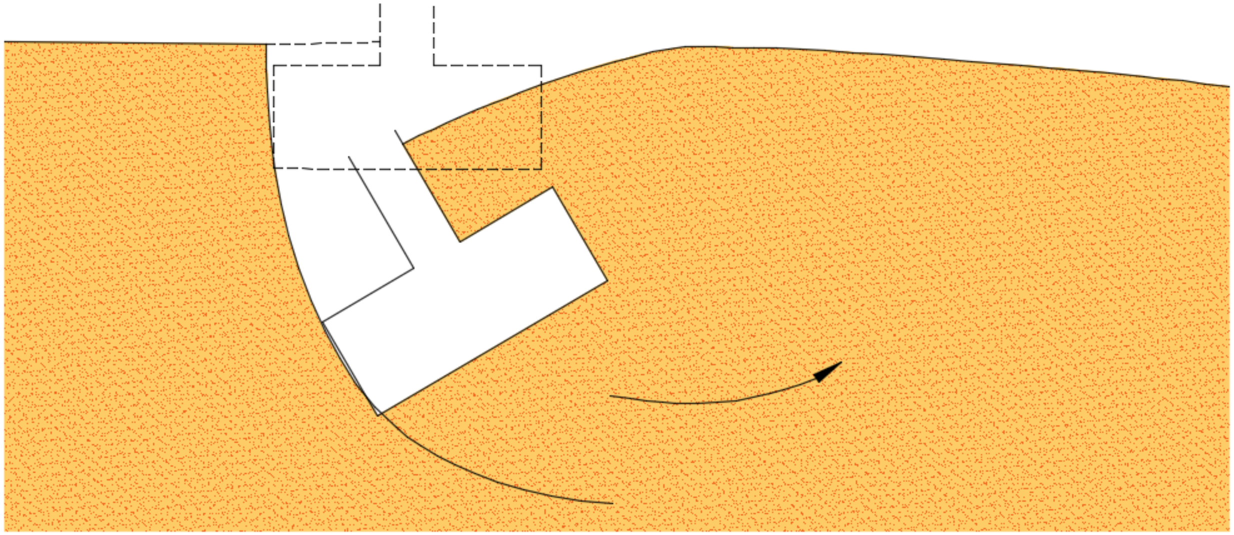
- ❑ Occurs over sand or clayey soil of medium compaction.
- ❑ Involves rupture of the soil only immediately below the footing.
- ❑ There is soil bulging on both sides of the footing, but the bulging is not as significant as in general shear. That's because the underlying soil compacted less than the soil in general shear.
- ❑ The failure surface of the soil will **gradually** (not sudden) extend outward from the foundation (not the ground surface) as shown by **solid lines** in the figure.
- ❑ So, local shear failure can be considered as a transitional phase between general shear and punching shear.



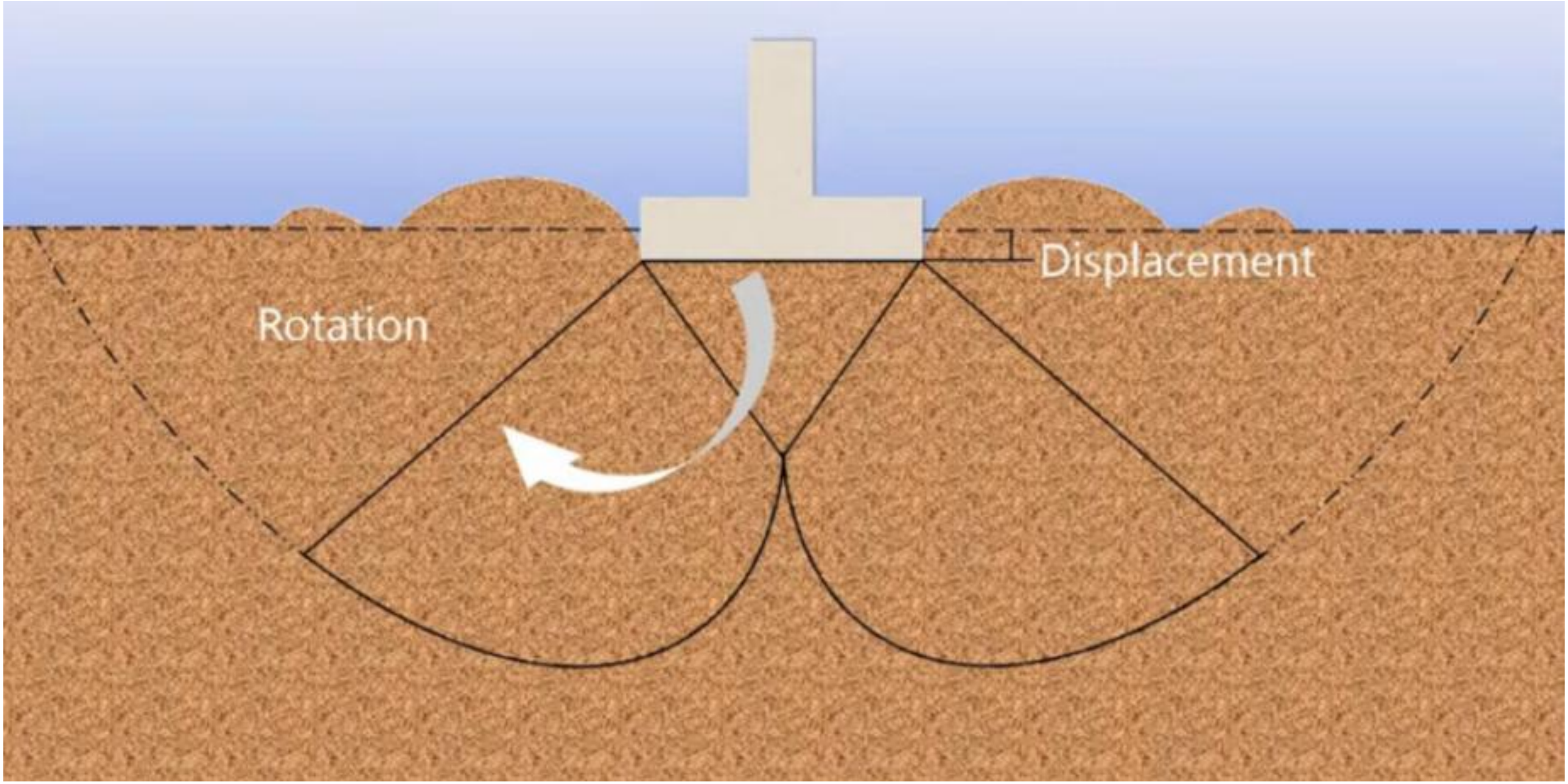


LOCAL SHEAR FAILURE

- Because of the transitional nature of local shear failure, the ultimate bearing capacity could be defined as the firs failure load (q_u,1) which occur at the point which have the first measure nonlinearity in the load/unit area-settlement curve (open circle), or at the point where the settlement starts rabidly increase (q_u) (closed circle).
- This value of (q_u) is the required (load/unit area) to extends the failure surface to the ground surface (dashed lines in the figure).
- In this type of failure, the value of (q_u) is not the peak value so, this failure called (Local Shear Failure).
- The actual local shear failure in field is proceed as shown in the figure below:



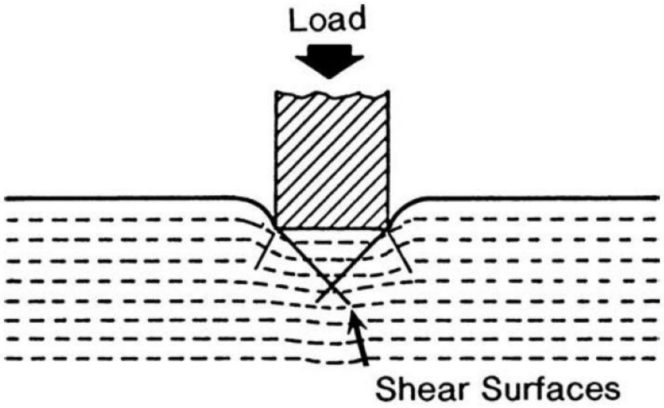
LOCAL SHEAR FAILURE

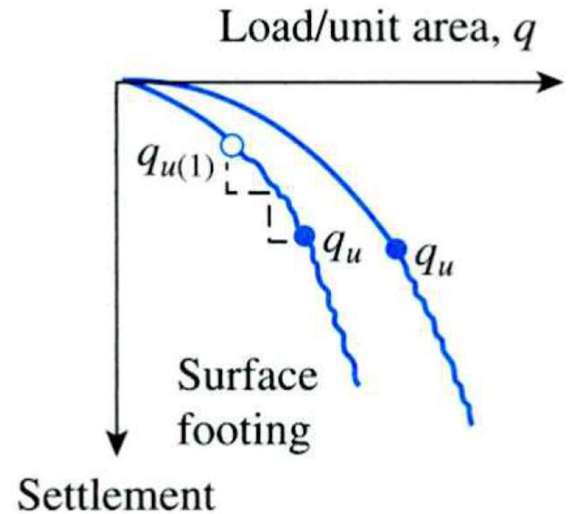


PUNSHING SHEAR FAILURE

The following are some characteristics of punching shear failure:

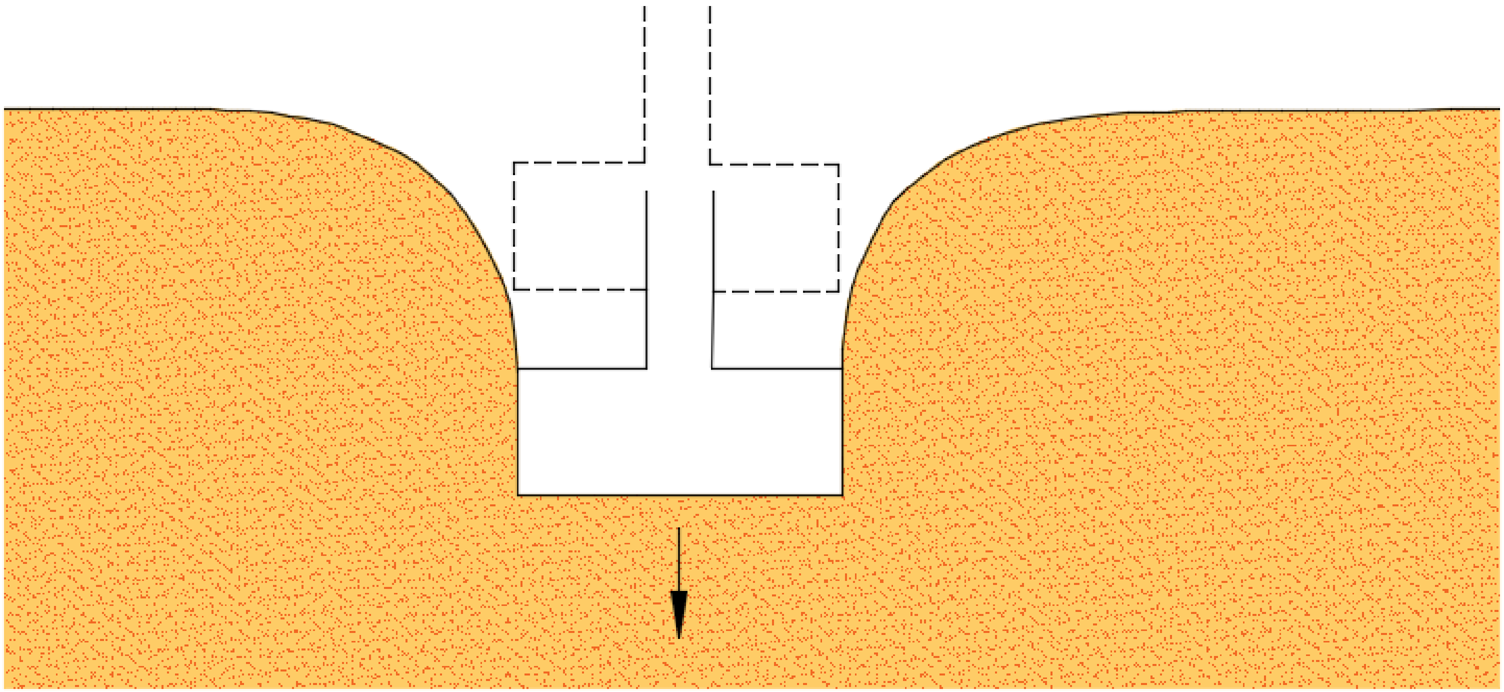
- Occurs over fairly loose soil.
- Punching shear failure does not develop the distinct shear surfaces associated with a general shear failure.
- The soil outside the loaded area remains relatively uninvolved and there is a minimal movement of soil on both sides of the footing.
- The process of deformation of the footing involves compression of the soil directly below the footing as well as the vertical shearing of soil around the footing perimeter.
- As shown in figure, the (q)-settlement curve does not have a dramatic break and the bearing capacity is often defined as the first measure nonlinearity in the (q)-settlement curve(q_u,1).



