3.3 Orthogonality

Orthogonal Vectors

DEFINITION:

Two nonzero vectors **u** and **v** in $\boldsymbol{R}^n$ are said to be ***orthogonal*** (or ***perpendicular***) if **u** · **v** = 0. We will also agree that the zero vector in $\boldsymbol{R}^n$ is orthogonal to *every* vector in $\boldsymbol{R}^n$

EXAMPLE 1

Show that **u** = $(-2, 3, 1, 4)$ and **v** = $(1, 2, 0, -1)$ are orthogonal vectors in $\boldsymbol{R}^4$.

THEOREM: Theorem of Pythagoras in R^n

If **u** *and* **v** *are orthogonal vectors in* $\boldsymbol{R^n}$ *with the Euclidean inner product,*

then

$$\|u+v\|^2 = (\|u\|^2 + \|v\|^2)$$

EXAMPLE 2

We showed in Example 1 that the vectors

$$\mathbf{u} = (-2, 3, 1, 4) \text{ and } \mathbf{v} = (1, 2, 0, -1)$$

are orthogonal. Verify the Theorem of Pythagoras for these vectors.

Exercise Set 3.3

▶ In Exercises 1–2, determine whether $\mathbf{u}$ and $\mathbf{v}$ are orthogonal vectors. ◀

1. (a) $\mathbf{u} = (6, 1, 4),\ \mathbf{v} = (2, 0, -3)$

(b) $\mathbf{u} = (0, 0, -1),\ \mathbf{v} = (1, 1, 1)$

(c) $\mathbf{u} = (3, -2, 1, 3),\ \mathbf{v} = (-4, 1, -3, 7)$

(d) $\mathbf{u} = (5, -4, 0, 3),\ \mathbf{v} = (-4, 1, -3, 7)$

2. (a) $\mathbf{u} = (2, 3),\ \mathbf{v} = (5, -7)$

(b) $\mathbf{u} = (1, 1, 1),\ \mathbf{v} = (0, 0, 0)$

(c) $\mathbf{u} = (1, -5, 4),\ \mathbf{v} = (3, 3, 3)$

(d) $\mathbf{u} = (4, 1, -2, 5),\ \mathbf{v} = (-1, 5, 3, 1)$