3.1 Vectors in 2-Space, 3-Space, and n-Space

DEFINITION 1 If n is a positive integer, then an ***ordered n-tuple*** is a sequence of n real numbers $(v_1, v_2, \ldots, v_n)$. The set of all ordered n-tuples is called ***n-space*** and is denoted by R^n.

Operations on Vectors in R^n

DEFINITION 2 Vectors $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ and $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ in R^n are said to be ***equivalent*** (also called ***equal***) if

$$v_1 = w_1, \quad v_2 = w_2, \ldots, \quad v_n = w_n$$

We indicate this by writing $\mathbf{v} = \mathbf{w}$.

EXAMPLE 1

Equality of Vectors : $(a,b,c,d) = (1,-4,2,7)$

DEFINITION 3 If $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ and $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ are vectors in R^n, and if k is any scalar, then we define

$$\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2, \ldots, v_n + w_n) \tag{10}$$

$$k\mathbf{v} = (kv_1, kv_2, \ldots, kv_n) \tag{11}$$

$$-\mathbf{v} = (-v_1, -v_2, \ldots, -v_n) \tag{12}$$

$$\mathbf{w} - \mathbf{v} = \mathbf{w} + (-\mathbf{v}) = (w_1 - v_1, w_2 - v_2, \ldots, w_n - v_n) \tag{13}$$

EXAMPLE 2

If v = (1 , −3 , 2) and w = (4 , 2 , 1), Find v + w , 2 v ,- w , v - w

THEOREM:

If **u**,**v**, and **w** are vectors in R^n, and if k and m are scalars, then:

(a) u+v=v+u

(b) (u+v)+w=u+(v+w)

(c) u+0=0+u=u

(d) u+(−u)=0

(e) k(u+v)=ku+kv

(f) (k+m)u=ku+mu

(g) k(mu)=(km)u

(h)1 u=u

THEOREM:

If **v** is a vector in $\boldsymbol{R^n}$ and k is a scalar, then:

(a) 0 v = 0

(b) k 0 = 0

(c) (−1) v = − v

Linear Combinations

DEFINITION 4 If $\mathbf{w}$ is a vector in R^n, then $\mathbf{w}$ is said to be a ***linear combination*** of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r$ in R^n if it can be expressed in the form

$$\mathbf{w} = k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + \cdots + k_r\mathbf{v}_r \tag{14}$$

where $k_1, k_2, \ldots, k_r$ are scalars. These scalars are called the ***coefficients*** of the linear combination. In the case where $r = 1$, Formula (14) becomes $\mathbf{w} = k_1\mathbf{v}_1$, so that a linear combination of a single vector is just a scalar multiple of that vector.

Exercise Set 3,1

9. Let $\mathbf{u} = (4, -1)$, $\mathbf{v} = (0, 5)$, and $\mathbf{w} = (-3, -3)$. Find the components of

   (a) $\mathbf{u} + \mathbf{w}$ (b) $\mathbf{v} - 3\mathbf{u}$

   (c) $2(\mathbf{u} - 5\mathbf{w})$ (d) $3\mathbf{v} - 2(\mathbf{u} + 2\mathbf{w})$

10. Let $\mathbf{u} = (-3, 1, 2)$, $\mathbf{v} = (4, 0, -8)$, and $\mathbf{w} = (6, -1, -4)$. Find the components of

    (a) $\mathbf{v} - \mathbf{w}$ (b) $6\mathbf{u} + 2\mathbf{v}$

    (c) $-3(\mathbf{v} - 8\mathbf{w})$ (d) $(2\mathbf{u} - 7\mathbf{w}) - (8\mathbf{v} + \mathbf{u})$

11. Let $\mathbf{u} = (-3, 2, 1, 0)$, $\mathbf{v} = (4, 7, -3, 2)$, and $\mathbf{w} = (5, -2, 8, 1)$. Find the components of

    (a) $\mathbf{v} - \mathbf{w}$ (b) $-\mathbf{u} + (\mathbf{v} - 4\mathbf{w})$

    (c) $6(\mathbf{u} - 3\mathbf{v})$ (d) $(6\mathbf{v} - \mathbf{w}) - (4\mathbf{u} + \mathbf{v})$

12. Let $\mathbf{u} = (1, 2, -3, 5, 0)$, $\mathbf{v} = (0, 4, -1, 1, 2)$, and $\mathbf{w} = (7, 1, -4, -2, 3)$. Find the components of

    (a) $\mathbf{v} + \mathbf{w}$ (b) $3(2\mathbf{u} - \mathbf{v})$

    (c) $(3\mathbf{u} - \mathbf{v}) - (2\mathbf{u} + 4\mathbf{w})$ (d) $\frac{1}{2}(\mathbf{w} - 5\mathbf{v} + 2\mathbf{u}) + \mathbf{v}$