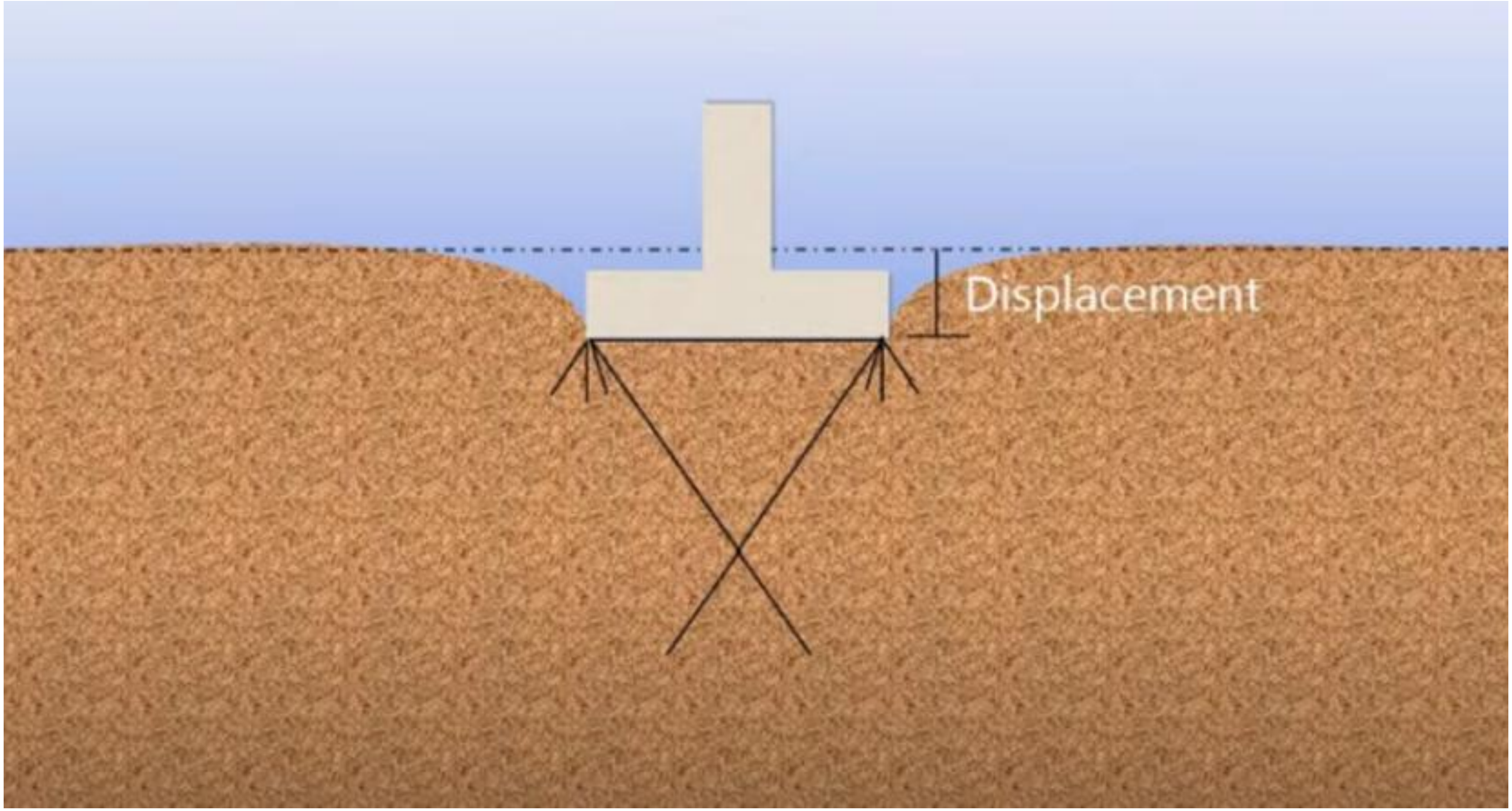


PUNSHING SHEAR FAILURE

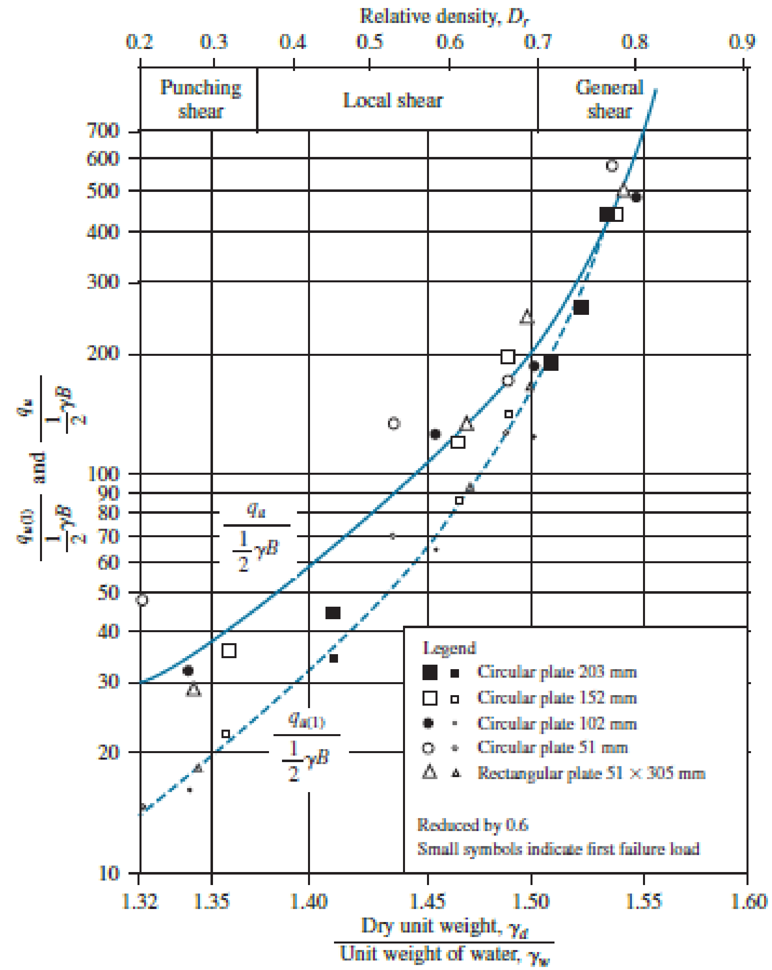
- Beyond the ultimate failure (load/unit area) (q_u,1), the (load/unit area)-settlement curve will be steep and practically linear.
- The actual punching shear failure in field is proceed as shown in the figure below:



PUNSHING SHEAR FAILURE



Foundation Failures in Sand



Variation of $q_{u(1)}/0.5\gamma B$ and $q_u/0.5\gamma B$ for circular and rectangular plates on the surface of a sand (

Modes of Foundation Failure in Sand

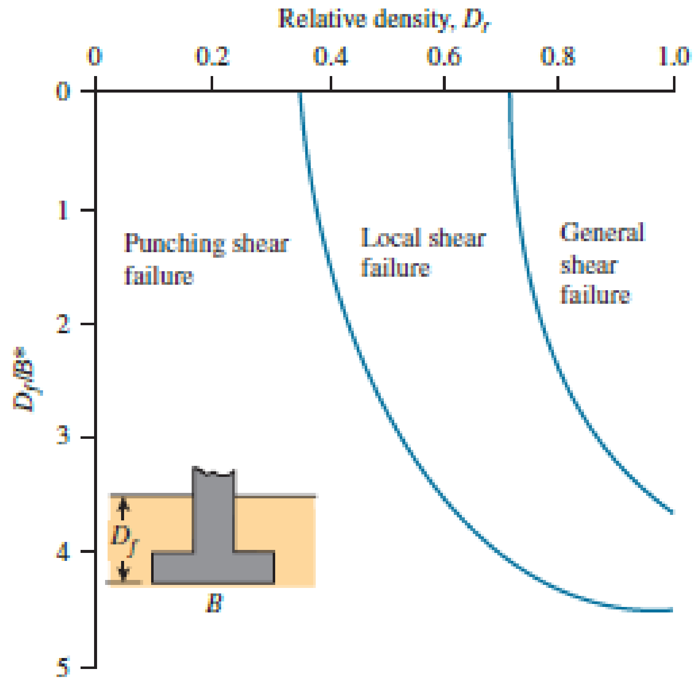
D_r = relative density of sand

D_f = depth of foundation measured from the ground surface

$$B^* = \frac{2BL}{B + L}$$

B = width of foundation

L = length of foundation



Modes of foundation failure in sand (Vesic ,1973)

TERZAGHI'S BEARING CAPACITY THEORY

Terzaghi was the first to present a comprehensive theory for evaluation of the ultimate bearing capacity of rough shallow foundation.
This theory is based on the following assumptions:

1. The foundation is considered to be shallow if ($D_f \leq B$).

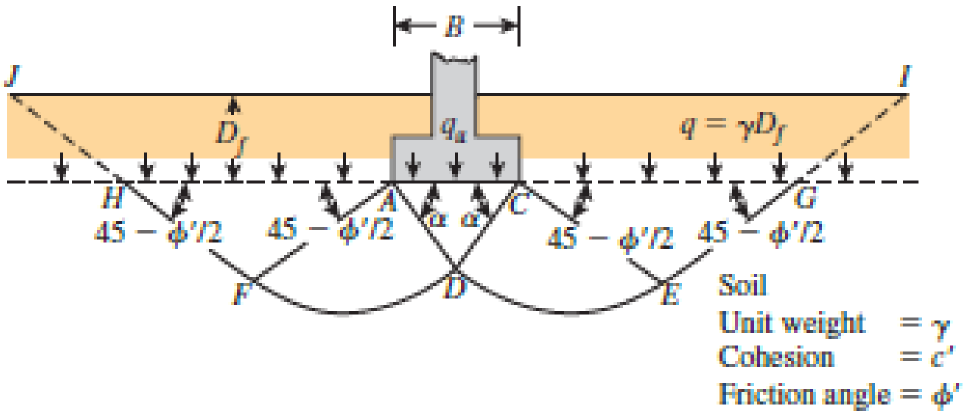
2. The foundation is considered to be strip or continuous if ($B/L \rightarrow 0.0$). (Width to length ratio is very small and goes to zero), and the derivation of the equation is to a strip footing.

3. The effect of soil above the bottom of the foundation may be assumed to be replaced by an equivalent surcharge ($q=\gamma D_f$). So, the shearing resistance of this soil along the failure surfaces is neglected (Lines GI and HJ in the figure)

4. The failure surface of the soil is similar to general shear failure (i.e. equation is derived for general shear failure) as shown in the figure.

Note:
1. In recent studies, investigators have suggested that, foundations are considered to be shallow if [$D_f \leq (3 \rightarrow 4)B$], otherwise, the foundation is deep.
2. Always the value of (q) is the effective stress at the bottom of the foundation.

TERZAGHI'S BEARING CAPACITY THEORY



Bearing capacity failure in soil under a rough rigid continuous (strip) foundation

The failure zone under the foundation can be separated into three parts:

1. **The triangular zone ACD immediately under the foundation**
2. **The radial shear zones ADF and CDE with the curves DE and DF being arcs of a logarithmic spiral**
3. **Two triangular Rankine passive zones AFH and CEG**

TERZAGHI'S BEARING CAPACITY EQUATION

The equation was derived for a strip footing and general shear failure:
(for continuous or strip footing)

$$q_u = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma$$

Where

q_u=Ultimate bearing capacity of the soil (KN/m^2)

c'=Cohesion of soil (KN/m^2)

q = Effective stress at the bottom of the foundation (KN/m^2)

N_c, N_q, N_γ= Bearing capacity factors (non-dimensional) and are functions only of the soil friction angle, ϕ'

The variations of bearing capacity factors and underlying soil friction angle are given in (Table 6.1) for general shear failure.

TERZAGHI'S BEARING CAPACITY FACTORS

TABLE 6.1 Terzaghi's Bearing Capacity Factors–

ϕ'	N_c	N_q	N_γ[a]	ϕ'	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a]From Kumbhojkar (1993)

TERZAGHI'S BEARING CAPACITY EQUATION

The equation above (for strip footing) was modified to be useful for both square and circular footings as following:

For square footing:

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma \quad \text{(square foundation)}$$

B=The dimension of each side of the foundation .

For circular footing:

$$q_u = 1.3c'N_c + qN_q + 0.3\gamma BN_\gamma \quad \text{(circular foundation)}$$

B=The diameter of the foundation .

Note:

These two equations are also for general shear failure, and all factors in the two equations (except, B,) are the same as explained for strip footing.

FACTOR OF SAFETY

Ultimate bearing capacity is the maximum value the soil can bear it.
i.e. if the bearing stress from foundation exceeds the ultimate bearing capacity of the soil, shear failure in soil will be occur.
so we must design a foundation for a bearing capacity less than the ultimate bearing capacity to prevent shear failure in the soil. This bearing capacity is **"Allowable Bearing Capacity"** and we design for it.
i.e. the applied stress from foundation must not exceed the allowable bearing capacity of soil.

$$q_{\text{all, gross}} = \frac{q_{\text{u, gross}}}{FS}$$

$q_{all,gross}$ = Gross allowable bearing capacity
$q_{u,gross}$ = Gross ultimate bearing capacity (Terzaghi equation)
FS = Factor of safety for bearing capacity ≥3

However, practicing engineers prefer to use the "**net** allowable bearing capacity" such that:

$$q_{\text{all (net)}} = \frac{q_{\text{u}} - q}{FS}$$

$$q = \gamma D_f$$

Example 6.1

EXAMPLE 6.1

A 2.0 m wide strip foundation is placed at a depth of 1.5 m within a sandy clay, where $c' = 10\ \text{kN/m}^2$, $\phi' = 26°$, and $\gamma = 19.0\ \text{kN/m}^3$. Determine the maximum wall load that can be allowed on the foundation with a factor of safety of 3, assuming general shear failure. Use gross values.

SOLUTION

From Eq. (6.10),

$$q_u = c'N_c + qN_q + 0.5\gamma BN_\gamma$$

From Table 6.1, $N_c = 27.09$, $N_q = 14.21$, and $N_\gamma = 9.84$. Thus,

$$q_u = (10)(27.09) + (19.0 \times 1.5)(14.21) + (0.5)(19.0)(2.0)(9.84) = 862.8\ \text{kN/m}^2$$

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{862.8}{3} = 287.6\ \text{kN/m}^2$$

Therefore, the maximum allowable load $Q = 287.6 \times 2 =$ **575 kN/m**.

■

Example 6.2

EXAMPLE 6.2

A design requires placing a square foundation at 1.0 m depth to carry a column load of 1500 kN. The soil properties are: $c' = 15$ kN/m^2, $\phi' = 24°$, and $\gamma = 18.5$ kN/m^3. What should be the width B of the foundation?

SOLUTION

From Eq. (6.19),

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma$$

From Table 6.1, $N_c = 23.36$, $N_q = 11.40$, and $N_\gamma = 7.08$.

$$q_u = (1.3)(15)(23.36) + (18.5 \times 1.0)(11.40) + (0.4)(18.5)(B)(7.08)$$
$$= 52.4\,B + 666.4 \text{ kN/m}^2$$

$$q_{all} = \frac{q_u}{FS} = \frac{52.4B + 666.4}{3} = 17.5B + 222.1$$

The applied pressure to the ground is $\frac{1500}{B^2}$ kN/m^2. Therefore, $\frac{1500}{B^2} = 17.5B + 222.1$.

By trial and error (or use of a graphics calculator), $B = 2.4$ m.

■

Modification of Bearing Capacity Equations for Water Table

Terzaghi equation gives the ultimate bearing capacity based on the assumption that the water table is located well below the foundation.

However, if the water table is close to the foundation, the bearing capacity will decrease due to the effect of water table, so, some modification of the bearing capacity equation will be necessary.

$$q_u = c'N_c + qN_q + \frac{1}{2}\gamma B N_\gamma$$

The values which will be modified are:

1. q (for soil above the foundation) in the second term of equation.

2. γ (for the underlying soil) in the third term of equation .

Modification of Bearing Capacity Equations for Water Table

There are three cases according to location of water table:

Case I. If the water table is located so that $0 \le D_1 \le D_f$, the factor q in the bearing capacity equations takes the form

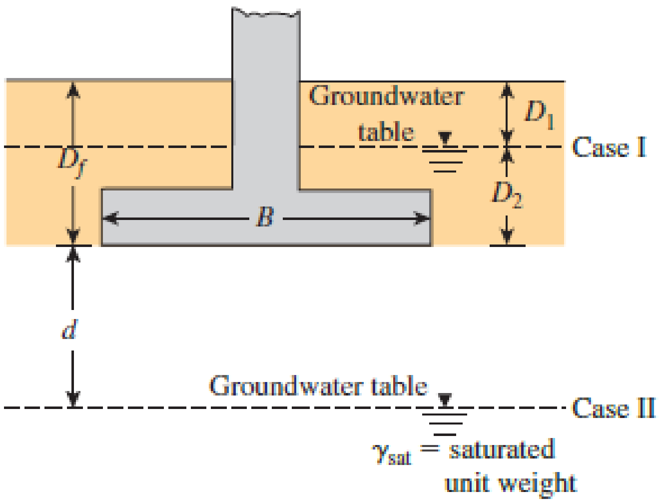
$$q = \text{effective surcharge} = D_1\gamma + D_2(\gamma_{sat} - \gamma_w)$$

where

γ_{sat} = saturated unit weight of soil

γ_w = unit weight of water

Also, the value of γ in the last term of the equations has to be replaced by $\gamma' = \gamma_{sat} - \gamma_w$.



Case II. For a water table located so that $0 \le d \le B$,

$$q = \gamma D_f$$

In this case, the factor γ in the last term of the bearing capacity equations must be replaced by the weighted average value of the effective unit weight within B below the foundation, which is given by

$$\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

Case III. When the water table is located so that $d \ge B$, the water will have no effect on the ultimate bearing capacity.

TERZAGI'S EQUATIONS SHORTCOMINGS

Terzagi's equations shortcomings:

- **They don't deal with rectangular foundations ($0<B/L<1$).**
- **The equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation.**
- **The inclination of the load on the foundation is not considered (if exist).**

The General Bearing Capacity Equation

To account for all these shortcomings, Meyerhof suggested the following form of the general bearing capacity equation:

$$q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

c' = cohesion

q = effective stress at the level of the bottom of the foundation

γ = unit weight of soil

B = width of foundation (= diameter for a circular foundation)

$F_{cs}, F_{qs}, F_{\gamma s}$ = shape factors

$F_{cd}, F_{qd}, F_{\gamma d}$ = depth factors

$F_{ci}, F_{qi}, F_{\gamma i}$ = load inclination factors

N_c, N_q, N_γ = bearing capacity factors

In the *case of inclined loading on a foundation*, the general equation provides the vertical component.

The General Bearing Capacity Equation

Notes:

1. This equation is valid for both general and local shear failure.
2. This equation is similar to original equation for ultimate bearing capacity (Terzaghi's equation) which was derived for continuous foundation, but the shape, depth, and load inclination factors are added to Terzaghi's equation to be suitable for any case may exist.

Bearing Capacity Factors: N_c, N_q, N_γ

The angle $\alpha=\phi'$ (according Terzaghi theory) was replaced by $\alpha=45+\phi'/2$. So, the bearing capacity factor will be changed.

The variations of bearing capacity factors (N_c, N_q, N_γ) and underlying soil friction angle (ϕ') are given in Table 6.2.

The General Bearing Capacity Equation

TABLE 6.2 Bearing Capacity Factors From Eqs. (6.30), (6.29), and (6.31)

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	11	8.80	2.71	1.44
1	5.38	1.09	0.07	12	9.28	2.97	1.69
2	5.63	1.20	0.15	13	9.81	3.26	1.97
3	5.90	1.31	0.24	14	10.37	3.59	2.29
4	6.19	1.43	0.34	15	10.98	3.94	2.65
5	6.49	1.57	0.45	16	11.63	4.34	3.06
6	6.81	1.72	0.57	17	12.34	4.77	3.53
7	7.16	1.88	0.71	18	13.10	5.26	4.07
8	7.53	2.06	0.86	19	13.93	5.80	4.68
9	7.92	2.25	1.03	20	14.83	6.40	5.39
10	8.35	2.47	1.22	21	15.82	7.07	6.20
22	16.88	7.82	7.13	37	55.63	42.92	66.19
23	18.05	8.66	8.20	38	61.35	48.93	78.03
24	19.32	9.60	9.44	39	67.87	55.96	92.25
25	20.72	10.66	10.88	40	75.31	64.20	109.41
26	22.25	11.85	12.54	41	83.86	73.90	130.22
27	23.94	13.20	14.47	42	93.71	85.38	155.55
28	25.80	14.72	16.72	43	105.11	99.02	186.54
29	27.86	16.44	19.34	44	118.37	115.31	224.64
30	30.14	18.40	22.40	45	133.88	134.88	271.76
31	32.67	20.63	25.99	46	152.10	158.51	330.35
32	35.49	23.18	30.22	47	173.64	187.21	403.67
33	38.64	26.09	35.19	48	199.26	222.31	496.01
34	42.16	29.44	41.06	49	229.93	265.51	613.16
35	46.12	33.30	48.03	50	266.89	319.07	762.89
36	50.59	37.75	56.31				

The General Bearing Capacity Equation

Shape Factors

Factor	Relationship	Reference
Shape	$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$ $F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$ $F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$	DeBeer (1970)

Notes:

1. If the foundation is continuous or strip →B/L=0.0
2. If the foundation is circular→B=L=diameter→B/L=1

The General Bearing Capacity Equation

Depth Factors

Factor	Relationship	Reference
Depth	$\frac{D_f}{B} \le 1$ For $\phi = 0$: $F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$ $F_{qd} = 1$ $F_{\gamma d} = 1$ For $\phi' > 0$: $F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$ $F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$ $F_{\gamma d} = 1$ $\frac{D_f}{B} > 1$ For $\phi = 0$: $F_{cd} = 1 + 0.4\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$ $F_{qd} = 1$ $F_{\gamma d} = 1$ For $\phi' > 0$: $F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$ $F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$ $F_{\gamma d} = 1$	Hansen (1970)

Important Notes:

1. If the value of (B) or (D_f) is required, you should do the following:

Assume ($D_f/B \le 1$) and calculate depth factors in term of (B) or (D_f).

Substitute in the general equation, then calculate (B) or (D_f).

After calculating the required value, you must check your assumption→($D_f/B \le 1$).

If the assumption is true, the calculated value is the final required value.

If the assumption is wrong, you must calculate depth factors in case of ($D_f/B > 1$) and then calculate (B) or (D_f) to get the true value.

2. For both cases ($D_f/B \le 1$) and ($D_f/B > 1$)

if $\phi > 0$→ calculate F_{qd} firstly, because F_{cd} depends on F_{qd}.

The General Bearing Capacity Equation

Inclination Factors

Factor	Relationship	Reference
Inclination	$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$ $F_{\gamma i} = \left(1 - \frac{\beta^\circ}{\phi'}\right)^2$ β = inclination of the load on the foundation with respect to the vertical	Meyerhof (1963); Hanna and Meyerhof (1981)

Note:

If $\beta^\circ = \phi \rightarrow F_{\gamma i} = 0.0$, so you don't need to calculate $F_{\gamma s}$ and $F_{\gamma d}$, because the last term in Meyerhof equation will be zero.

Example 6.3

EXAMPLE 6.3

Solve Example 6.1 using Eq. (6.28).

EXAMPLE 6.1

A 2.0 m wide strip foundation is placed at a depth of 1.5 m within a sandy clay, where $c' = 10\ \text{kN/m}^2$, $\phi' = 26°$, and $\gamma = 19.0\ \text{kN/m}^3$. Determine the maximum wall load that can be allowed on the foundation with a factor of safety of 3, assuming general shear failure. Use gross values.

Q = 575 KN

SOLUTION

From Eq. (6.28), the ultimate bearing capacity is given by

$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5\gamma B\,N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

For $\phi' = 26°$, from Table 6.2, $N_c = 22.25$, $N_q = 11.85$, and $N_\gamma = 12.54$. Since the load is vertical, the inclination factors are unity.

For strip foundation, $L > B$ and, hence, all three shape factors are unity.

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 26(1 - \sin 26)^2 \times \frac{1.5}{2.0} = 1.23$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c\tan\phi'} = 1.23 - \frac{1 - 1.23}{22.25\tan 26} = 1.25$$

$$F_{\gamma d} = 1$$

Hence,

$$q_u = (10)(22.25)(1)(1.25)(1) + (19.0\times1.5)(11.85)(1)(1.23)(1.0) + (0.5)(19.0)(2.0)(12.54)(1)(1)(1)$$

$$= 931.8\ \text{kN/m}^2$$

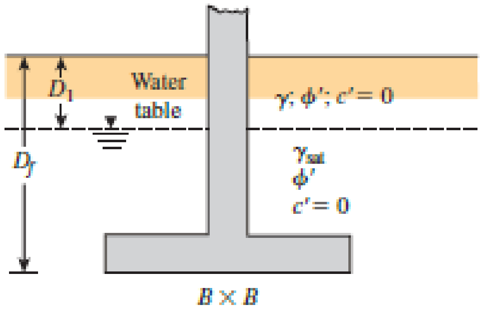
$$q_{all} = \frac{q_u}{FS} = \frac{931.8}{3} = 310.6\ \text{kN/m}^2$$

Therefore, the maximum allowable load $Q = 310.6 \times 2 =$ **621 kN/m.**

Example 6.4

EXAMPLE 6.4

A square foundation ($B \times B$) has to be constructed as shown in Figure 6.11. Assume that $\gamma = 16.5$ kN/m^3, $\gamma_{sat} = 18.55$ kN/m^3, $\phi' = 34°$, $D_f = 1.22$ m, and $D_1 = 0.61$ m. The gross allowable load, Q_{all}, with FS = 3 is 667.2 kN. Determine the size of the foundation. Use Eq. (6.28).



SOLUTION

We have

$$q_{all} = \frac{Q_{all}}{B^2} = \frac{667.2}{B^2} \text{ kN/m}^2 \quad \text{(a)}$$

From Eq. (6.28) (with $c' = 0$), for vertical loading, we obtain

$$q_{all} = \frac{q_u}{\text{FS}} = \frac{1}{3}\left(qN_q F_{qs} F_{qd} + \frac{1}{2}\gamma' B N_\gamma F_{\gamma s} F_{\gamma d}\right)$$

For $\phi' = 34°$, from Table 6.2, $N_q = 29.44$ and $N_\gamma = 41.06$. Hence,

$$F_{qs} = 1 + \frac{B}{L}\tan\phi' = 1 + \tan 34 = 1.67$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4 = 0.6$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \frac{D_f}{B} = 1 + 2\tan 34(1 - \sin 34)^2 \frac{4}{B} = 1 + \frac{1.05}{B}$$

$$F_{\gamma d} = 1$$

and

$$q = (2)(16.5) + 2(18.55 - 9.81) = 15.4 \text{ kN/m}^2$$

So

$$q_{all} = \frac{1}{3}\left[(15.4)(29.44)(1.67)\left(1 + \frac{1.05}{B}\right) + \left(\frac{1}{2}\right)(18.5 - 9.81)(B)(41.06)(0.6)(1)\right] \quad \text{(b)}$$

$$= 252.38 + \frac{265}{B} + 35.89B$$

Combining Eqs. (a) and (b) results in

$$\frac{667.2}{B^2} = 252.38 + \frac{265}{B} + 35.89B$$

By trial and error, we find that $B \approx$ **1.3 m**. ■

Example 6.5

EXAMPLE 6.5

A square column foundation (Figure 6.12) is to be constructed on a sand deposit. The allowable load Q will be inclined at an angle $\beta = 20°$ with the vertical. The standard penetration numbers N_{60} obtained from the field are as follows. Determine Q. Use FS = 3, Eq. (3.13), Eq. (3.29), and Eq. (6.28).

Depth (m)	N_{60}
1.5	3
3.0	6
4.5	9
6.0	10
7.5	10
9.0	8

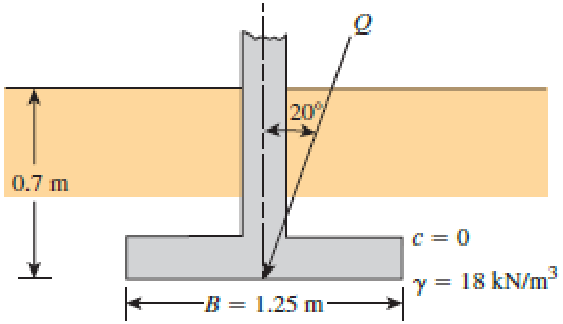


FIGURE 6.12

Example 6.5

SOLUTION

From Eq. (3.29),

$$\phi' \text{ (deg)} = 27.1 + 0.3(N_1)_{60} - 0.00054[(N_1)_{60}]^2$$

The following is an estimation of ϕ' in the field using Eq. (3.29).

Depth (m)	σ_o'(kN/m^2)	C_N from Eq. (3.13)	N_{60}	$(N_1)_{60}$	ϕ' (°) from Eq. (3.29)
1.5	27.0	1.92	3	5.8	28.8
3.0	54.0	1.36	6	8.2	29.5
4.5	81.0	1.11	9	10.0	30.0
6.0	108.0	0.96	10	9.6	29.9
7.5	135.0	0.86	10	8.6	29.6
9.0	162.0	0.79	8	6.3	29.0

Average $\phi' = 29.5° \approx 30°$

With $c' = 0$, the ultimate bearing capacity [Eq. (6.28)] becomes

$$q_u = qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$$q = (0.7)(18) = 12.6 \text{ kN/m}^2$$

$$\gamma = 18 \text{ kN/m}^3$$

From Table 6.2 for $\phi' = 30°$,

$$N_q = 18.4$$

$$N_\gamma = 22.4$$

Example 6.5

From Table 6.3 (*Note:* $B = L$),

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi' = 1 + 0.577 = 1.577$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 0.6$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + \frac{(0.289)(0.7)}{1.25} = 1.162$$

$$F_{\gamma d} = 1$$

$$F_{qi} = \left(1 - \frac{\beta°}{90°}\right)^2 = \left(1 - \frac{20}{90}\right)^2 = 0.605$$

$$F_{\gamma i} = \left(1 - \frac{\beta°}{\phi'}\right)^2 = \left(1 - \frac{20}{30}\right)^2 = 0.11$$

Hence,

$$q_u = (12.6)(18.4)(1.577)(1.162)(0.605) + \left(\frac{1}{2}\right)(18)(1.25)(22.4)(0.6)(1)(0.11)$$

$$= 273.66 \text{ kN/m}^2$$

$$q_{all} = \frac{q_u}{FS} = \frac{273.66}{3} = 91.22 \text{ kN/m}^2$$

Now,

$$Q\cos 20 = q_{all}B^2 = (91.22)(1.25)^2$$

$$Q \approx \mathbf{151.7\ kN}$$

■

EFFECT OF SOIL COMPRESSIBILITY

The change of failure mode is due to soil compressibility.

Vesic (1973) proposed the following modification to the general bearing capacity equation:

$$q_u = c'N_cF_{cs}F_{cd}F_{cc} + qN_qF_{qs}F_{qd}F_{qc} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma c}$$

F_{cc}, F_{qc}, and $F_{\gamma c}$ are soil compressibility factors.

EFFECT OF SOIL COMPRESSIBILITY

Steps for calculating the soil compressibility factors:

Step 1. Calculate the *rigidity index*, I_r, of the soil at a depth approximately $B/2$ below the bottom of the foundation, or

$$I_r = \frac{G_s}{c' + q' \tan \phi'}$$

where

G_s = shear modulus of the soil

q' = effective overburden pressure at a depth of $D_f + B/2$

Step 2. The critical rigidity index, $I_{r(cr)}$, can be expressed as

$$I_{r(cr)} = \frac{1}{2}\left\{\exp\left[\left(3.30 - 0.45\frac{B}{L}\right)\cot\left(45 - \frac{\phi'}{2}\right)\right]\right\}$$

The variations of $I_{r(cr)}$ with B/L are given in Table 6.8.

TABLE 6.8 Variation of $I_{r(cr)}$ with ϕ' and B/L

ϕ' (deg)	$I_{r(cr)}$					
	$B/L = 0$	$B/L = 0.2$	$B/L = 0.4$	$B/L = 0.6$	$B/L = 0.8$	$B/L = 1.0$
0	13.56	12.39	11.32	10.35	9.46	8.64
5	18.30	16.59	15.04	13.63	12.36	11.20
10	25.53	22.93	20.60	18.50	16.62	14.93
15	36.85	32.77	29.14	25.92	23.05	20.49
20	55.66	48.95	43.04	37.85	33.29	29.27
25	88.93	77.21	67.04	58.20	50.53	43.88
30	151.78	129.88	111.13	95.09	81.36	69.62
35	283.20	238.24	200.41	168.59	141.82	119.31
40	593.09	488.97	403.13	332.35	274.01	225.90
45	1440.94	1159.56	933.19	750.90	604.26	486.26

EFFECT OF SOIL COMPRESSIBILITY

Steps for calculating the soil compressibility factors:

Step 3. If $I_r \geq I_{r(cr)}$, then

$$F_{cc} = F_{qc} = F_{\gamma c} = 1$$

However, if $I_r < I_{r(cr)}$, then

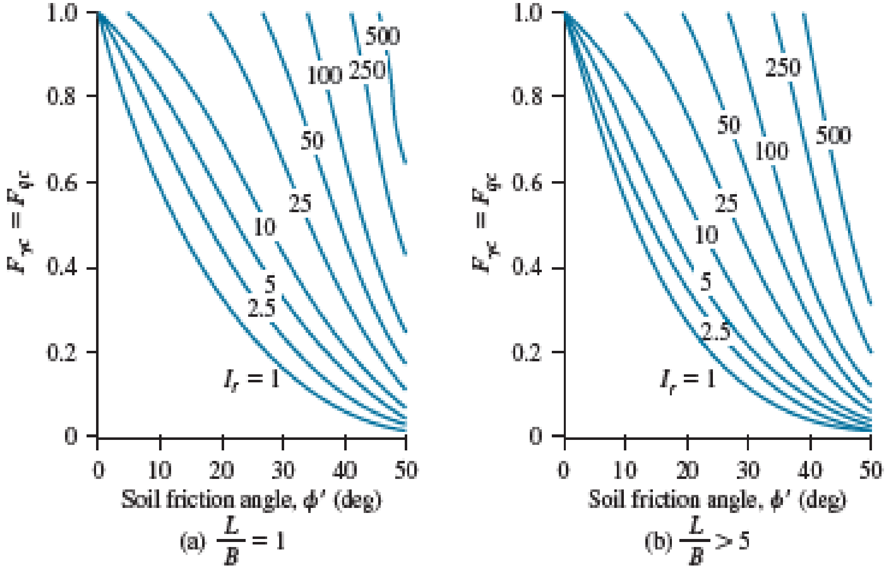
$$F_{\gamma c} = F_{qc} = \exp\left\{\left(-4.4 + 0.6\frac{B}{L}\right)\tan\phi' + \left[\frac{(3.07\sin\phi')(\log 2I_r)}{1+\sin\phi'}\right]\right\}$$

$\phi = 0,$

$$F_{cc} = 0.32 + 0.12\frac{B}{L} + 0.60\log I_r$$

For $\phi' > 0,$

$$F_{cc} = F_{qc} - \frac{1 - F_{qc}}{N_q \tan\phi'}$$



Variation of $F_{\gamma c} = F_{qc}$ with I_r and ϕ'

EXAMPLE 6.6

EXAMPLE 6.6

For a shallow foundation, $B = 0.6$ m, $L = 1.2$ m, and $D_f = 0.6$ m. The known soil characteristics are

Soil:

$$\phi' = 25°$$
$$c' = 48 \text{ kN/m}^2$$
$$\gamma = 18 \text{ kN/m}^3$$
$$\text{Modulus of elasticity, } E_s = 620 \text{ kN/m}^2$$
$$\text{Poisson's ratio, } \mu_s = 0.3$$

Calculate the ultimate bearing capacity.

SOLUTION

From Eq. (6.43),

$$I_r = \frac{G_s}{c' + q' \tan \phi'}$$

However,

$$G_s = \frac{E_s}{2(1 + \mu_s)}$$

So

$$I_r = \frac{E_s}{2(1 + \mu_s)[c' + q' \tan \phi']}$$

EXAMPLE 6.6

Now,

$$q' = \gamma\left(D_f + \frac{B}{2}\right) = 18\left(0.6 + \frac{0.6}{2}\right) = 16.2 \text{ kN/m}^2$$

Thus,

$$I_r = \frac{620}{2(1 + 0.3)[48 + 16.2 \tan 25]} = 4.29$$

From Eq. (6.44),

$$I_{r(\text{cr})} = \frac{1}{2}\left\{\exp\left[\left(3.3 - 0.45\frac{B}{L}\right)\cot\left(45 - \frac{\phi'}{2}\right)\right]\right\}$$

$$= \frac{1}{2}\left\{\exp\left[\left(3.3 - 0.45\frac{0.6}{1.2}\right)\cot\left(45 - \frac{25}{2}\right)\right]\right\} = 62.41$$

Since $I_{r(\text{cr})} > I_r$, we use Eqs. (6.45) and (6.47) to obtain

$$F_{\gamma c} = F_{qc} = \exp\left\{\left(-4.4 + 0.6\frac{B}{L}\right)\tan\phi' + \left[\frac{(3.07 \sin\phi')\log(2I_r)}{1 + \sin\phi'}\right]\right\}$$

$$= \exp\left\{\left(-4.4 + 0.6\frac{0.6}{1.2}\right)\tan 25 + \left[\frac{(3.07 \sin 25)\log(2 \times 4.29)}{1 + \sin 25}\right]\right\} = 0.347$$

and

$$F_{cc} = F_{qc} - \frac{1 - F_{qc}}{N_c \tan\phi'}$$

For $\phi' = 25°$, $N_c = 20.72$ (see Table 6.2); therefore,

$$F_{cc} = 0.347 - \frac{1 - 0.347}{20.72 \tan 25} = 0.279$$

EXAMPLE 6.6

Now, from Eq. (6.42),

$$q_u = c'N_cF_{cs}F_{cd}F_{cc} + qN_qF_{qs}F_{qd}F_{qc} + \tfrac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma c}$$

From Table 6.2, for $\phi' = 25°$, $N_c = 20.72$, $N_q = 10.66$, and $N_\gamma = 10.88$. Consequently,

$$F_{cs} = 1 + \left(\frac{N_q}{N_c}\right)\left(\frac{B}{L}\right) = 1 + \left(\frac{10.66}{20.72}\right)\left(\frac{0.6}{1.2}\right) = 1.257$$

$$F_{qs} = 1 + \frac{B}{L}\tan\phi' = 1 + \frac{0.6}{1.2}\tan 25 = 1.233$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\,\frac{0.6}{1.2} = 0.8$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 25\,(1 - \sin 25)^2\left(\frac{0.6}{0.6}\right) = 1.311$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c\tan\phi'} = 1.311 - \frac{1 - 1.311}{20.72\tan 25}$$

$$= 1.343$$

and

$$F_{\gamma d} = 1$$

Thus,

$$q_u = (48)(20.72)(1.257)(1.343)(0.279) + (0.6 \times 18)(10.66)(1.233)(1.311)$$

$$(0.347) + \left(\tfrac{1}{2}\right)(18)(0.6)(10.88)(0.8)(1)(0.347) = \mathbf{549.32\ kN/m^2}$$

ECCENTRICALLY LOADED FOUNDATION

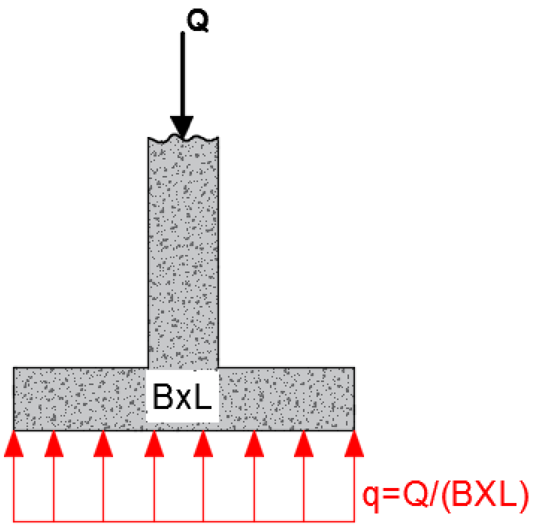
If the load applied on the foundation is in the center of the foundation without eccentricity, the bearing capacity of the soil will be uniform at any point under the foundation (as shown in figure) because there is no any moments on the foundation, and the general equation for stress under the foundation is:

$$\text{Stress } q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y}$$

In this case, the load is in the center of the foundation and there are no moments so,

$$\text{Stress } q = \frac{Q}{A}$$

(uniform at any point below the foundation)

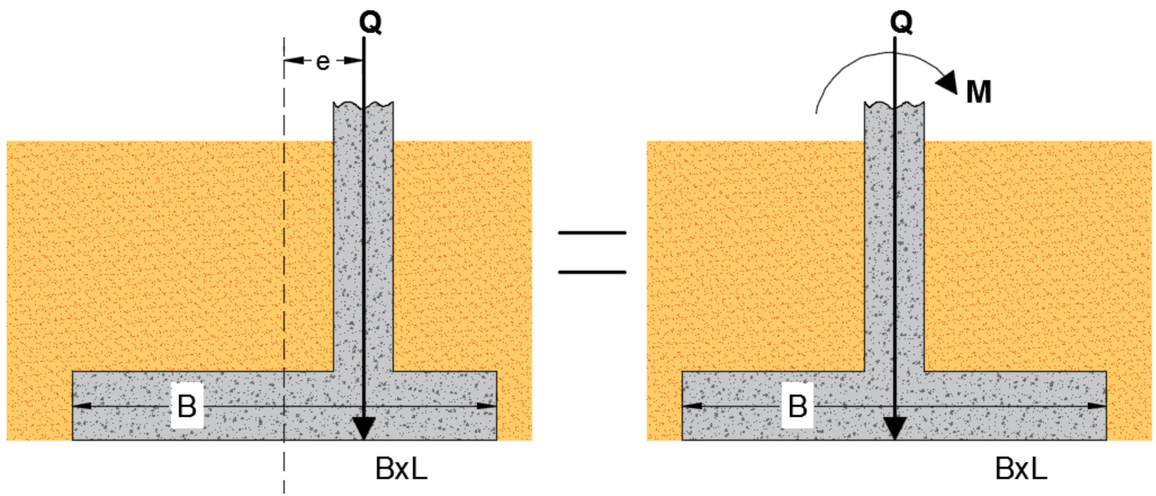


ECCENTRICALLY LOADED FOUNDATION

However, in several cases, as with the base of a retaining wall or neighbor footing, the loads does not exist in the center, so foundations are subjected to moments in addition to the vertical load (as shown in the figure).
In such cases, the distribution of pressure by the foundation on the soil is not uniform because there is a moment applied on the foundation and the stress under the foundation will be calculated from:

$$\text{Stress } q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \qquad \text{(two way eccentricity)}$$

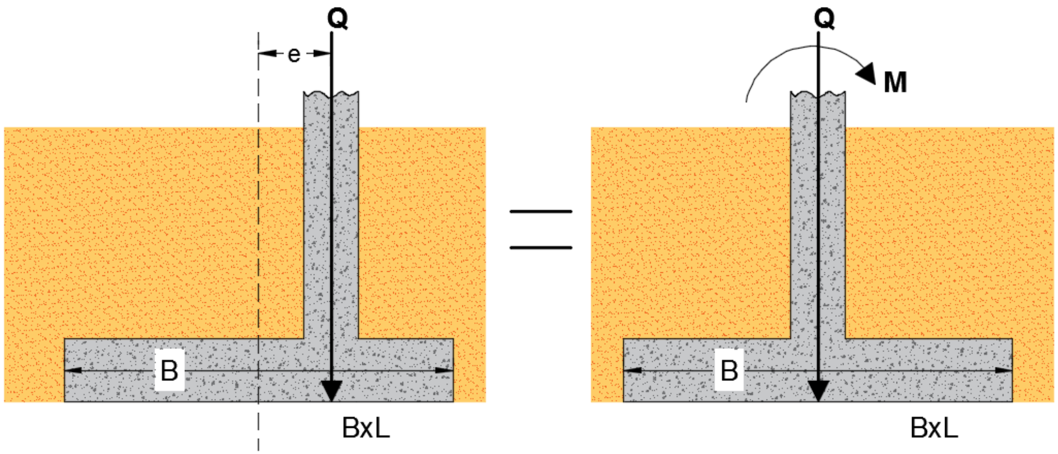
$$\text{Stress } q = \frac{Q}{A} \pm \frac{Mc}{I} \qquad \text{one way eccentricity}$$



ONE WAY ECCENTRICITY

$$\text{Stress } q = \frac{Q}{A} \pm \frac{Mc}{I}$$

one way eccentricity



Since the pressure under the foundation is not uniform, there are maximum and minimum pressures (under the two edges of the foundation) and we concerned about calculating these two pressures.

Assume the eccentricity is in direction of (B)
A=B×L
M=Q×e
c=B/2 (maximum distance from the center)

$$I = \frac{B^3 L}{12}$$ **(I is about the axis that resists the moment)**

ONE WAY ECCENTRICITY

$$q = \frac{Q}{B*L} \pm \frac{Q*e*B}{\frac{2B^3*L}{12}} = \frac{Q}{B*L} \pm \frac{Q*e*6}{B^2*L}$$

$$q = \frac{Q}{B*L}(1 \pm \frac{6e}{B})$$

There are three cases for calculating maximum and minimum pressures according to the values of (e and B/6)

Case I. (For e<B/6):

$$q_{max} = \frac{Q}{B*L}(1+\frac{6e}{B})$$

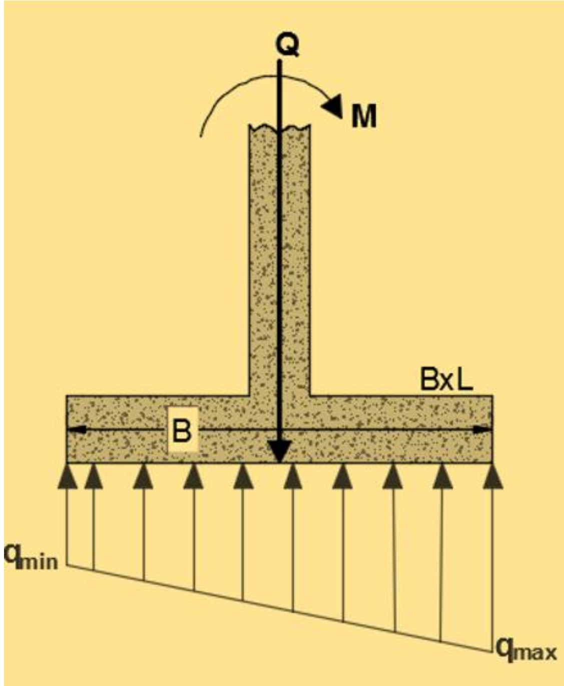
$$q_{min} = \frac{Q}{B*L}(1-\frac{6e}{B})$$

If eccentricity in (L) direction (For e<L/6):

$$q_{max} = \frac{Q}{B*L}(1+\frac{6e}{L})$$

$$q_{min} = \frac{Q}{B*L}(1-\frac{6e}{L})$$

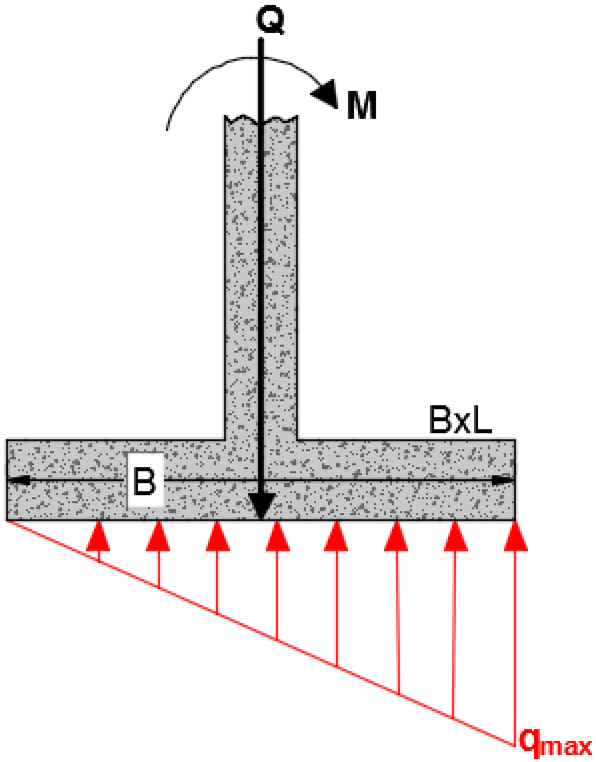
In this case, q_{min} is positive



ONE WAY ECCENTRICITY

Case II. (For e=B/6):

$$q_{max} = \frac{Q}{B*L}(1+\frac{6e}{B})$$
$$q_{min} = \frac{Q}{B*L}(1-1) = 0$$



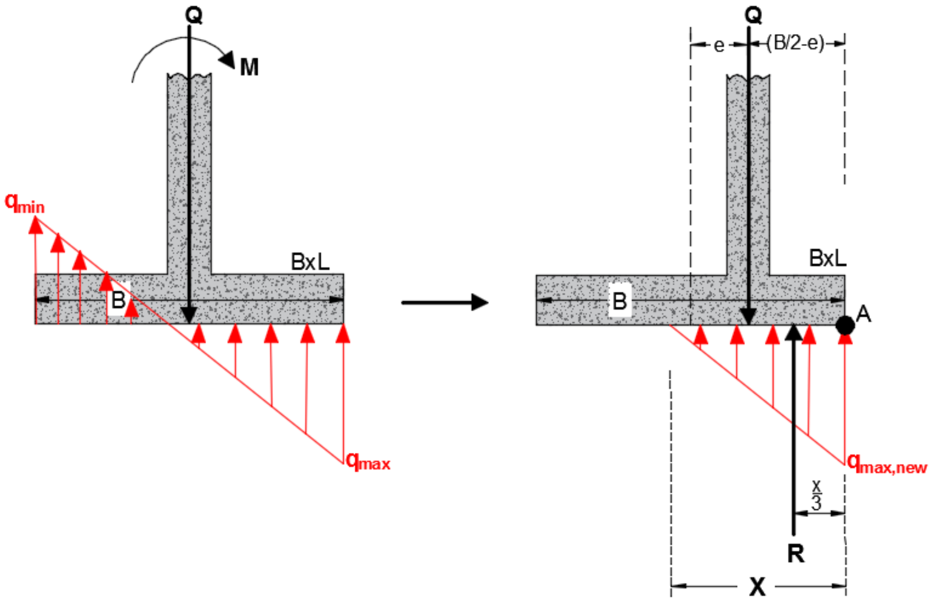
If eccentricity in (L) direction (For e= L/6):

$$q_{max} = \frac{Q}{B*L}(1+\frac{6e}{L})$$
$$q_{min} = \frac{Q}{B*L}(1-1) = 0$$

ONE WAY ECCENTRICITY

Case III. (For e>B/6):

As shown in the figure, the value of (q_{min}) is negative (i.e. tension in soil), but we know that soil can't resist any tension, thus, negative pressure must be prevented by making (q_{min}=0) at distance (x) from point (A) as shown in the figure, and determine the new value of (q_{max}) by static equilibrium as following:



R=area of triangle*L

$=0.5*q_{max,new}*X*L$ (1)

$\Sigma F_y=0.0 \rightarrow R=Q$ (2)

$\Sigma M@A=0.0 \rightarrow Q*(B/2-e)=R*X/3$

(but from Eq.2→R=Q)→X=3(B/2−e)

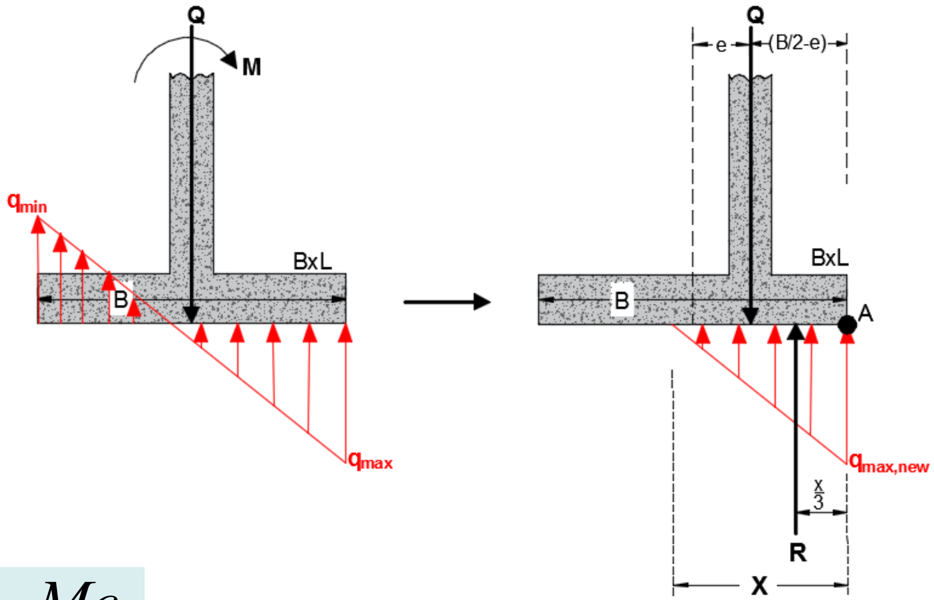
Substitute by X in Eq. (1) →

$R=Q=0.5*q_{max,new}*3(B/2-e)*L$

$\rightarrow q_{max,new}=4Q/[3L(B-2e)]$

ONE WAY ECCENTRICITY

Case III. (For $e>B/6$):



If eccentricity in (L) direction (For e> L/6):

$$q_{max,new}=4Q/[3B(L-2e)]$$

Note:

If the foundation is circular

$$q=\frac{Q}{A}\pm\frac{Mc}{I}$$

$$A=\frac{\pi D^2}{4}$$

$$c=\frac{D}{2}$$

$$I=\frac{\pi D^4}{64}$$

Calculate q_{max} and q_{min}

Ultimate Bearing Capacity under Eccentric Loading One-Way Eccentricity

Effective Area Method:

If the load does not exist in the center of the foundation, or if the foundation subjected to moment in addition to the vertical loads, the stress distribution under the foundation is not uniform. So, to calculate the ultimate (uniform) bearing capacity under the foundation, new area should be determined to make the applied load in the center of this area and to develop uniform pressure under this new area. This new area is called Effective Area.

Meyerhof's Effective Area Method

Step 1. Determine the effective dimensions of the foundation (Figure 6.20a):

$$B' = \text{effective width} = B - 2e$$
$$L' = \text{effective length} = L$$

Note that if the eccentricity were in the direction of the length of the foundation, the value of L' would be equal to $L - 2e$. The value of B' would equal B. The smaller of the two dimensions (i.e., L' and B') is the effective width of the foundation.

Step 2. Use Eq. (6.28) for the ultimate bearing capacity:

$$q'_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

To evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$, use the relationships given in Table 6.3 with *effective length* and *effective width* dimensions instead of L and B, respectively. To determine F_{cd}, F_{qd}, and $F_{\gamma d}$, use the relationships given in Table 6.3. However, do not replace B with B'.

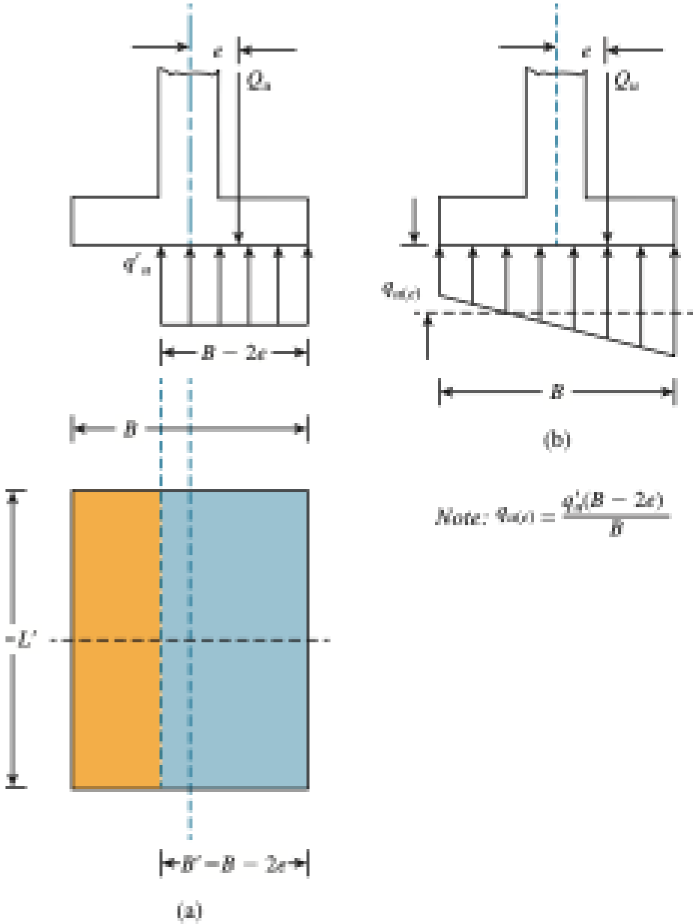
Step 3. The total ultimate load that the foundation can sustain is

$$Q_u = q'_u \overbrace{(B')(L')}^{A'}$$

where A' = effective area.

Step 4. The factor of safety against bearing capacity failure is

$$\text{FS} = \frac{Q_u}{Q}$$



GURE 6.20 Definition of q'_u and $q_{u(e)}$

Meyerhof's Effective Area Method

It is important to note that q'_u is the ultimate bearing capacity of a foundation of width $B' = B - 2e$ with a centric load (Figure 6.20a). However, the actual distribution of soil reaction at ultimate load will be of the type shown in Figure 6.20b. In Figure 6.20b, $q_{u(e)}$ is the average load per unit area of the foundation. Thus,

$$q_{u(e)} = \frac{q'_u(B - 2e)}{B}$$

EXAMPLE 6.7

EXAMPLE 6.7

A continuous foundation is shown in Figure 6.24. If the load eccentricity is 0.2 m, determine the ultimate load, Q_u, per unit length of the foundation. Use Meyerhof's effective area method.

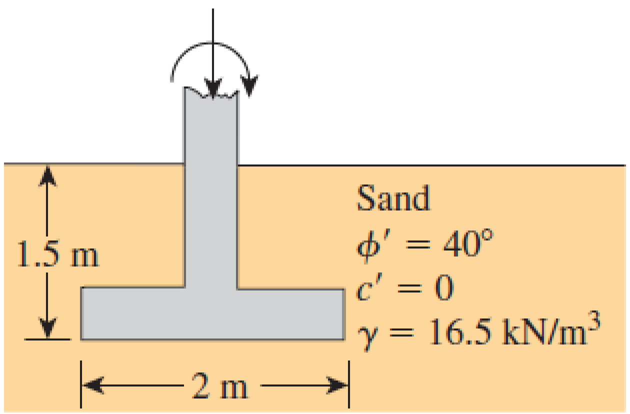


FIGURE 6.24 A continuous foundation with load eccentricity

EXAMPLE 6.7

SOLUTION

Meyerhof's Effective Area Method

For $c' = 0$, Eq. (6.55) gives

$$q'_u = qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma' B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

where $q = (16.5)(1.5) = 24.75 \text{ kN/m}^2$.

For $\phi' = 40°$, from Table 6.2, $N_q = 64.2$ and $N_\gamma = 109.41$. Also,

$$B' = 2 - (2)(0.2) = 1.6 \text{ m}$$

Because the foundation in question is a continuous foundation, B'/L' is zero. Hence, $F_{qs} = 1$, $F_{\gamma s} = 1$. From Table 6.3,

$$F_{qi} = F_{\gamma i} = 1$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \frac{D_f}{B} = 1 + 0.214\left(\frac{1.5}{2}\right) = 1.16$$

$$F_{\gamma d} = 1$$

and

$$q'_u = (24.75)(64.2)(1)(1.16)(1) + \left(\frac{1}{2}\right)(16.5)(1.6)(109.41)(1)(1)(1) = 3287.39 \text{ kN/m}^2$$

Consequently,

$$Q_u = (B')(1)(q'_u) = (1.6)(1)(3287.39) \approx \mathbf{5260\ kN}$$

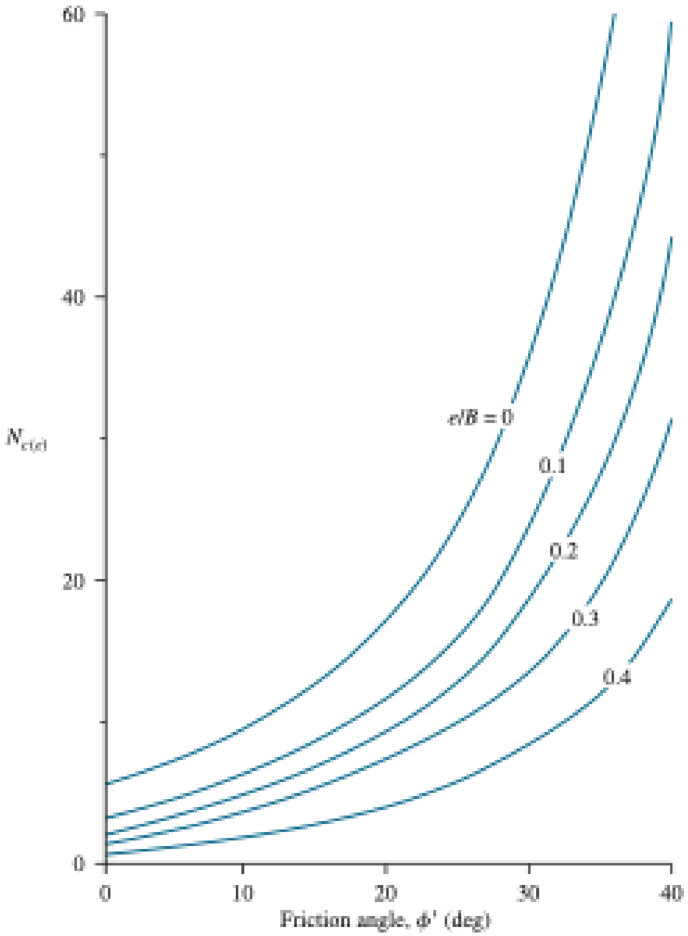
Prakash and Saran Theory

The ultimate load *per unit length of a continuous foundation*

$$Q_u = q_{u(e)}B = B\left[c'N_{c(e)} + qN_{q(e)} + \frac{1}{2}\gamma BN_{\gamma(e)}\right]$$

where $N_{c(e)}, N_{q(e)}, N_{\gamma(e)}$ = bearing capacity factors under eccentric loading.

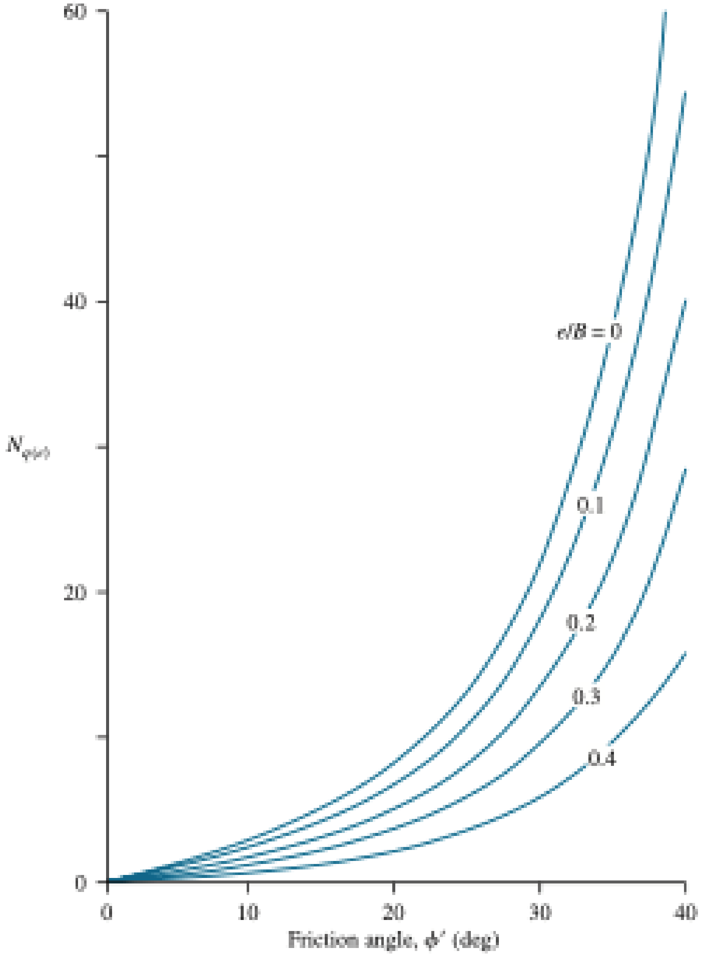
Prakash and Saran Theory



$\phi' = 40°$	
e/B	$N_{c(e)}$
0	94.83
0.1	66.60
0.2	54.45
0.3	36.3
0.4	18.15

FIGURE 6.21 Variation of $N_{c(e)}$ with ϕ'

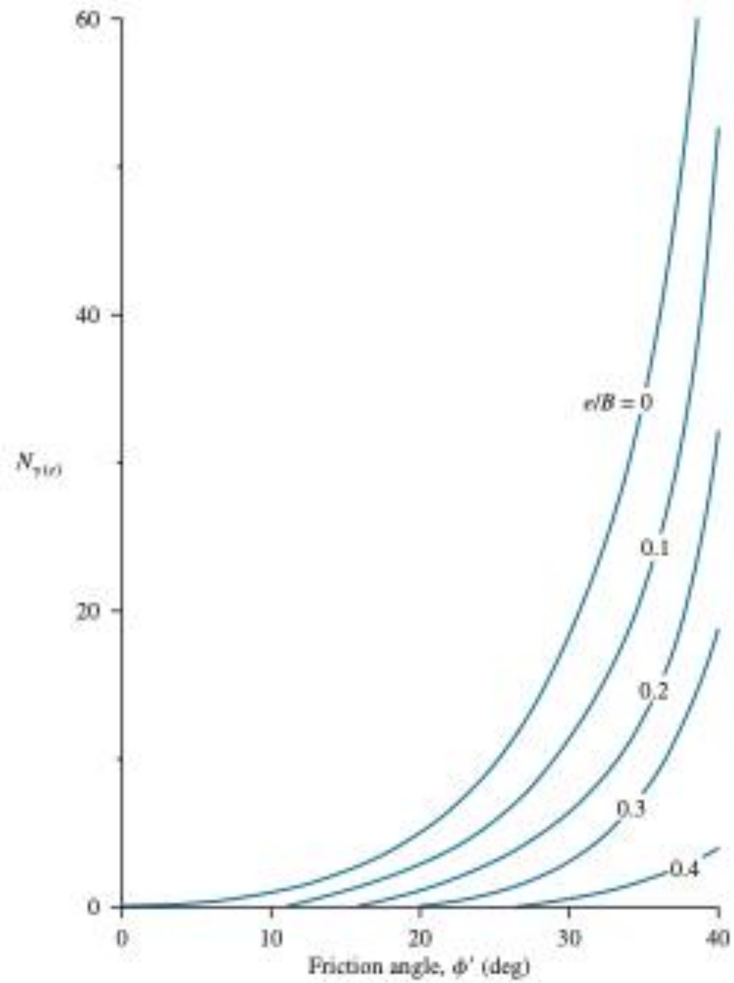
Prakash and Saran Theory



$\phi' = 40°$	
e/B	$N_{\gamma(e)}$
0	81.27
0.1	56.09
0.2	45.18
0.3	30.18
0.4	15.06

FIGURE 6.22 Variation of $N_{\gamma(e)}$ with ϕ'

Prakash and Saran Theory



$\phi' = 40°$	
e/B	$N_{\gamma(e)}$
0	115.80
0.1	71.80
0.2	41.60
0.3	18.50
0.4	4.62

FIGURE 6.23 Variation of $N_{\gamma(e)}$ with ϕ'

EXAMPLE 6.8

EXAMPLE 6.8

Solve Example 6.7 using Eq. (6.58).

Prakash and Saran Theory

EXAMPLE 6.7

A continuous foundation is shown in Figure 6.24. If the load eccentricity is 0.2 m, determine the ultimate load, Q_u, per unit length of the foundation. Use Meyerhof's effective area method.

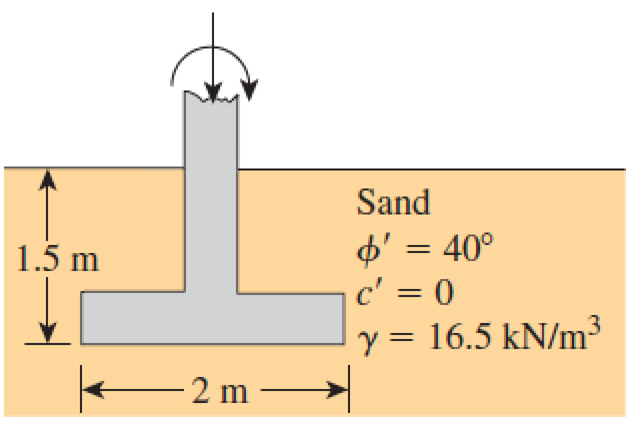


FIGURE 6.24 A continuous foundation with load eccentricity

EXAMPLE 6.8

Prakash and Saran Theory

SOLUTION

Since $c' = 0$,

$$Q_u = B\left[qN_{q(e)} + \frac{1}{2}\gamma BN_{\gamma(e)}\right]$$

$$\frac{e}{B} = \frac{0.2}{2} = 0.1$$

For $\phi' = 40°$ and $e/B = 0.1$, Figures 6.22 and 6.23 give $N_{q(e)} \approx 56.09$ and $N_{\gamma(e)} \approx 71.8$. Hence,

$$Q_u = 2[(24.75)(56.09) + \left(\tfrac{1}{2}\right)(16.5)(2)(71.8)] = \mathbf{5146\ kN}$$

Reduction Factor Method

Granular Soil

Purkayastha and Char (1977) (*continuous foundations on sand*)

$$R_k = 1 - \frac{q_{u(e)}}{q_{u(\text{centric})}}$$

where

R_k = reduction factor

$q_{u(e)}$ = average ultimate bearing capacity of eccentrically loaded continuous foundations (See Figure 6.20.)

q_u = ultimate bearing capacity of centrally loaded continuous foundations

The magnitude of R_k can be expressed as

$$R_k = a\left(\frac{e}{B}\right)^k$$

where a and k are functions of the embedment ratio D_f/B (Table 6.9).

TABLE 6.9 Variations of a and k [Eq. (6.64)]

D_f/B	a	k
0.00	1.862	0.73
0.25	1.811	0.785
0.50	1.754	0.80
1.00	1.820	0.888

Reduction Factor Method

Granular Soil

$$q_{u(e)} = q_u(1 - R_k) = q_u\left[1 - a\left(\frac{e}{B}\right)^k\right]$$

where

$$q_u = qN_qF_{qd} + \frac{1}{2}\gamma BN_\gamma F_{\gamma d}$$

The relationships for F_{qd} and $F_{\gamma d}$ are given in Table 6.3.

Based on several laboratory model tests, Patra et al. (2012a) have concluded that

$$q_{u(e)} \approx q_u\left(1 - \frac{2e}{B}\right)$$

The ultimate load *per unit length* of the foundation can then be given as

$$Q_u = Bq_{u(e)}$$

EXAMPLE 6.9

EXAMPLE 6.9

Solve Example 6.7 using Eq. (6.67).

Reduction Factor Method

EXAMPLE 6.7

A continuous foundation is shown in Figure 6.24. If the load eccentricity is 0.2 m, determine the ultimate load, Q_u, per unit length of the foundation. Use Meyerhof's effective area method.

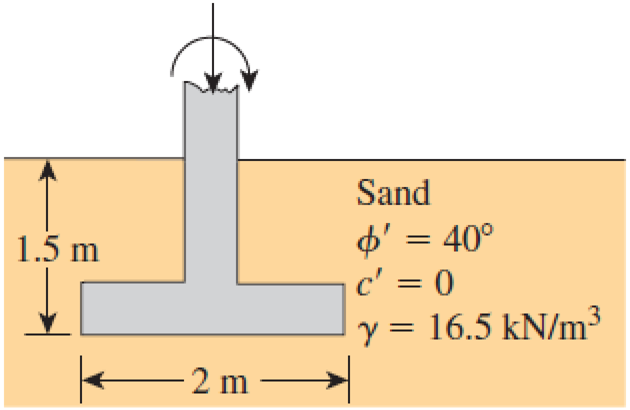


FIGURE 6.24 A continuous foundation with load eccentricity

EXAMPLE 6.9

Reduction Factor Method

SOLUTION

With $c' = 0$,

$$q_{u(e)} = qN_qF_{qd} + \frac{1}{2}\gamma BN_\gamma F_{\gamma d}$$

For $\phi' = 40°$, $N_q = 64.2$ and $N_\gamma = 109.41$ (see Table 6.2). Hence,

$$F_{qd} = 1.16 \text{ and } F_{\gamma d} = 1 \text{ (see Example 6.7)}$$

$$q_u = (24.75)(64.2)(1.16) + \frac{1}{2}(16.5)(2)(109.41)(1)$$

$$= 1843.18 + 1805.27 = 3648.45 \text{ kN/m}^2$$

From Eq. (6.67),

$$q_{u(e)} = q_u\left(1 - \frac{2e}{B}\right)$$

$$= 3648.45\left[1 - 2\left(\frac{0.2}{2}\right)\right]$$

$$= 2918.76 \text{ kN/m}^2$$

$$Q_u = Bq_{u(e)} = (2)(2918.76) \approx \mathbf{5838\ kN}$$

The Ultimate Load Qu

Method	Qu
Meyerhof’s Effective Area Method	5260 kN
Prakash and Saran Theory	5146 kN
Reduction Factor Method	5838 kN

Ultimate Bearing Capacity under Eccentric Loading
Two-Way Eccentricity

This condition is equivalent to a load Q_u placed eccentrically on the foundation with $x = e_B$ and $y = e_L$ (Figure 6.25d). Note that

$$e_B = \frac{M_y}{Q_u}$$

and

$$e_L = \frac{M_x}{Q_u}$$

If Q_u is needed, it can be obtained from Eq. (6.56); that is,

$$Q_u = q'_u A'$$

where, from Eq. (6.55),

$$q'_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

and

$$A' = \text{effective area} = B'L'$$

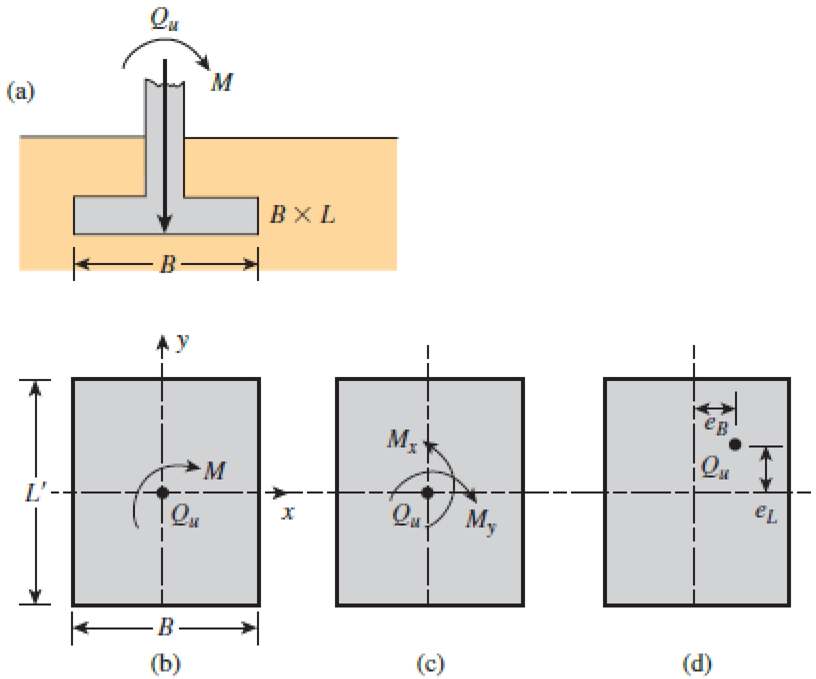


FIGURE 6.25 Analysis of foundation with two-way eccentricity

Ultimate Bearing Capacity under Eccentric Loading
Two-Way Eccentricity

Case I. $e_L/L \geq \frac{1}{6}$ and $e_B/B \geq \frac{1}{6}$. The effective area for this condition is shown in Figure 6.26, or

$$A' = \tfrac{1}{2}B_1L_1$$

where

$$B_1 = B\left(1.5 - \frac{3e_B}{B}\right)$$

and

$$L_1 = L\left(1.5 - \frac{3e_L}{L}\right)$$

The effective length L' is the larger of the two dimensions B_1 and L_1. So the effective width is

$$B' = \frac{A'}{L'}$$

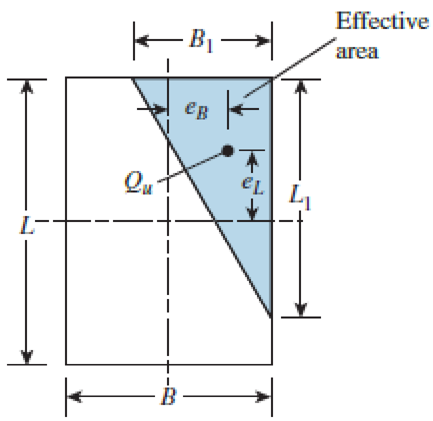


FIGURE 6.26 Effective area for the case of $e_L/L \geq \frac{1}{6}$ and $e_B/B \geq \frac{1}{6}$

Ultimate Bearing Capacity under Eccentric Loading
Two-Way Eccentricity

Case II. $\frac{1}{6} < e_L/L < 0.5$ and $e_B/B < \frac{1}{6}$. The effective area for this case, shown in Figure 6.27a, is

$$A' = \tfrac{1}{2}(L_1 + L_2)B$$

The magnitudes of L_1 and L_2 can be determined from Figure 6.27b. The effective width is

$$B' = \frac{A'}{L_1 \text{ or } L_2 \quad \text{(whichever is larger)}}$$

The effective length is

$$L' = L_1 \text{ or } L_2 \quad \text{(whichever is larger)}$$

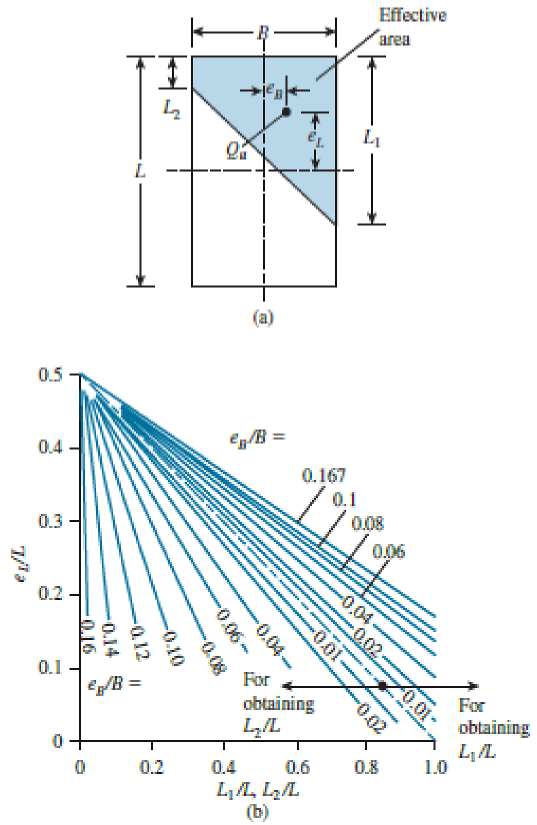


FIGURE 6.27 Effective area for case II where $\frac{1}{6} < e_L/L < 0.5$ and $e_B/B < \frac{1}{6}$

Ultimate Bearing Capacity under Eccentric Loading
Two-Way Eccentricity

Case III. $e_L/L < \frac{1}{6}$ and $\frac{1}{6} < e_B/B < 0.5$. The effective area, shown in Figure 6.28a, is

$$A' = \tfrac{1}{2}(B_1 + B_2)L$$

The effective width is

$$B' = \frac{A'}{L}$$

The effective length is

$$L' = L$$

The magnitudes of B_1 and B_2 can be determined from Figure 6.28b.

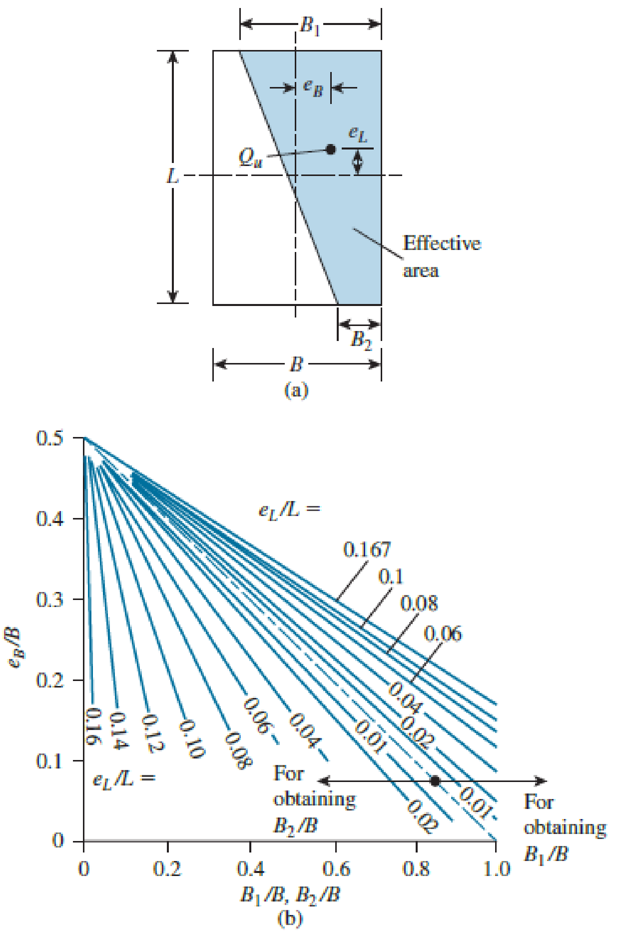


FIGURE 6.28 Effective area for case III where $e_L/L < \frac{1}{6}$ and $\frac{1}{6} < e_B/B < 0.5$

Ultimate Bearing Capacity under Eccentric Loading
Two-Way Eccentricity

Case IV. $e_L/L < \frac{1}{6}$ and $e_B/B < \frac{1}{6}$. Figure 6.29a shows the effective area for this case. The ratio B_2/B, and thus B_2, can be determined by using the e_L/L curves that slope upward. Similarly, the ratio L_2/L, and thus L_2, can be determined by using the e_L/L curves that slope downward. The effective area is then

$$A' = L_2B + \tfrac{1}{2}(B + B_2)(L - L_2)$$

The effective width is

$$B' = \frac{A'}{L}$$

The effective length is

$$L' = L$$

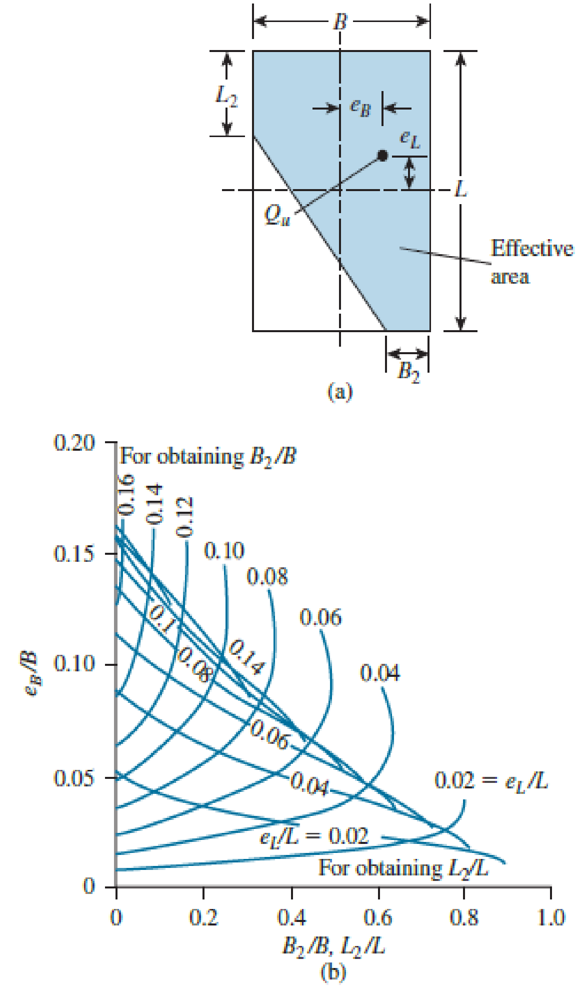


FIGURE 6.29 Effective area for case IV where $e_L/L < \frac{1}{6}$ and $e_B/B < \frac{1}{6}$

Ultimate Bearing Capacity under Eccentric Loading
Two-Way Eccentricity

Case V. (Circular Foundation) In the case of circular foundations under eccentric loading (Figure 6.30a), the eccentricity is always one way. The effective area A' and the effective width B' for a circular foundation are given in a nondimensional form in Table 6.10. Once A' and B' are determined, the effective length can be obtained as

$$L' = \frac{A'}{B'}$$

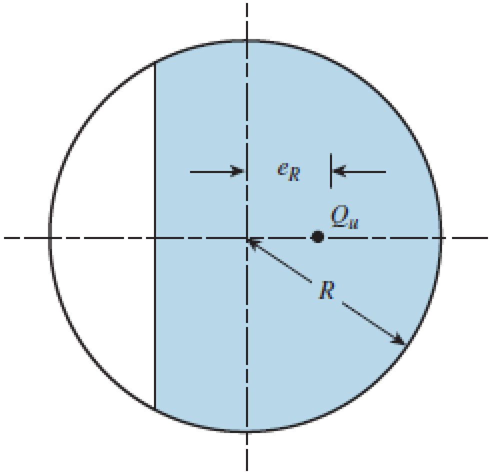


FIGURE 6.30 Effective area for circular foundation

TABLE 6.10 Variation of A'/R^2 and B'/R with e_R/R for Circular Foundations

e_R/R	A'/R^2	B'/R
0.1	2.8	1.85
0.2	2.4	1.32
0.3	2.0	1.2
0.4	1.61	0.80
0.5	1.23	0.67
0.6	0.93	0.50
0.7	0.62	0.37
0.8	0.35	0.23
0.9	0.12	0.12
1.0	0	0

EXAMPLE 6.10

EXAMPLE 6.10

A square foundation is shown in Figure 6.31, with $e_L = 0.3$ m and $e_B = 0.15$ m. Assume two-way eccentricity, and determine the ultimate load, Q_u.

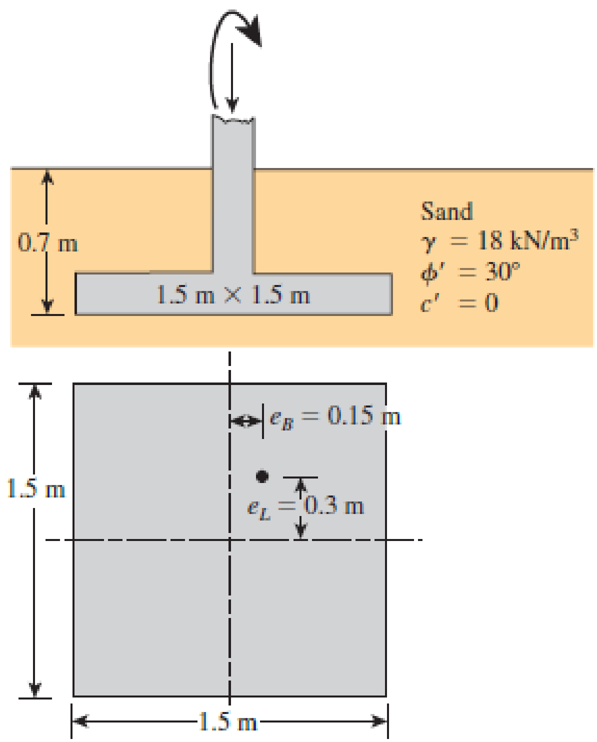


FIGURE 6.31 An eccentrically loaded foundation

EXAMPLE 6.10

SOLUTION

We have

$$\frac{e_L}{L} = \frac{0.3}{1.5} = 0.2$$

and

$$\frac{e_B}{B} = \frac{0.15}{1.5} = 0.1$$

This case is similar to that shown in Figure 6.27a. From Figure 6.27b, for $e_L/L = 0.2$ and $e_B/B = 0.1$,

Case II. $\frac{1}{6} < e_L/L < 0.5$ and $e_B/B < \frac{1}{6}$.

$$\frac{L_1}{L} \approx 0.85; \qquad L_1 = (0.85)(1.5) = 1.275 \text{ m}$$

and

$$\frac{L_2}{L} \approx 0.21; \qquad L_2 = (0.21)(1.5) = 0.315 \text{ m}$$

From Eq. (6.75),

$$A' = \tfrac{1}{2}(L_1 + L_2)B = \tfrac{1}{2}(1.275 + 0.315)(1.5) = 1.193 \text{ m}^2$$

From Eq. (6.77),

$$L' = L_1 = 1.275 \text{ m}$$

From Eq. (6.76),

$$B' = \frac{A'}{L'} = \frac{1.193}{1.275} = 0.936 \text{ m}$$

Note from Eq. (6.55) with $c' = 0$,

$$q'_u = qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

where $q = (0.7)(18) = 12.6 \text{ kN/m}^2$.

For $\phi' = 30°$, from Table 6.2, $N_q = 18.4$ and $N_\gamma = 22.4$. Thus from Table 6.3,

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{0.936}{1.275}\right)\tan 30° = 1.424$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{0.936}{1.275}\right) = 0.706$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + \frac{(0.289)(0.7)}{1.5} = 1.135$$

and

$$F_{\gamma d} = 1$$

So

$$Q_u = A'q'_u = A'(qN_qF_{qs}F_{qd} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d})$$
$$= (1.193)[(12.6)(18.4)(1.424)(1.135)$$
$$+ (0.5)(18)(0.936)(22.4)(0.706)(1)] \approx \mathbf{606\ kN}$$

EXAMPLE 6.11

EXAMPLE 6.11

Consider the foundation shown in Figure 6.31 with the following changes:

$$e_L = 0.18 \text{ m}$$

$$e_B = 0.12 \text{ m}$$

For the soil, $\gamma = 16.5 \text{ kN/m}^3$.

$$\phi' = 25°$$

$$c' = 25 \text{ kN/m}^2$$

Determine the ultimate load, Q_u.

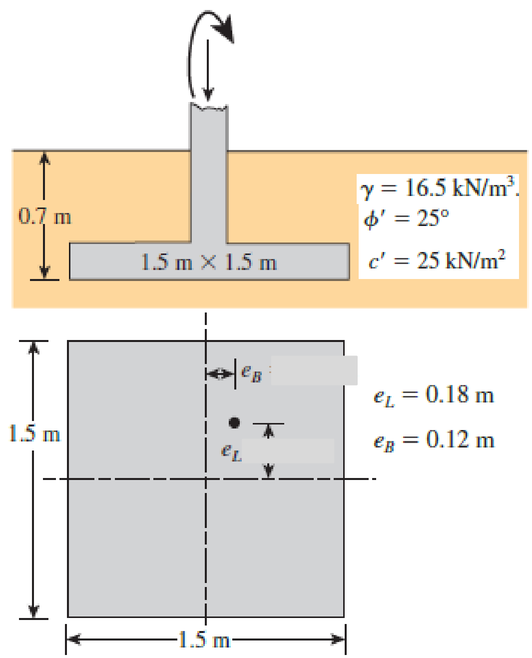


FIGURE 6.31 An eccentrically loaded foundation

EXAMPLE 6.11

SOLUTION

Case IV. $e_L/L < \frac{1}{6}$ and $e_B/B < \frac{1}{6}$

$$\frac{e_L}{L} = \frac{0.18}{1.5} = 0.12; \qquad \frac{e_B}{B} = \frac{0.12}{1.5} = 0.08$$

This is the case shown in Figure 6.29a. From Figure 6.29b,

$$\frac{B_2}{B} \approx 0.1; \qquad \frac{L_2}{L} \approx 0.32$$

So

$$B_2 = (0.1)(1.5) = 0.15 \text{ m}$$

$$L_2 = (0.32)(1.5) = 0.48 \text{ m}$$

From Eq. (6.81),

$$A' = L_2B + \frac{1}{2}(B + B_2)(L - L_2) = (0.48)(1.5) + \frac{1}{2}(1.5 + 0.15)(1.5 - 0.48)$$

$$= 0.72 + 0.8415 = 1.5615 \text{ m}^2$$

$$B' = \frac{A'}{L} = \frac{1.5615}{1.5} = 1.041 \text{ m}$$

$$L' = 1.5 \text{ m}$$

From Eq. (6.55),

$$q'_u = c'N_cF_{cs}F_{cd} + qN_qF_{qs}F_{qd} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}$$

For $\phi' = 25°$, Table 6.2 gives $N_c = 20.72$, $N_q = 10.66$, and $N_\gamma = 10.88$. From Table 6.3,

$$F_{cs} = 1 + \left(\frac{B'}{L'}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1.041}{1.5}\right)\left(\frac{10.66}{20.72}\right) = 1.357$$

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{1.041}{1.5}\right)\tan 25 = 1.324$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{1.041}{1.5}\right) = 0.722$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 25(1 - \sin 25)^2\left(\frac{0.7}{1.5}\right) = 1.145$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'} = 1.145 - \frac{1 - 1.145}{20.72 \tan 25} = 1.16$$

$$F_{\gamma d} = 1$$

Hence,

$$q'_u = (25)(20.72)(1.357)(1.16) + (16.5 \times 0.7)(10.66)(1.324)(1.145) + \frac{1}{2}(16.5)(1.041)(10.88)(0.722)(1)$$

$$= 815.39 + 186.65 + 67.46 = 1069.5 \text{ kN/m}^2$$

$$Q_u = A'q_u' = (1069.5)(1.5615) = \mathbf{1670\ kN}$$

Simple Approach for Bearing Capacity with Two-Way Eccentricity

Meyerhof's suggestion, by neglecting the light yellow area in the figure, the load is acting at the centroid of the remaining area. The light yellow area consists of two strips of widths $2e_B$ and $2e_L$, as shown in the figure. The effective area of the foundation is thus $B' \times L'$. As discussed in Section 6.12, in the bearing capacity equation and in computing the shape factors, B' and L' should be used. In computing the depth factors, B should be used. In computing the column load, the effective area of the foundation must be used.

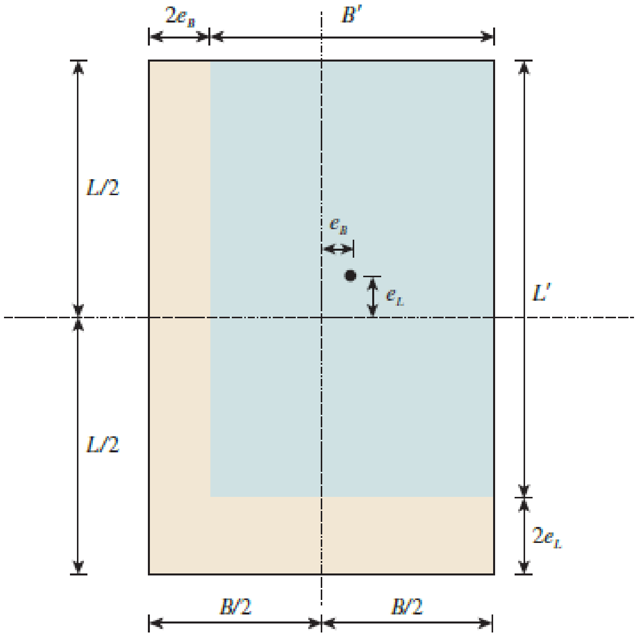


FIGURE 6.32 Effective area of a foundation with two-way eccentricity

EXAMPLE 6.12

EXAMPLE 6.12

A 2 m × 3 m foundation is expected to carry a column load with eccentricities $e_B =$ 0.15 m and $e_L = 0.2$ m. It is placed in a soil where $c' = 10.0$ kN/m^2, $\phi' = 22°$, and $\gamma = 18.0$ kN/m^3, at 1.0 m depth. Determine the maximum load the foundation can carry with factor of safety of 3.

SOLUTION

$B = 2.0$ m, and $L = 3.0$ m
$e_B = 0.15$ m, and $e_L = 0.20$ m
$B' = B - 2e_B = 2.0 - 2 \times 0.15 = 1.70$ m; $L' = L - 2e_L = 3.0 - 2 \times 0.20 = 2.60$ m

For $\phi' = 22°$, from Table 6.2, $N_c = 16.88$, $N_q = 7.82$, and $N_\gamma = 7.13$.

Shape factors:

$$F_{cs} = 1 + \left(\frac{B'}{L'}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1.70}{2.60}\right)\left(\frac{7.82}{16.88}\right) = 1.30$$

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{1.70}{2.60}\right)\tan 22 = 1.26$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{1.7}{2.6}\right) = 0.74$$

EXAMPLE 6.12

Depth factors:

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + 2\tan 22(1 - \sin 22)^2\frac{1}{2} = 1.16$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'} = 1.16 - \frac{1 - 1.16}{16.88 \tan 22} = 1.18$$

$$F_{\gamma d} = 1.00$$

With the load being vertical, $F_{ci} = F_{qi} = F_{\gamma i} = 1$. From Eq. (6.55),

$$q'_u = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + 0.5\gamma B' F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

$$q'_u = (10.0)(16.88)(1.30)(1.18)(1) + (18 \times 1)(7.82)(1.26)(1.16)(1) + 0.5(18)(1.70)(0.74)(1.0)(1)$$

$$= 258.9 + 205.7 + 11.3 = 475.9 \text{ kN/m}^2$$

The effective area $A' = B' \times L' = 1.70 \times 2.60 = 4.42 \text{ m}^2$. Therefore, $Q_u = 475.9 \times 4.42 = 2103.5$ kN. The allowable load, with FS = 3, is 2103.5/3 = **701 kN.**

■

Bearing Capacity of a Continuous Foundation Subjected to Eccentrically Inclined Loading

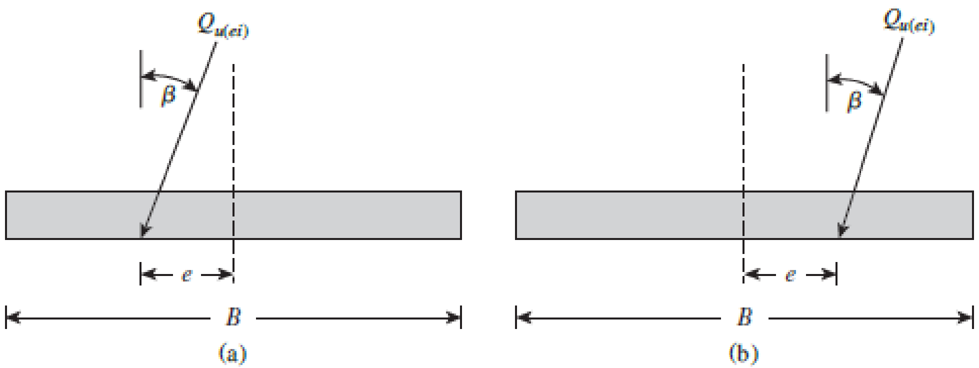


FIGURE 6.33 Continuous foundation subjected to eccentrically inclined load: (a) partially compensated case and (b) reinforced case

Partially Compensated Case

Meyerhof's effective area method can be used to determine the ultimate load $Qu(ei)$.

$$q'_u = c'N_cF_{cd}F_{ci} + qN_qF_{qd}F_{qi} + \frac{1}{2}\gamma N_\gamma B'F_{\gamma d}F_{\gamma i}$$

$q'_u =$ the vertical component of the soil reaction.

for continuous foundations, $F_{cs} = F_{qs} = F_{\gamma s} = 1$, and $B' = B - 2e$.

$$Q_{u(ei)} = \frac{(q'_u)(B')(1)}{\cos\beta} = \frac{q'_u(B - 2e)}{\cos\beta}$$

Bearing Capacity of a Continuous Foundation Subjected to Eccentrically Inclined Loading

Patra et al. (2012a) proposed a reduction factor to estimate $Q_{u(ei)}$ for a foundation on granular soil:

$$Q_{u(ei)} = q_u B(RF)$$

where RF = reduction factor

q_u = ultimate bearing capacity of the foundation with centric vertical loading (i.e., $e = 0, \beta = 0$)

The reduction factor can be expressed as

$$RF = \left(1 - 2\frac{e}{B}\right)\left(1 - \frac{\beta^\circ}{\phi'}\right)^{2-(D_f/B)}$$

$$Q_{u(ei)} = q_u B\left(1 - 2\frac{e}{B}\right)\left(1 - \frac{\beta^\circ}{\phi'}\right)^{2-(D_f/B)}$$

Reinforced Case (Granular Soil)

Patra et al. (2012b) conducted several model tests on continuous foundations on granular soil and gave the following correlation to estimate $Q_{u(ei)}$

$$Q_{u(ei)} = q_u B\left(1 - 2\frac{e}{B}\right)\left(1 - \frac{\beta^\circ}{\phi'}\right)^{1.5-0.7(D_f/B)}$$

EXAMPLE 6.13

EXAMPLE 6.13

A continuous foundation is shown in Figure 6.34. Estimate the inclined ultimate load, $Q_{u(el)}$, per unit length of the foundation. Use Eqs. (6.85) and (6.86).

$$q'_u = c'N_cF_{cd}F_{ci} + qN_qF_{qd}F_{qi} + \frac{1}{2}\gamma N_\gamma B'F_{\gamma d}F_{\gamma i}$$

$$Q_{u(el)} = \frac{(q'_u)(B')(1)}{\cos\beta} = \frac{q'_u(B-2e)}{\cos\beta}$$

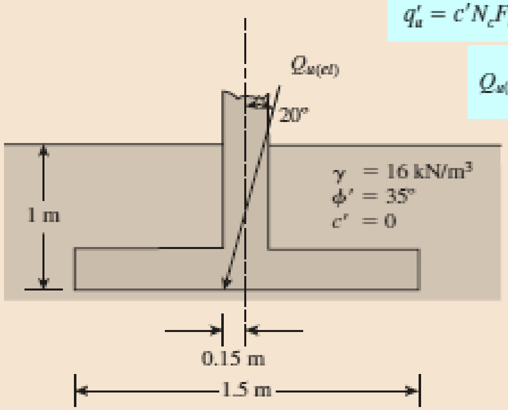


FIGURE 6.34

SOLUTION

From Eq. (6.85) with $c' = 0$, we have

$$q'_u = qN_qF_{qd}F_{qi} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma d}F_{\gamma i}$$

$$q = \gamma D_f = (16)(1) = 16 \text{ kN/m}^2$$

and

$$B' = B - 2e = 1.5 - (2)(0.15) = 1.2 \text{ m}$$

From Table 6.2 for $\phi' = 35°$, $N_q = 33.3$, and $N_\gamma = 48.03$, we have

$$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 35(1-\sin 35)^2\left(\frac{1}{1.5}\right) = 1.17$$

$$F_{\gamma d} = 1$$

$$F_{qi} = \left(1 - \frac{\beta°}{90°}\right)^2 = \left(1 - \frac{20}{90}\right)^2 = 0.605$$

$$F_{\gamma i} = \left(1 - \frac{\beta°}{\phi'}\right)^2 = \left(1 - \frac{20}{35}\right)^2 = 0.184$$

$$q'_u = (16)(33.3)(1.17)(0.605) + \left(\frac{1}{2}\right)(16)(1.2)(48.03)(1)(0.184) = 461.98 \text{ kN/m}^2$$

and

$$Q_{u(el)} = \frac{q'_u(B-2e)}{\cos\beta} = \frac{(461.98)(1.2)}{\cos 20} = 589.95 \text{ kN} \approx \mathbf{590 \text{ kN/m}}$$

■

EXAMPLE 6.14

EXAMPLE 6.14

Solve Example 6.13 using Eq. (6.89).

$$Q_{u(el)} = q_u B\left(1 - 2\frac{e}{B}\right)\left(1 - \frac{\beta^{\circ}}{\phi'}\right)^{2-(D_f/B)}$$

SOLUTION

From Eq. (6.28) with $c = 0$, we have

$$F_{qs} = F_{\gamma s} = 1 \text{ (continuous foundation)}$$

$$F_{qi} = F_{\gamma i} = 1 \text{ (vertical centric loading)}$$

and

$$q_u = qN_q F_{qd} + \frac{1}{2}\gamma B N_\gamma F_{\gamma d}$$

From Example 6.13, $q = 16\text{ kN/m}^2$, $N_q = 33.3$, $N_\gamma = 48.03$, $F_{qd} = 1.17$, and $F_{\gamma d} = 1$. Hence,

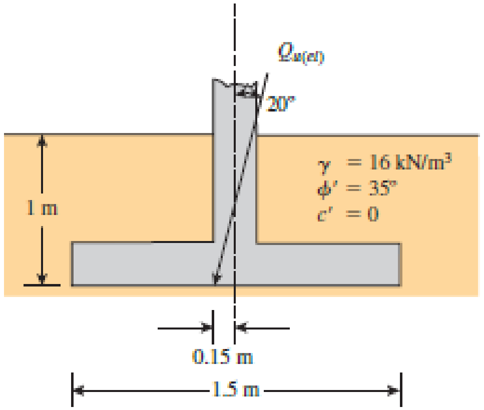
$$q_u = (16)(33.3)(1.17) + \left(\frac{1}{2}\right)(16)(1.5)(48.03)(1) = 1199.74\text{ kN/m}^2$$

and

$$Q_{u(el)} = q_u B\left[1 - 2\left(\frac{e}{B}\right)\right]\left(1 - \frac{\beta^{\circ}}{\phi'}\right)^{2-(D_f/B)}$$

$$= (1199.74)(1.5)\left[1 - (2)\left(\frac{0.15}{1.5}\right)\right]\left[1 - \left(\frac{20}{35}\right)\right]^{2-\left(\frac{1}{1.5}\right)}$$

$$\approx \mathbf{465\text{ kN/m}}$$



IMPORTANT NOTES

1. The soil above the bottom of the foundation are used only to calculate the term (q) in the second term of bearing capacity equations (Terzaghi and Meyerhof) and all other factors are calculated for the underlying soil.

2. Always the value of (q) is the effective stress at the level of the bottom of the foundation.

3. For the underlying soil, if the value of (c=cohesion=0.0) you don't have to calculate factors in the first term in equations (N_c in Terzaghi's equations) and (N_c, F_{cs}, F_{cd}, F_{ci} in Meyerhof equation).

4. For the underlying soil, if the value of ($\phi=0.0$) you don't have to calculate factors in the last term in equations (N_γ in Terzaghi's equations) and (N_γ, $F_{\gamma s}$,$F_{\gamma d}$,$F_{\gamma i}$ in Meyerhof equation).

5. If the load applied on the foundation is inclined with an angle ($\beta=\phi$). The value of ($F_{\gamma i}$) will be zero, so you don't have to calculate factors in the last term of Meyerhof equation (N_γ, $F_{\gamma s}$,$F_{\gamma d}$).

IMPORTANT NOTES

6. Always if we want to calculate the eccentricity, it's calculated as following:

$$e = \frac{\text{Overall Moment}}{\text{Vertical Loads}}$$

7. If the foundation is square, strip or circular, you may calculate (q_u) from Terzaghi or Meyerhof equations (should be specified in the problem).

8. But, if the foundation is rectangular, you must calculate (q_u) from Meyerhof general equation.

9. If the foundation width (B) is required, and there exist water table below the foundation at distance (d), you should assume $d \leq B$, and calculate B, then make a check for your assumption.

THE END